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# THE ASTRONOMICAL TABLES OF AL-KHWĀRIZMĪ

Translation with Commentaries of the Latin Version edited by H. SUTER supplemented by Corpus Christi College MS 283

BY

O. NEUGEBAUER

København 1962
i kommission hos Ejnar Munksgaard

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# SIGLA

From Suter, p. X f.:

C Chartres, Bibliothèque publique No. 214 (173)
M Paris, Bibliothèque Mazarine No. 3642 (1258)
N Madrid, Biblioteca Nacional No. 10016
O Oxford, Bodleian Library, Cod. Auct. F. I. 9 (= Bernard No. 4137 = Summary Catal. II,2 p. 862f. No. 4137)

Additional:

CC Oxford, Corpus Christi College MS 283 (in the Bodleian Library); original version
D Interlinear version in CC
S Text as established by Suter from C, M, N, O
T Vindob. 2385, Toledan Tables

Hugo Sanctallensis:

G Cambridge, Gonville and Caius College MS 456
Sa Oxford, Bodleian Library, Savile 15, f. 205
Se Oxford, Bodleian Library, Arch. Seld. B 34

# Preface

Based on the preliminary studies of the late A. BJØRNBO and of R. BESTHORN, H. SUTER[1] in 1914 published the astronomical tables of al-Khwārizmī (about A.D. 820) in the version of the Spanish astronomer Maslama al-Majrīṭī (about A.D. 1000), preserved in a Latin translation, presumably by Adelard of Bath (early 12th century). SUTER gave a critical edition of the Latin text, and extremely valuable commentaries of his own. He refrained, however, from a translation with the motivation[2] that it would be of little use to anyone who is not familiar with ancient and mediaeval astronomy, except when followed by ample commentaries. This being certainly true I shall not only present here a translation of what SUTER calls with good reason an "oft schwer verständlicher Text" but try to correlate this translation with a detailed discussion of the tables themselves. Indeed the close combination of text and tables is absolutely necessary for any serious use of al-Khwārizmī's work and its comparison with related treatises. And the Latin text is often so obscure or corrupt that an understanding can only be reached through constant reference to the tables and the underlying theory.

What I attempt here is simply to continue where BJØRNBO, BESTHORN and, above all, SUTER have left off. My first and main goal is to give a passably readable, more or less literal, translation as I understand the text and thus make Suter's work more accessible. Yet, as usual, the careful study of details leads to results in unexpected directions. I might mention the insight that one has to distinguish between an Islamic calendaric scheme for the dates of the mean syzygies, not necessarily always leading to identical results with the computed dates (p. 114); or the discovery that Kepler's method for obtaining approximate values for the parallaxes is identical with (and, of course, independent of) the Hindu procedure which is older by about one millenium (p. 124).

However limited my ambitions were, it was nevertheless natural to extend the investigation occasionally to authors like Azarqiel and Ibn Ezra whose works now have

[1] Kgl. Danske Vidensk. Selsk. Skrifter, 7. Raekke, hist. og filos. Afd. 3,1.
[2] Suter, p. XXIV.

become available thanks to the efforts of J. M. MILLÁS-VALLICROSA.[3] It was not my intention, however, to follow all the ramifications of the transmission of al-Khwārizmī's tables in late mediaeval astronomy. I only hope that the existence of my translation will help to facilitate the study of related works which have survived in abundance. Such investigations will be the more fruitful and significant the more they take into account the astronomical methodology of the texts and the values of the characteristic parameters which underly the computation of the numerical tables. Astronomical texts allow the use of much more powerful tools than the traditional comparison of their "radices" and of a few explicit data. It is for this reason that I have given a special index of parameters (p. 242).

Some new material is collected in the Appendix. C. H. HASKINS identified another, though incomplete, manuscript of al-Khwārizmī's work in Corpus Christi College, Oxford, but too late to be utilized for Suter's edition. Obviously I had to take this text into account for whatever variants it had to offer. The results of my investigations are published here (p. 135 ff). The main interest of the new text consists, however, in interlinear numbers and marginal notes which, on the one hand, lead us back to, among others, the Toledan Tables, and, on the other hand, provide some new facets of the process of transmission of astronomical knowledge from Spain to England.

No edition of the Toledan Tables is available.[4] I therefore used a manuscript, cod. Vindob. 2385, which according to Zinner's description in Osiris 1 (1936) p. 748 seems to be representative for the Latin translation by Gerard of Cremona (d. 1187). The Oesterreichische Nationalbibliothek in Wien kindly supplied me with a microfilm of this manuscript.

In my translation of al-Khwārizmī's introduction to his tables, I follow the running text as printed by SUTER except for such variants which represent the correct version but had been put in the apparatus – and the same holds for numbers in tables. For all other variants Suter's edition has to be consulted.

Finally I must underline that I use the expression "al-Khwārizmī's text and tables" only as a convenient name for the whole of the textual material, be it genuine or not, contained in Suter's publication. In fact a sharper scrutiny of text and tables shows that the lack of uniformity goes much deeper than is obvious from the addition of new tables or the change in geographical elements. In all parts of the work one can detect the juxtaposition of rules known from Hindu treatises and of tables which represent the much more advanced level of the astronomy of Ptolemy or Theon. When and where such disparate units were combined it is impossible to say on the basis of the extant text. Only one thing that seems fairly certain to me is that we are much farther removed from the original *zīj* of al-Khwārizmī than one would like to assume at first sight. This impression seems to be supported by a

[3] In particular his "Estudios sobre Azarquiel" (Madrid–Granada, 1943–1950) contains numerous references to al-Khwārizmī's tables.

[4] No manuscript of the Arabic original has been identified so far. The Latin versions are very numerous – probably the number of extant manuscripts might approach one hundred.

treatise to which HASKINS has first drawn our attention and which Mr. G. TOOMER and I have studied in detail. It contains a commentary by al-Muṭannā (perhaps 10th cent.) to the *zīj* of al-Khwārizmī, put into Latin by HUGO SANCTALLENSIS in the twelfth century,[5] and is apparently based on a more coherent treatise than the version of al-Majrīṭī.

In preparing this study I had the benefit of discussing the whole manuscript with my colleague Dr. H. B. VAN HOESEN which resulted in many improvements of my initial transcriptions and translations. A final reading of my text by Prof. A. J. SACHS also contributed to greater clarity and precision of formulation. Several difficult passages in the Corpus Christi MS were collated by Dr. G. L. HUXLEY in Oxford. I am very thankful to the President and Fellows of Corpus Christi College for granting me permission to publish photographs of two pages of their MS 283. The Bodleian Library has provided me with films of related MSS whenever needed, thus greatly facilitating my studies.

I know well that much more could have been learned about the complex history of late mediaeval astronomy by venturing farther out in the uncharted and unlimited ocean of extant source material. I deemed it wise, however, not to embark on a voyage which so easily could become a voyage of no return.

[5] Cf. below p. 54 n. 16. A critical edition with translation and commentary by Mr. E. MILLÁS is in preparation.

## Translation of the Text and Commentaries

In the numbering of chapters and tables I follow Suter. Within the single chapters, however, I have changed indentations whenever I felt that it would facilitate the reading.

In translating technical terms I ordinarily use the term which one would use in translating a Greek text; thus, e.g., I say anomaly for *argumentum* and not argument, but argument of latitude for *argumentum latitudinis*. Similarly I do not follow pedantically the distinction between Arabic, etc., and Latin terms; thus, e.g., I translate both *bught* and *stupor* by velocity. For all questions of terminology and variants Suter's text and apparatus must be consulted.

I also use the notation *a*;*b*,*c*° for expressions like "*a* degrees, *b* minutes, and *c* seconds" whenever this seems convenient.

### (Introduction). Here begin the Tables[1] of al-Khwārizmī, Translated from the Arabic by Athelard of Bath[2]

This book contains the positions of the seven planets and of the ascending nodes (of the moon), reckoned from noon of day 4 (Wednesday) to noon of day 5 (Thursday). And the same rule holds for the remaining days, namely (that they are

[1] *zīj*, which is the equivalent of Greek κανών.

[2] Cf. the corresponding entry at the end of Tab. 116 (below p. 132). A variant of this title is found in MS N: "Here begins the book *zīj*, i.e. the tables, of al-Khwārizmī, translated from the Arabic by Adelard of Bath, and put in order by Robert of Chester". A photograph of this page is reproduced in Millás-Vallicrosa, Bibl. Cat. Tol. pl. XVII. See also Haskins SHMS p. 123 and p. 123/124 note 32.

reckoned) from noon of the preceding to noon of the following (day). Since noon of day 4 was the beginning of the Hijra and of the month Muḥarram, also the (present) *zīj* takes this for its beginning.[3]

And thus, in this book of al-Khwārizmī, the corrections[4] for the planets and (the reckoning) of time are made with reference to the central place of the earth, called Arin, from which the four ends of the world have equal distance, namely 90 degrees or a quadrant.[5] To describe all regions of the earth and to establish all (local) times would be tedious and unfeasible since, for innumerable times and for boundless regions, the meridians have been recorded (with respect to) Arin, in such a way that it would not be difficult to determine from this radix by geometrical and arithmetical rules the other places and times.

The following, however, must be required from the reader: if he approaches (this work) devoid (of knowledge) of these arts, he might consider the results of these lines of computations (made) according to the subsequent rules rather as (merely) determined (by rules) than as necessary.[6] Whereas, if he has been (well) trained (in the use of) these (rules) – and has also made a trial at Ptolemy's Almagest – then he will not doubt that all that follows here originates from necessity. And so much for this.

Now, since time is here related to the motion of the planets, we must first deal with it (i.e., the different forms of time reckoning).

[3] Cf. the commentary to this introduction.

[4] I take here *examinatio* in its technical meaning "correction" or "equation". Cf. the similar passage at the end of ch. 23 (below p. 45).

[5] Cf. the commentary (p. 11).

[6] *magis certum quam necessarium habeat*; cf. the remark near the end of ch. 4 (p. 15) and p. 16 note 11.

## Commentary to the Introduction

As epoch of his tables al-Khwārizmī adopts noon of July 14 A.D. 622 (cf. fig. 1 which represents the time axis with alternating heavily drawn intervals for night, light intervals for daytime). Thus he reckons Muḥarram 1 of the first year of the era Hijra as extending from noon of Wednesday July 14 (julian day 1948438) to noon of Thursday July 15 (julian day 1948439). In the civil calendar with its evening epoch Thursday begins at sunset of July 14 and extends to sunset of July 15. Thus

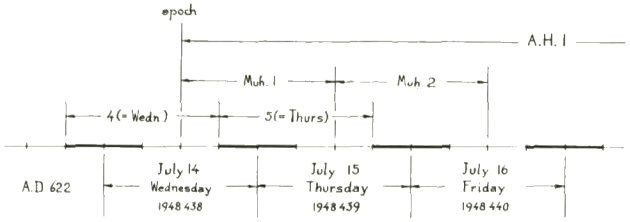


Fig. 1.

it is customary[1] to say that the Muslim astronomers reckon the era Hijra from A.D. 622 July 15 which corresponds to Muḥarram 1.

For Arin,[2] the zero meridian of Hindu astronomy, cf. Suter p. 33. In al-Khwārizmī's tables Arin is assumed to be located at the midpoint of the hemisphere which extends from the Ocean in the West to the Ocean in the East and from pole to pole.[3]

[1] Cf., e.g., Ginzel, Hdb. I p. 258f. Historians usually count the era Hijra as beginning with A.D. 622 July 16 (e.g., Schram, Tafeln p. XXX, p. 283ff. or Wüstenfeld-Mahler, Vergleichungstabellen).
[2] Present Ujjain, 76° east of Greenwich, 23° north.
[3] Cf. also p. 86 n. 3.

## Chapter 1. Description of the Arabic Year

The Arabic year is regulated by the motion of the moon. They call "year" the time in which the moon, having left the sun, returns to it twelve times, which takes 354 days plus 1/5 and 1/6 days, i.e., 11/30 days.[1] These additional (fractions) are not counted if they amount to 1/2 day or less; then the year has 354 days. If, however, more than 1/2 day has accumulated, then one whole day will be counted, and hence the year has 355 days. Such a year is called *elkebice* in Arabic.

In this way the Arabic year consists of 12 months, the first of which, *elmuharram*, has 30 days, the second, *zafar*, 29, and so forth, alternately, as the table below indicates,[2] except for the last, *dulheia*, which, by its position, should have 29 days, but is often counted with 30 days because of the above mentioned accumulation of fractions. Thus it happens that the beginning of the Arabic year with its variable ending sometimes falls in winter, sometimes in summer.

[1] Or 354;22 days.
[2] Such a table is partially preserved only in MS N; cf. Suter p. XV. Cf., however, Tab. 2 (p. 84).

## Chapter 2. Description of the Roman Year

The year of the Romans contains, according to the motion of the sun, 365 and 1/4 days. This additional (fraction) is not counted if it amounts to 1/4 or 1/2 day. If it is more than 1/2 day, then one whole day is taken and the year has 366 days and is called *bissextilis* by the Romans, *elkebice* by the Arabs. The Roman year contains 12 months, the first of which, October, has 31 days, November 30, December 31 1/4, and the *bissextus* is put at the end of December, such that this month is reckoned as 31 days in three years, but 32 in the fourth; January 31, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30.

From this definition of the year it is clear, that the solar year is found to be 11 days longer than the lunar year; because 30 fifths and sixths (of one day) make 11 days.[1]

If now, with an increasing number of the above mentioned years, someone

[1] This argument is, of course, incorrect, since it concerns only the Arabic 30-year cycle between lunar months and days, but not a lunisolar cycle.

wishes to know how many Arabic years are evenly contained in how many Roman years, he may resolve both into days; then their correspondence will become evident.

Since this book is intended to be useful for different nations, of which not all follow the same time (reckoning), a table[2] is given in the following,[3] which contains the times of kings as well as of events, such that, when these rules are given and noted according to the time of the Hijra, time and years can be reduced to any definition of different nations.

Since[4] we must introduce for the requirements of this treatise the extent of the year, we must also discuss its division. Thus the year is divided into 12 months, the month into a different number of days, the day into 24 hours, the hour into 60 *dakaicas*, the *dakaica* into 60 *zenias*, i.e., seconds.[5]

Now, since, as mentioned above, time is reckoned differently by different (people), we must add the times of the different reigns.[6] From *athofen*, i.e. the flood of Noah, to *elkarnain*, i.e. Alexander, 2793 years, 2 months, and 5 days; from Alexander to *acofra*, i.e. (our Spanish) era, 273 years, 9 months, 17 days; from (our) era to the incarnation of the Lord 38 years; from the incarnation of the Lord to the Hijra 621 years, 6 months, 15 days.

Since this book, as mentioned before, determines the positions of the planets and the ascending node according to Arabic years and months, and since we are concerned about its usefulness for their Latin (forms), we note first of all the table given below, by means of which one can obtain from any Roman years and months and days, beginning with the very year in which this book was translated into our language, for unlimited time, the correspondence with Arabic years, months, and days.

The entry for these tables[7] is as follows: If you ask what number a year has, beginning with the Hijra, and month and day, one has to enter (the table) through the 30-year groups of the year of the Lord to the corresponding Arabic years and note down the corresponding entry; then through single years to its corresponding (Arabic year) and take it down; and similarly with the months. With all these (numbers) collected, the smaller (units) must be reduced to bigger ones, and as often as 12 months are reduced to one year, so many times 6 days must be added to the number of days. But if months remain which do not complete one year, then nothing is to be added for one (month), one day for two (months), for three again one, for four two. In this way you will obtain the Arabic years, months, and days which are the equivalent of the Roman years, months, and days.[8]

[2] *pagina* is here often used for "table"; cf., e.g., the title of Tab. 2, Suter p. 110.

[3] Tab. 1: Suter p. 109.

[4] From here to the end of ch. 2: omitted by MS O.

[5] Marginal note in C and N: "Note that, according to this book, the sun does not complete its course in one single year; but so much is missing as it moves while 3 degrees and 2 minutes of the outermost circle (of the sphere of the fixed stars) rise. During 120 years this accumulates to one day and a small fraction of one day". Cf. the commentary (C) to this chapter.

[6] Cf. commentary (A) to this chapter.

[7] Cf. commentary (B) to this chapter.

[8] In MS N follows here a table enumerating the Arabic months; cf. Suter p. XV, 2).

### Commentary to Chapter 2

(A). For simplicity of reference we denote the epoch dates of different eras by Greek letters as follows:[1]

(β) flood
($\varepsilon_1$) Alexander
(ʒ) Spanish era
(η) Christian era
(ι) Hijra

The statement that the interval from (η) to (ι) amounts to 621 years, 6 months, 15 days obviously means that (η) is taken as A.D. 1 Jan. 1 and (ι) as 622 July 15 which is the standard date for the astronomical reckoning of the Moḥammedan era.[2] Also correct is the date of (ʒ), 38 years before (η), thus −37 Jan. 1.[3] Going back from this epoch of the Spanish era by 273 years, 9 months, and 17 days, which we reckon as 273 years 287 days, we arrive for ($\varepsilon_1$) at −311 March 20. Thus the era "Alexander" is here the Seleucid era with Nisan as first month; we shall see that the chronological table, which is put at the beginning of al-Khwārizmī's work[4] counts the years of "Alexander" from −311 Oct. 1, thus following the Syrian norm for the Seleucid era, in which Tishri is the first month of the year. Inconsistencies of this kind are very common in mediaeval chronological tables, which never can be taken at their face value.

Again from Tab. 1 we learn[5] that the era of the "flood" (β) is actually the Indian era of the Kaliyuga, with −3101 Febr. 17 as epoch. It also becomes clear from Tab. 1 that "years" of eras which begin before "Alexander" are understood to be Egyptian years, while "years" of all eras beginning with "Alexander" or later are meant to be julian years. In the present chapter the "flood" supposedly precedes ($\varepsilon_1$) by 2793 years, 2 months, 5 days. Going back 2793 Egyptian years and 65 days from −311 March 20 = julian day 1607544 we obtain for (β) julian day 588034 or −3103 Dec. 13 instead of −3101 Febr. 17.

This error in our text can be explained as the result of a schematic use of Tab. 1. There we find the following entries

line 15: (β) to (ι): $3725^{y}\ 11^{m}\ 19^{d}$
line 7: ($\varepsilon_2$) to (ι): $932^{y}\ 9^{m}\ 17^{d}$.

Thus by subtraction

(β) to ($\varepsilon_2$): $2793^{y}\ 2^{m}\ 2^{d}$

[1] Cf. also fig. 25 p. 83.
[2] Cf. above p. 10.
[3] Cf. Ginzel, Hdb. III, p. 176f.
[4] Tab. 1 and 3a. The same norm is also used by Petrus Alfonsus in our MS. CC (below p. 143). It may be remarked that also the tables of Azarqiel operate with the Seleucid era, Syrian style, and julian years; cf. Millás-Vallicrosa, Azarquiel p. 116 and Tab. 7 to 14 (p. 158ff.).
[5] Below p. 82.

and this is the figure given in chapter 2, except that the number of days is 5 instead of 2. Actually this procedure is incorrect, first, because (β) to (ι) is reckoned in Egyptian years, but ($\varepsilon_2$) to (ι) in julian years; secondly, because the epoch for the years of "Alexander" in chapter 2 is ($\varepsilon_1$) = −311 March 20, but in Tab. 1 ($\varepsilon_2$) = −311 Oct. 1. This is a good example of how new epoch dates are created by the indiscriminate use of tables whose composite character was not realized. Also modern scholars have come to incorrect results by unwittingly combining in their chronological computations different types of years.[6]

(**B**). The preserved text contains no table which corresponds to the description given, but Tab. 3 at least comes near to it. There 30-year groups of Arabic years are equated with (julian) "*anni Alexandri*", i.e. years of the Seleucid era, not "years of the Lord" (i.e. Christian era).

In all these computations "months" are counted as 30 days, e.g. in the third section of Tab. 3.[7] Consequently, if "12 months" are reduced to one Arabic year of 354 days, one has to add 6 days to the days of the Arabic calendar. Similarly in the counting of excess months: since every second month of the Arabic calendar is only 29 days long, one has to add to the Arabic date one day for every two months.

(**C**). ad p. 12 note 5. Of course, it makes no sense to speak without specification about the time it takes 3;2° of the sphere of the fixed stars to rise. What the text means is: if 360° of the equator rise in 1 day, then 3;2° correspond to $0;0,30,20^d$. Thus one year is assumed to contain $365;15,30,20^d$. The same length of the sidereal year has been found by Kennedy ascribed to al-Khwārizmī on f. 8 of Cod. Arab. 927 of the Escorial, supposedly the Mumtaḥan zīj.[8] The accumulated correction for 120 years amounts then to $2,0 \cdot 0;0,30,20 = 1;0,40^d = 1\ 1/90^d$ or "one day and a small fraction", as the scholion expresses it.

[6] E.g. Taqizadeh [2[ p. 122 who found for (β) the year 3104 B.C.
[7] Cf. also chapter 9.
[8] Cf. Kennedy, Survey, No. 51 and p. 147, No. 7 of the list.

## Chapter 3. On the Headings of the Columns[1]

So much must be added to what we have explained so far, that it may become clear what each heading means, since almost all tables of this book are headed by the names of the months, and of the accumulated and single Arabic years.

When the philosophers investigated the orbits of the planets, and the circumferences of the orbits,[2] and when they divided the circles in signs, the signs in degrees, the degrees in minutes, the minutes in seconds, then they wanted to determine, how many of these divisions each planet traversed in single times. Thus time also had to be divided, and so they divided the year into months, the months into days, the days into hours, the hours into minutes. Having found the number of years

[1] Cf. for this translation Suter p. 36.
[2] This might be a formulation influenced by the Hindu tradition according to which the circumferences, instead of the radii, of eccenters and epicycles were recorded.

or months in which each planet traversed its orbit, they came from years to months, from months to days, from days to hours, from hours to minutes. Having thus obtained the result for one year, they went from one year up to 30 years. Having thus found how much the said planets travelled in 30 years, by adding 30 to 30 up to 60, they measured in this way the motions of the stars to an infinity of years, of months, of days, and hours.

In this fashion the first lines were headed by (the words) minutes of hours, hours, and days, and months, and single years and accumulated years, and then they added according to the motions of the single planets the signs, degrees, minutes, and seconds; and they called the years from 1 to 30 "single" but "accumulated" however many of 30-year periods they collected. However, we go on to the following.

## Chapter 4. On the Beginning of the Arabic Months according to the Table Below

The procedure with the table below,[1] by means of which one finds the beginning of the Arabic months, is as follows. One notes down the *notula*[2] (i.e. number of weekday) of the accumulated years, then the *notula* of the single years according to the present time,[3] finally the *notula* of the month, the beginning of which we want to know. After adding these three *notulae* one has to subtract multiples of seven from their total and the remainder will give the required first day of the month. If the remainder is 1, then the first *feria* (Sunday) is indicated, if 2 the second (Monday), and so forth.

The arrangement of the *notulae* of the months is as follows. Since *feria* 1 is given to the first month, which is reckoned as 30 days, *feria* 3 will belong to the 2nd month; since this (month) is reckoned as 29 days, *feria* 4 will belong to the 3rd month, and so forth. Because on the basis of the given number of days the *notula* of the last month is 4, the first single year is denoted by 4. The *notula* of the second single year is obtained by adding 4 to this 4, and subtracting 7 from this total, and so forth, unless when the sum increases by 1 through the accumulation of the above mentioned 1/5 and 1/6 (day)[4] – then this order is changed. Since by this rule the *notula* of the last single year is 5 – either by the addition of 4 and subtraction of 7, or by the accumulation of fractions – the first of the accumulated years is denoted by 5. From this originated the *notula* of the second (entry of the) accumulated years by adding 5 and subtracting 7 from the total, and so forth. Thus certainty will be provided for the table, according to the above mentioned method of reckoning.

The first day of *Almuharram* and the third *feria* fell in the year 1126 of the incarnation of the Lord on the 26th day of January, but the Arabic year was 520.[5]

[1] Table 2.
[2] Actually called *notae* in Tab. 2.
[3] This means: use as entry the number of the current year.
[4] Cf. chapter 1 (p. 11): one lunar year = $354^{d} + 1/5 + 1/6$.
[5] Indeed one finds in Tab. 2 the *notula* 1 for each of the three entries: 510, 10, and Muḥarram, thus the total 3 (= Tuesday); cf. Suter p. 37 and p. 38. For a discussion of this date, cf. Haskins SHMS p. 23 n. 19 and below p. 231.

## Chapter 5. On the Determination of the Intercalating Year[1]

The method for finding by means of the following tables[2] the intercalating year[1] and the *feria* (i.e. weekday) with which any Roman month begins is this: go from the accumulated Arabic years,[3] then from the single years,[4] to the corresponding Alexandrian years; similarly from the Arabic months[5] to the corresponding (numbers). In the accumulated total one has to reduce the smaller (units) to larger ones, i.e., fractions to days, days to months, months to years. Since, however, the months listed below are counted as containing 30 days, we must subtract 5 1/4 days as often[6] as we reduce 12 months to one year, because 12 times 30 is only 360, whereas the Roman year contains 365 1/4 days. The sum obtained in this way has to be counted from October[7] to the present month; and so many days beyond 30 as there are assigned to some months[8] must be subtracted from this sum. If, however, a February intervenes, which contains 28 days, 2 days must be added to the sum because of this. In this way both the present Roman month and day (which corresponds to the given Arabic date) will be manifest.

From the sum (of Roman years) obtained in the said way, 28 years must be subtracted as many times as possible. In order to reduce the labor of this subtraction a table is given[9] for the consecutive multiples of 28. The number which you find in this table closest to the given number, but smaller than it, you subtract and keep the remainder in your memory. According to this remainder enter in the table[10] of bissextile (years), which the Arabs call *kebicen*, and from it it will be apparent how the intercalation is related to the present year. And it will not be hidden with which *feria* each month begins.

The justification[11] for the tables for the present problem, first months, then single years, finally accumulated years, is this: Since the Arabic months which are placed (in Tab. 3) behind the single years are to be reduced to single years and solar months, each of which contains, as we have said, 30 days, the number 1 is given beside the first lunar month, which is also 30 days long. To the second (lunar month) is given (in the table) 1 (month) from the preceding (month) and 29 days

[1] Cf. for this terminology Suter p. 38. What is meant is not "intercalary year" but a type of year which uses intercalations.

[2] Tab. 3.

[3] Tab. 3, left part.

[4] Tab. 3, middle section.

[5] Tab. 3, right part.

[6] Here is added on the margin by a later hand (cf. Suter p. XVIII, 6): "i.e. from remaining months and days from which is made up a year that is added to the other years; because a year is not made up from 12 months of 30 days but 5 and a quarter day have to be added. To the months are given here 30 days only, wherefore one has sometimes to add, sometimes to subtract in order to get (the number of days) which the Roman months really contain".

[7] Cf. chapter 2.

[8] Namely October, December, January, etc. (cf. ch. 2).

[9] Table 3, last column, and, more extensively, Table 1, first section.

[10] Tab. 3a.

[11] *certitudo*, i.e. proof or reason.

from itself. Following this procedure, 11 (months) and 24 (days) are given to the last (Arabic) month. Since in this way the last month is associated with 11 (months) and 24 (days), these same numbers are also assigned[12] to the first single year. Since, according to this process, to the last line of the single years is assigned 29 (years) 1 (month) 8 3/4 (days) also the first line of the accumulated years is associated with this total. This becomes evident as follows: If one subtracts from it (the entry for 30 years) the *radix*, the said number will be found.[13] And because the accumulated years increase by 30-year steps, one adds for each line the same amount to the preceding line, i.e., the number of the last line of the single years, namely 29 and 1 and 8 and 3/4. In this way the table can be extended indefinitely.

The justification[11] for the table of intercalations[14] is obtainable from the contents of days which are written above each month (in Tab. 3a). Since October begins with *feria* 2 and since it is 31 days long, November will begin with *feria* 5. In the same way for the others.

[12] In the middle section of Tab. 3.
[13] Indeed, one has in Table 3, left section:

| | | | | | |
|---|---|---|---|---|---|
| *radix* | 932$^{y}$ | 9$^{m}$ | 17 | 0$^{d}$ | |
| 30 | 961 | 10 | 25 | 3/4 | |

thus the difference of 29$^{y}$ 1$^{m}$ 8 3/4$^{d}$.
[14] Tab. 3a.

## Chapter 6. On the Division of the Circles[1]

It is necessary for our present purposes that we first discuss the circles[1] of the planets. The circle, which the Arabs call *felek*, is divided in 12 signs, each sign in 30 degrees, the degree in 60 *dakaica* which some call parts, the *dakaica* in 60 seconds, the second in 60 thirds, and in this way decreases, as far as one wishes, the size of the (divisions of the) circle, indefinitely.

[1] Or orbits (*rota*).

## Chapter 7 [Version O[1]]. On the Determination of the *Elwazat*, i. e. the Mean Places of the Planets

We must now show how the mean place of each planet can be found.

One enters the tables of mean motions[2] for any given time with the number of accumulated years; then with the number of single years passed by, that is to say not counting the current year; similarly for the months, days, hours, and fractions, adding the corresponding (numbers), first the signs to the signs, the degrees to the degrees, the minutes to the minutes, and the seconds to the seconds; then reducing seconds to minutes, minutes to degrees, degrees to signs.

[1] Also contained in CC, except for small variants (cf. below p. 214 ff.). Version O of chapter 7 is also found in some manuscripts of the Toledan Tables; cf. Zinner [1[ p. 765.
[2] Tables 4 to 20. The same term *medialitas* is also found used for mean motion during given intervals of time and for mean longitudes thus obtained. In my translation I usually avoid this ambiguity.

The remainder, resulting from these reductions, is to be retained. If, e.g., there are 70 seconds, 10 will remain; if 70 minutes, also 10; if 40 degrees, 10 will correctly remain. If, however, by the reduction 12 signs accumulate, then we ignore them. What remains will be the mean place of each planet, that is to say of the planet under investigation, for the noon of the given day, and for the locality of Arin, for which this instruction was made.

If we are removed in longitude from this (place, namely Arin), then the distance between our place and the locality of Arin must be taken into account. Thus having established how many degrees and perhaps minutes our place is distant from Arin, one hour is to be reckoned for each 15 degrees. An amount, however, which does not reach 15 (degrees) is to be multiplied by 4, resulting in "moments" of hours.

What is accumulated from all this in hours, is to be taken as the difference between our noon and noon of the locality of Arin. And one must understand that, if the given place is in the eastern part, one must subtract for each planet the amount of the above mentioned hours (which represent the difference from) noon of the locality of Arin, but add, if it is in the western part. In this way the place of the planets is established for noon of our place.

[Marginal note:[3]] Note that the longitude of a climate is counted from the eastern or western *elmuchre*[4] to the (given) locality, the latitude, however, from Arin to the (given) locality . . . . . .[5]

[3] Only in O, not in CC.
[4] Limits of the inhabited world; cf. Suter p. 245.
[5] Unintelligible words: *aequidistanter*(?) *Elcuscus* (= *qaus*, arc; cf. Suter p. 244) *vero longitudinem arguit*(?).

## Chapter 7. [Version C and M]. On the *Elwacat*, i. e. the Mean (Place) of the Planets, first for the Locality of Arin, then for Noon of Any Region

You enter for any (given) moment in the table (called) *elwacat*, for any particular planet,[1] with the accumulated years, then with the single years, then with the months and days and hours and minutes,[2] all of them completed. The total of the (numbers) found there you reduce, beginning with the smallest (units), i.e. seconds.

If 12 signs accumulate by this reduction you ignore them and note down (only) the remainder of signs. Otherwise you write instead of 12 the symbol of the *cifra*[3] ʒ. Similarly for the other (units). The whole remainder obtained in this way will be the *elwacat* of the given planet for noon of the locality of Arin, for which this instruction was made.

[1] Tables 4 to 20.
[2] *momenta*.
[3] *figuram cifrae*. For the form of the zero symbol cf. Irani [1].

Thereafter subtract the longitude of your place from the longitude of the locality of Arin, which is 90 degrees, and count one hour for every 15 degrees of the remainder. An amount which does not reach 15 multiply by 4 and the product will be the number of "moments".

With these hours and with these minutes you enter the mean-motion tables. What you find for each you add to the mean motion with respect to Arin if your place is to the west (of Arin), but you subtract from it, if to the east. In this way the mean place of each planet will be established for your location.

## Chapter 7. [Concluding Section][1]

Also note that, according to the Chaldeans,[2] 4000 camel paces make one mile, and that 33 1/3[3] miles on the earth (correspond to) 1/2 degree on the heaven, such that the whole circumference of the earth contains 24000 miles. The reason[4] for it is: if one travels from any place directly south, then, after going 66 2/3 miles, a star observed at the first place, will be seen (at the second place) one degree higher, (if observed) at the same minute of the hour. Consequently 1 degree and 1/2 make 100 miles, thus 15 (degrees) 1000 miles, 1 sign 2000 (miles), thus 12 (signs) 24000 (miles).[5]

[1] Omitted in CC, on the margin of C and O, in the text of M; cf. Suter p. 43.
[2] Meaning the Arabs; cf. Suter p. 43 and below p. 22 note 11.
[3] Text: *tertia, id est thuld.* Cf. Suter p. 246 s.v.
[4] Following Suter and Nallino (Scritti V p. 415 note 4) in reading *causa* instead of *contra* or *cura.*
[5] For this result concerning the circumference of the earth see, e.g., Nallino, Scritti V p. 293f., 297ff., 412ff. I do not know, however, of a division of one mile into 4000 camel paces; ordinarily one mile equals 4000 cubits.

## Chapter 8. How to find the Place of the Sun[1]

The (true) place of the sun is to be found as follows. First one must find for the given hour its *elwacat*, i.e. the mean (position). Then, from what was found, the *elaug*, i.e. the apogee,[2] of the sun must be subtracted, namely 2 signs, 17 degrees, 55 minutes. The remainder is called *elheza*, i.e. anomaly.[3] With the number of this anomaly, one enters the (table called) *tadil*, i.e. equation.[4] Whatever the corresponding (number) is, (found) according to this rule, should be noted down, both mentally and in writing.

Then, if the said anomaly was more than six signs, the equation must be added to the mean position; if, however, less, it has to be subtracted from it. In this way you will find the sign of the sun, and the degrees, and the minutes, and the seconds.

[1] This heading is omitted in CC.
[2] *sublimatio.*
[3] *argumentum.*
[4] This is the equation of center (*examinatio*), tabulated in Tab 21 to 26.

From this you should distribute, beginning at the first degree of Aries, sign for sign, degree for degree, minute for minute, second for second.[5]

If minutes remain in the anomaly, one has to enter (the table of the equation) with the preceding (integer) degree[6] and add what is to be added, or subtract, corresponding to the (remaining) minutes[7] of the equation. Seconds below 30 should be ignored, but above 30 they should be completed to one whole (minute). What results from the above mentioned division[8] is the true place (of the sun) which you wish to determine.

Understand too that, if the anomaly was six signs, followed by a *cifra* for the degrees, i.e. zero, then the mean place itself[9] indicates the (true) place which you wanted.

[5] In short: the result gives the longitude, counted from ♈ 0°, in signs, degrees, etc.
[6] The entries in Tab. 21 to 26 proceed by integer degrees.
[7] *partes*.
[8] For interpolation.
[9] No correction is necessary in the trivial case of the sun being exactly in the perigee.

### Commentary to Chapters 7 and 8

The mean longitude $\bar{\lambda}$ of the sun for a given moment can be determined by means of Tab. 4 and 5. From it one finds the true longitude $\lambda$ from

$$\lambda = \bar{\lambda} + \Theta$$

where the "equation of center" $\Theta$ is to be found in Tab. 21 to 26, column 3, as function of the distance

$$\bar{\kappa} = \bar{\lambda} - \lambda_A$$

of the mean sun from the apogee, which is located at

$$\lambda_A = ♊\ 17;55.$$

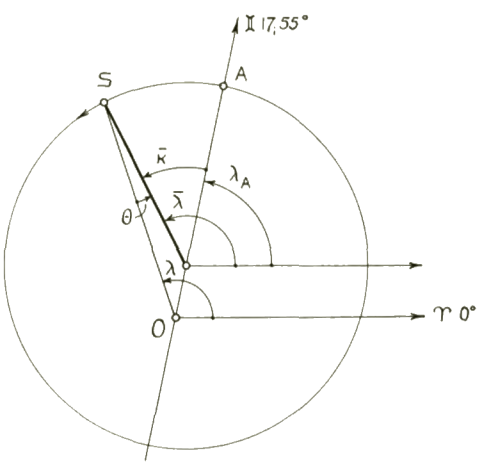


Fig. 2.

The sign of $\Theta$ is negative for $0 < \bar{\kappa} < 180$ and positive for $180 < \bar{\kappa} < 360$ (cf. fig. 2).

Note that the chapters 7 to 17 are also preserved in manuscript CC fol. 142$^{v}$ and 143$^{r}$ (cf. below p. 214 ff.)

## Chapter 9. How to Find the Place of the Moon[1]

The (true) place of the moon is found as follows. For a given hour one has to enter (in the table for) the mean position,[2] as well as its anomaly.[3] With the anomaly, one has to take the corresponding (value) in (the table for) the equation.[4] Thereafter the fractions should be reduced to integers, in the above-mentioned fashion. Then one has to consider the effect of the equation on the anomaly. If the anomaly was greater than six signs, the equation must be added to the mean position; if less, it must be subtracted from it.

[1] This heading omitted in CC.
[2] Tab. 6 to 8, column 2.
[3] *argumentum* or *elheza*; column 3 in Tab. 6 to 8.
[4] Tab. 21 to 26, column 4.

### Commentary to Chapter 9

The fact that the lunar motion is here described with only one inequality makes it evident that we are actually dealing with a type of astronomy uninfluenced by the Almagest.

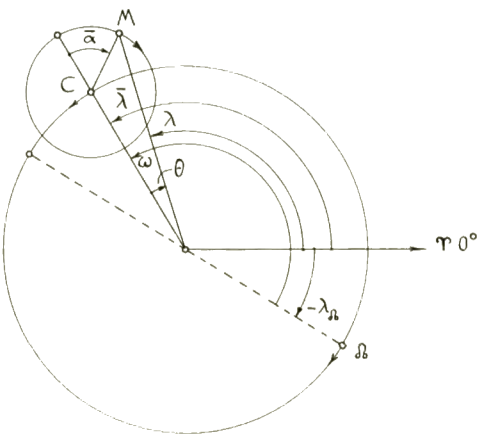


Fig. 3.

The process of computation can be explained by means of an epicyclic model[1] (fig. 3). For the given moment one finds from Tables 6 to 8 the mean longitude $\bar{\lambda}$, the mean anomaly $\bar{\alpha}$, and from $\bar{\alpha}$ the equation $\Theta$ in tables 21 to 26.

[1] There exists, of course, an equivalent eccenter-model.

## Chapter 10. How to Find the Place of Saturn, of Jupiter, and of Mars[1]

The (true) places of Saturn, Jupiter, and Mars are to be determined as follows. First one must find for each of them, and also for the sun, the mean position.[2] Then one has to subtract the mean position of the planet from the mean position of the sun. Whatever the remainder, it is to be called *elheca*.[3]

With this argument one has to enter the *examinatio sublimationis definitae*[4] if the *elheca* is less than six signs; if it is greater, one must enter[5] both the *sublimatio definita*[4] and the *examinatio*[6] of the same[5] argument and add[7] this *sublimatio* to the *examinatio* of the (same) argument; this total[8] is to be called *sublimatio examinata*. This *sublimatio examinata* is to be subtracted from the mean position of the planet, and the remainder is called *centrum*.

With the *centrum* one has now to enter the *examinatio centri*.[9] Thereafter, if the *centrum* itself exceeds six signs, the *examinatio* must be added to the *centrum* itself, but subtracted from the *argumentum* itself. If, however, the *centrum* is less than six signs, its *examinatio* must be subtracted from the *centrum*, but added to the *argumentum*. These quantities, in this fashion augmented or diminished, are called *centrum examinatum* and *argumentum examinatum*, respectively.

Thereafter one must enter with the *argumentum examinatum* the *examinatio argumenti*[10] and keep the result. If the *argumentum examinatum* exceeds six signs, the *examinatio* is to be subtracted from the *centrum*, but if less, it is to be added to it. The result can be called *elhacil* by the Chaldeans,[11] by us the definitive or the final *centrum*. To this final *centrum* must be added the *sublimatio examinata*. This total shows you the (true) position of the planet.

Remember to keep this ready at hand. If there remain some excess minutes when you enter the tables for the *examinatio* of the planets, either for the *argumentum* or for the *centrum*, with their signs and degrees, then you must enter once more with the same sign but with one degree added. If the previously found *examinatio* adds some minutes to the later one, or, on the contrary, the second adds to the first, one must subtract or add as much as corresponds to these remaining minutes which are in excess over the previous *examinatio*. What results from this, evaluate as follows: what was given in minutes, that much counts as *altheniae*, i.e. seconds, for each minute.

[1] This heading omitted in CC.
[2] Tab. 9 to 14 for the outer planets, Tab. 4 and 5 for the sun.
[3] The anomaly $\alpha_1$ in fig. 4 ff. (p. 23 ff.): also called *argumentum*.
[4] Tab. 27 to 44, col. 3.
[5] I here follow the text of the MS O; cf. note 7.
[6] Tab. 27 to 44, col. 4.
[7] I combine the version O with the other ones by reading "*et in sublimationem definitam et in examinationem ipsius argumenti intrandum est, conjugendum itaque ...*"
[8] Of columns 3 and 4.
[9] Tables 27 to 44, col. 5.
[10] Tables 27 to 44, col. 4. In Tab. 27 this column is indeed called *examinatio argumenti*, but in the remaining tables *examinatio ipsius elheza*.
[11] Version O: "in Arabic".

### Commentary to Chapter 10

Al-Khwārizmī's procedure for finding the true longitude of a planet for a given moment $t$ is based on Hindu methods known to us from the Sūrya-Siddhānta, Khaṇḍa-Khādyaka etc. These Hindu methods, in turn, are a modification of the Greek eccenter-epicycle models, designed to obtain essentially the same results while avoiding the explicit use of eccenters and epicycles by projecting the corresponding deviations from mean motion upon a concentric circle.[1] It seems plausible that these modifications are due to Hindu astronomers; it is in any case certain that the underlying model was not influenced by Ptolemy's great innovation of a motion regulated from an eccentric equant.[2] The procedure followed by al-Khwārizmī is a slight modification of the procedures we know from Hindu sources.

[1] I have explained these methods in my paper "Transmission of Planetary Theories in Ancient and Medieval Astronomy" (New York 1955, also Scripta Mathematica 22, 1956, p. 165–192) where the references to the sources can be found.

[2] Also Ptolemy's special treatment of Mercury is absent.

### Analysis of the Procedure for the Outer Planets

**Step 1.** Find for the given moment $t$ the mean longitude $\bar{\lambda}_\odot$ of the sun and $\bar{\lambda}$ of the planet (according to the rules of ch. 7). Then form

$$\alpha_1 = \bar{\lambda}_\odot - \bar{\lambda} \qquad \text{where } 0 \leq \alpha_1 < 360^\circ. \tag{1}$$

This means that the radius to the planet P′ on the epicycle is always parallel to the direction from O to the mean sun (cf. fig. 4).

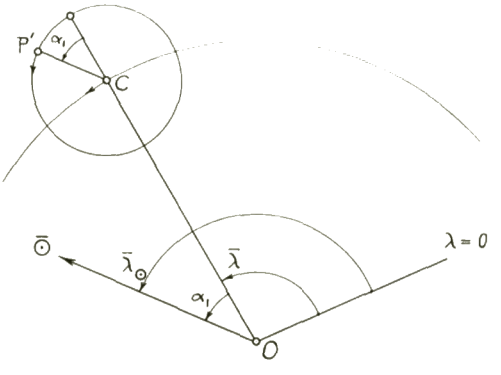


Fig. 4.

Terminology: $\bar{\lambda}_\odot$ = *elwacat* of the sun
$\bar{\lambda}$ = *elwacat* of the planet
$\alpha_1$ = *elheca* or *argumentum*.

Note: The values of $\alpha_1$ are partially tabulated, namely for days and hours, in Tab. 9, 11, 13, in the section headed *heza*. Thus it suffices to compute the mean longitudes only for years and months.

**Step 2.** Enter Tables 27 to 44 with $\alpha_1$ as argument and find

$$\text{if} \quad 0 \leq \alpha_1 \leq 180: \quad \text{in column 3} \quad s(\alpha_1)$$

$$\text{if } 180 \leq \alpha_1 < 360: \left\{ \begin{array}{l} \text{in column 3} \quad s(\alpha_1) \\ \text{in column 4} \quad |\sigma_1(\alpha_1)| \end{array} \right\} \text{ and form } s+|\sigma_1|$$

Terminology: $s$ = *examinatio sublimationis definitae*
$\sigma_1$ = *examinatio argumenti* (or *ipsius elheza*)

In this operation two parameters of the original Greek model are combined: the longitude $\lambda_A$ of the apogee of the deferent[1] and the epicyclic equation $\sigma_1$ due to

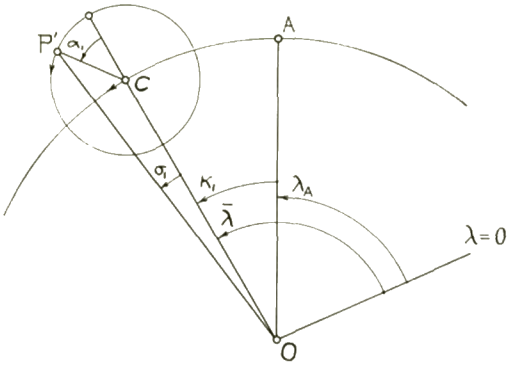


Fig. 5.

the anomaly $\alpha_1$ (cf. fig. 5). The correction $\sigma_1$ is known in Hindu astronomy as the *shīghra* correction (or correction for "conjunction"), obviously positive for $0 < \alpha_1 \leq 180$ and negative for $180 \leq \alpha_1 \leq 360$. Column 4 gives only the absolute value $|\sigma_1|$ of $\sigma_1$, whereas column 3 tabulates

$$s = \lambda_A - 1/2\,|\sigma_1|\,.$$

For $0 \leq \alpha_1 \leq 180$ this means that $s = \lambda_A - 1/2\,\sigma_1$ and for $180 \leq \alpha_1 < 360$ we have

$$s+|\sigma_1| = \lambda_A + 1/2\,|\sigma_1| = \lambda_A - 1/2\,\sigma_1.$$

Thus the role of step 2 means to form, for all values of $\alpha_1$,

$$s(\alpha_1) = \lambda_A - 1/2\,\sigma_1(\alpha_1)\,. \tag{2a}$$

With $s(\alpha_1)$ thus determined we are told to form

$$\kappa_2 = \bar{\lambda} - s \tag{2b}$$

[1] For the numerical values assigned to $\lambda_A$ for the different planets cf. ch. 18 (p. 41) and p. 99.

the result being called *centrum*. According to Greek terminology the κέντρον would be the distance of the center of the epicycle from the apogee (cf. fig. 5):

$$\kappa_1 = \bar{\lambda} - \lambda_A \,. \tag{2c}$$

Thus we can say that step 2 consists in finding

$$\kappa_2 = \kappa_1 + 1/2\ \sigma_1 . \tag{2d}$$

This allows of a simple geometric interpretation (fig. 6). If Q′ is the point of the deferent into which the observer projects the planet P′, then the midpoint of the epicycle radius CP′ is approximately projected into the point C′ which has the angular distance $1/2\ \sigma_1$ from C. Thus $\kappa_2$ is the distance of this point C′ from the apogee A.

It would be equally correct to say that $\kappa_2$ is the distance of C from a modified apogee A′ which has the longitude $s = \lambda_A - 1/2\ \sigma_1$ and thus precedes A by $1/2\ \sigma_1$ (fig. 6).

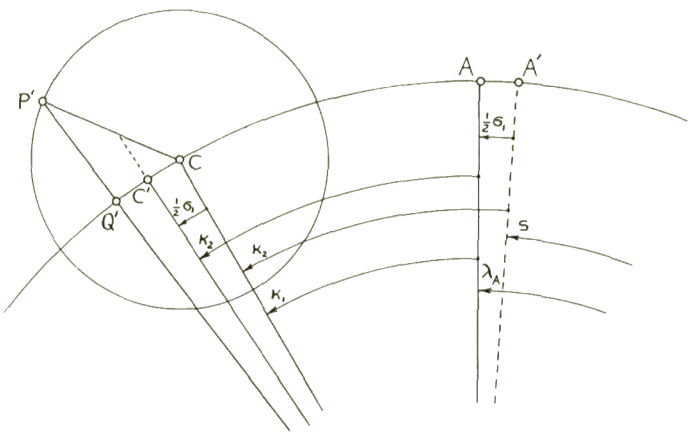


*Fig. 6.*

**Step 3.** With $\kappa_2$, found in step 2, as argument we now find in Tab. 27 to 44, column 5, the *examinatio centri* $|\mu_1(\kappa_2)|$ (which corresponds to the Hindu *manda*-correction, or correction for "apsis") and form

$$\kappa_3 = \kappa_2 + \mu_1(\kappa_2) \tag{3a}$$

where

$$\left.\begin{aligned} \mu_1 &= -|\mu_1| \quad \text{for} \quad 0 \leq \kappa_2 < 180 \\ \mu_1 &= |\mu_1| \quad \text{for} \quad 180 < \kappa_2 \leq 360. \end{aligned}\right\} \tag{3b}$$

The result $\kappa_3$ is called *centrum examinatum*, i.e. corrected *centrum*.

This correction $\mu_1$ is a correction for eccentricity. In the Greek formulation of the problem the center C of the epicycle would not move on a concentric deferent but on an eccenter which is moved by an amount *e* in the direction OA (cf. fig. 7). In our

case $\mu_1$ ($\kappa_2$) is the correction which would correspond to a position of the center of the epicycle in C′, i.e. to a position which is already corrected by an intermediate value of the epicyclic equation (fig. 8).

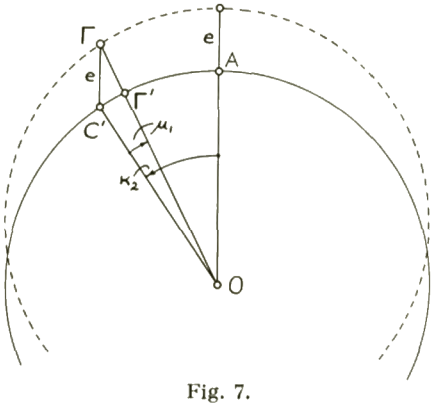


Fig. 7.

Exactly as in the previous step we may also refer $\kappa_3$ to the modified apogee A′. With respect to this point, $\kappa_3$ is the distance of a point C″ which lies $\mu_1$ ahead of C (cf. fig. 8) since C″ has the distance $\kappa_3$ from A′ and C has the distance $\kappa_2$ from A′, thus $CC'' = \kappa_2 - \kappa_3 = -\mu_1$.

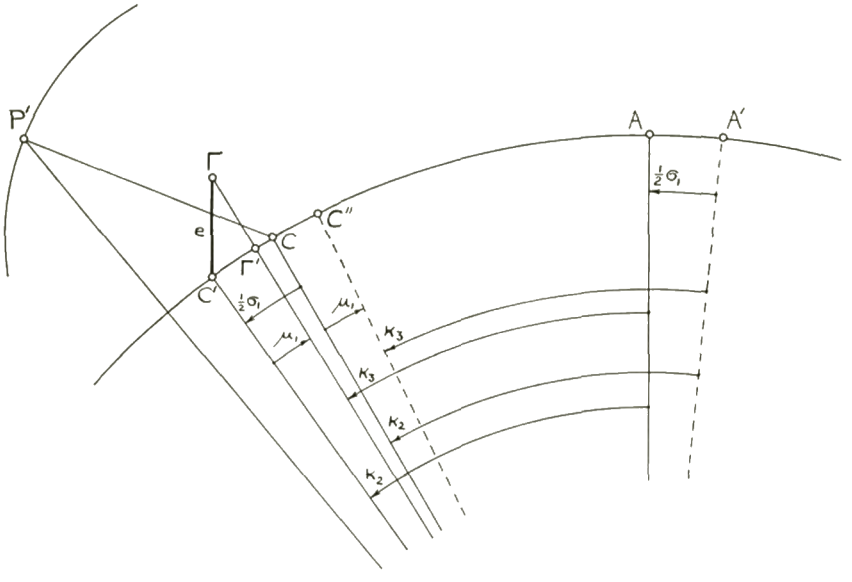


Fig. 8.

**Step 4.** With $\mu_1$, found in step 3, and with signs defined by (3a), one now forms the *argumentum examinatum*

$$\alpha_2 = \alpha_1 - \mu_1. \tag{4}$$

Fig. 9 shows that this means that we have now moved the center of the epicycle from C to the point C″, defined in step 3. From (4) it follows that the point P″ with the epicyclic anomaly $\alpha_2$ lies on C″P″ || CP′ or again parallel to the direction from O to the mean sun.

**Step 5.** For the new position P″ of the planet we determine the σ-correction by entering, with $\alpha_2$ as argument, column 4 where we find (cf. fig. 9):

$$|\sigma_2| = |\sigma_2(\alpha_2)|. \tag{5a}$$

Applying the rule

$$\left.\begin{aligned} \sigma_2 &= |\sigma_2| \qquad \text{for} \quad 0 \leqq \alpha_2 \leqq 180 \\ \sigma_2 &= -|\sigma_2| \qquad \text{for} \quad 180 \leqq \alpha_2 \leqq 360 \end{aligned}\right\} \tag{5b}$$

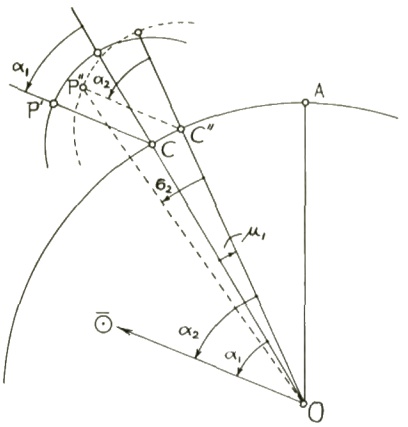


Fig. 9.

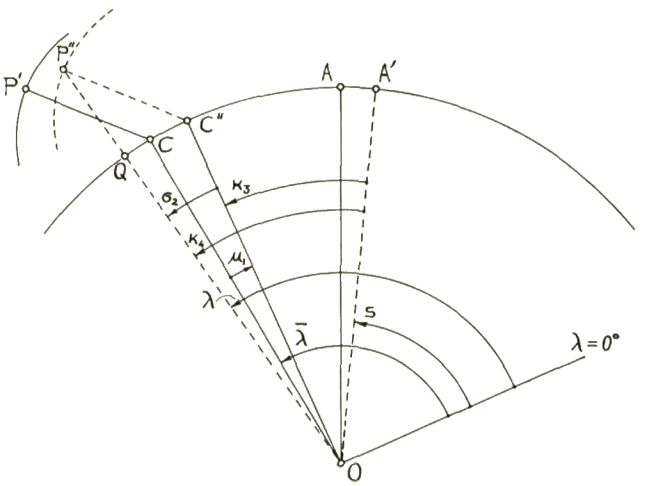


Fig. 10.

one forms the *elhazil* or final *centrum*

$$\kappa_4 = \kappa_3 + \sigma_2. \tag{5c}$$

Then the true longitude of the planet is given by

$$\lambda = \kappa_4 + s \tag{6}$$

where $s$ is the quantity found in step 2.

The meaning of this step becomes clear if we substitute in (6) the relations (5c), (2b), and then (3a):

$$\begin{aligned} \lambda &= \kappa_4 + s = \kappa_3 + \sigma_2 + \bar{\lambda} - \kappa_2 \\ &= \kappa_2 + \mu_1 + \sigma_2 + \bar{\lambda} - \kappa_2 \\ &= \bar{\lambda} + \mu_1 + \sigma_2. \end{aligned}$$

Fig. 10 shows that $\lambda$ is the longitude of the planet at P″. At first the center of the epicycle was at C, corresponding to a mean longitude $\bar{\lambda}$. Then was found the correction $\mu_1$ which represents a compromise between a correction for epicyclic anomaly and eccentricity (cf. fig. 8, p. 26). This brings the epicycle to C″ at $\bar{\lambda}+\mu_1$ and the planet to P″. The new epicyclic equation $\sigma_2$ added to $\bar{\lambda}+\mu_1$ then gives the longitude $\lambda$ of the planet.

That this procedure results indeed in an approximation to the correct solution can be seen as follows. The exact solution would be obtained by first placing the

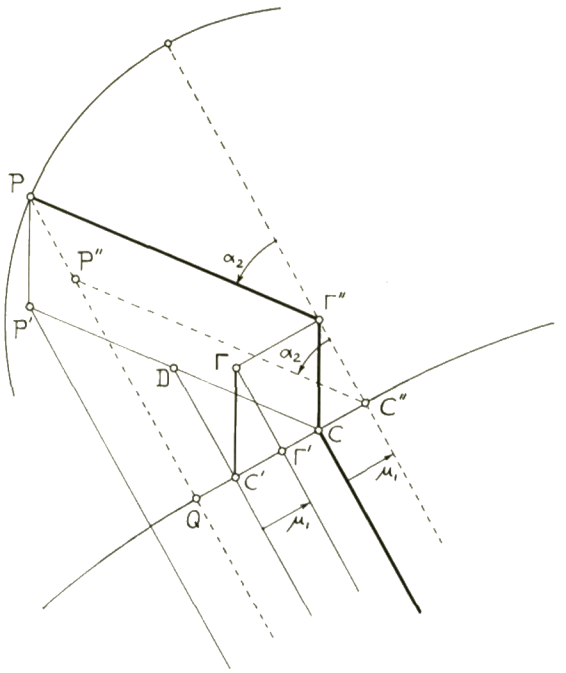


Fig. 11.

center C of the epicycle at the point of mean longitude $\bar{\lambda}$ (cf. fig. 11). To take care of the eccentricity $e$ of the deferent we then move C to $\Gamma''$ where $C\,\Gamma'' = e$ is parallel to the apsidal line OA. $\Gamma''$ is now the correct center of the epicycle. The planet P on the epicycle is located on the radius $\Gamma''P$ which is parallel to the direction to the mean sun. If we connect P with O we meet the original concentric deferent at a point Q. The longitude $\lambda$ of Q is also the longitude of P and all we have to show is that the five steps of al-Khwārizmī's procedure bring us also to the point Q, or at least near it.

This is indeed the case if we are permitted to consider the arc of the deferent near C as practically a straight line and the radii connecting its points with O as parallel. Then we can describe the single steps as follows. Step 1 projects the midpoint D of CP′ into C′. In step 2 we draw $C'\Gamma \parallel C\,\Gamma'' = e$. The projection of $C'\Gamma'$

upon the deferent is the correction $\mu_1$ found in step 3. According to our assumption $C'\Gamma\Gamma''C$ can be considered as a parallelogram. Therefore $\mu_1$ can also be represented by $CC''$ where $C''$ is the projection of $\Gamma''$ onto the deferent. But this agrees exactly with the definition of $C''$ in step 3. In step 4 we have drawn $C''P''$ parallel to $CP'$, hence also parallel to $\Gamma''P$. Because $CP' = C''P'' = \Gamma''P$ we would find $P''$ located on PQ if PQO were parallel to $\Gamma''C''O$. But this is exactly what we assumed to be approximately correct. Thus $P''$ has, within the accuracy of this approximation, the same longitude as P and Q.

## Chapter 11. How to Find the Places of Venus and Mercury[1]

The (true) places of Venus and Mercury can be found in the same way as has been described above (for the outer planets), excepting what is called their *argumentum* in this book, because their mean position is identical with (the mean position of) the sun.

[1] Heading omitted in CC.

### Commentary to Chapter 11

For the outer planets the epicyclic anomaly $\alpha_1$ is given by $\alpha_1 = \bar{\lambda}_\odot - \bar{\lambda}$ (cf. fig. 4 p. 23). For the inner planets we have $\bar{\lambda} = \bar{\lambda}_\odot$ and the anomaly $\alpha_1$ is independent

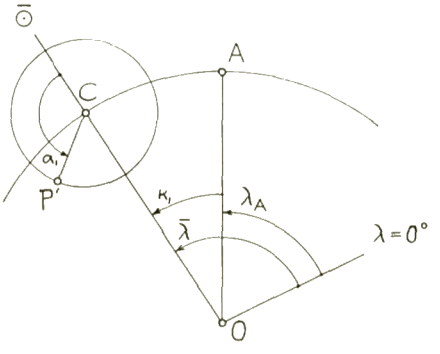


Fig. 12.

of $\bar{\lambda}_\odot$ (cf. fig. 12). For the given moment, $\bar{\lambda}$ and $\alpha_1$ can be found from the tables 4, 5 and 15 to 18 respectively in the customary fashion in steps of 30 Arabic years, single years, etc.

The remaining computation follows the pattern of ch. 10, using Tab. 45 to 56, cols. 1 to 5:

col. 3: $s = \lambda_A - 1/2\ \sigma_1(\alpha_1)$ form: $\kappa_2 = \bar{\lambda} - s$

col. 5: $\mu_1(\kappa_2)$ form: $\kappa_3 = \kappa_2 + \mu_1$

form: $\alpha_2 = \alpha_1 - \mu_1$

col. 4: $\sigma_2\,(\alpha_2)$ form: $\kappa_4 = \kappa_3 + \sigma_2$

and finally

$$\lambda = \kappa_4 + s = \bar{\lambda} + \mu_1 + \sigma_2$$

as in the case of the outer planets.

## Chapter 12. How to Find the Place of the Ascending Node (of the Moon)[1]

The place of the ascending node (of the moon) is to be found as follows. Note its mean longitude[2] and subtract it from 12 signs. You declare the remainder as the place of the ascending node.

[1] Heading omitted in CC.
[2] From Tab. 19 and 20 which give $-\lambda_{\mathfrak{Ω}}$. Cf. also fig. 3 p. 21.

## Chapter 13. On the Station and Direct and Retrograde Motion of the Planets[1]

In the following way it is possible to know at what times the planets are in direct motion, when retrograde, and when stationary: one enters with the number of the equated center[2] for every one of the 5 planets the table of the first station[3] and notes down the corresponding number. Then the first station is to be subtracted from 12 signs; what remains is called *elmukaam eltheni*, i.e. second station. Then one takes the equated argument of the planet; if this equals either one of the two stations, then the planet is located in this station. If it is between the two, namely, greater than the first but less than the second, then (the planet is) retrograde. Otherwise (the planet) is known without doubt to be in direct motion.

[1] Heading omitted in CC.
[2] *centrum examinatum* has here a different meaning from that in chapter 10, step 3. Cf. the commentary below.
[3] Tab. 27 to 56, column 6.

### Commentary to Chapter 13

The general principle for ascertaining the type of motion of a planet is the following one. In column 6 of Tab. 27 to 56 are listed the epicyclic anomalies at which a planet becomes stationary, depending on its distance from the apogee of the deferent, which in turn determines the distance of the epicycle from the observer. If the given epicyclic anomaly $\alpha$ is less than the value $\alpha_s$ for the first station, or greater than the value $\alpha_t = 360 - \alpha_s$ for the second station, then the motion of the planet is direct; otherwise the planet is retrograde or, in case of exact agreement with $\alpha_s$ or $\alpha_t$, stationary.

The model that underlies the rules given in this chapter is again the simple

eccentric epicycle (fig. 13). The independent variable which determines the position of the stations is the equated distance from the apogee

$$\kappa = \bar{\kappa} + \mu$$

where $\mu = \mu(\bar{\kappa})$ is the correction for eccentricity, tabulated in column 5 of our table. The resulting $\kappa$ is called *centrum examinatum* or "equated center". It must not be identified with the centrum examinatum $\kappa_3 = \kappa_2 + \mu_1$ found in step 3 of chapter 10. This is evident if one recalls that $\kappa_2 = \kappa_1 + 1/2\,\sigma_1$ where $\sigma_1$ is a function of the arbitrary position of the planet on the epicycle. Obviously the location of the stations can not depend on the variable position of the planet on the epicycle.

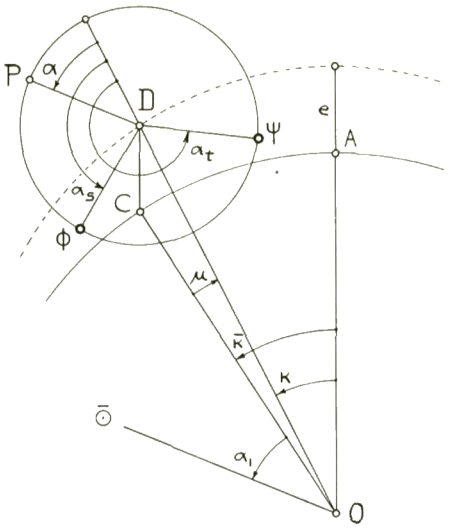


Fig. 13.

After the anomalies $\alpha_s$ and $\alpha_t$ have been found for the value of $\kappa$ in question, one has to compare the actual anomaly $\alpha$ of the planet with $\alpha_s$ and $\alpha_t$. Again $\alpha$ must be the "equated anomaly"

$$\alpha = \alpha_1 - \mu(\bar{\kappa}) \quad \begin{cases} \mu \leqq 0 \text{ for } & 0 \leqq \bar{\kappa} \leqq 180 \\ \mu \geqq 0 \text{ for } & 180 \leqq \bar{\kappa} \leqq 360 \end{cases}$$

such that DP (fig. 13) is parallel to the direction from O to the mean sun. Thus *argumentum examinatum* means here the true epicyclic anomaly $\alpha$ and not $\alpha_2 = \alpha_1 - \mu_1$ found in step 4 of chapter 10. In other words, *examinatio* means here simply "correction" or "equation", without determening the specific type of modification which leads to this correction.

As is shown in the discussion of Tab. 27 to 56, column 6 (p. 101), the numerical tables are taken from the Handy Tables of Theon. Whether this goes back to al-Khwārizmī himself or not is impossible to say.

## Chapter 14. On the Interval of Time according to the Above-mentioned Stations[1]

If you wish to know for how long they (the planets) have been retrograde, after one of the planets has become retrograde, you have to take the first station and the equated argument[2] (already) found and find the argument which is appropriate to this investigation (i.e. the proper difference of anomaly). From it one can conclude in what time the specific planet traverses the excess of the equated argument over the first station; we shall say that it has been retrograde during that time.

How much (more) time it is moving (retrograde) can be found as follows. One considers how much the second station exceeds the equated argument. Then one can conclude from the argument which is appropriate to this investigation how much time it takes in passage. Thus will be found what you ask.

If you want to know about the direct motion, how much time has passed by (from the beginning) of its (forward) direction until the present moment, you consider how much its equated argument exceeds the second station and in how much time it traverses (this interval). This is indeed what you want.[3]

If (the planet) was not far from the first station, and the question is, how far it is distant from it; (then) one has to consider how much the first station exceeds the equated argument and in what time it traverses (this interval). In this way what you are asking for results.

In the present *zīj* is listed for each of the three superior planets its above-mentioned argument.[4] For Venus and Mercury only the arguments which are listed in this book are needed for this purpose.

[1] Heading omitted in CC.
[2] I follow here the text of C and M.
[3] Following O. The version M can perhaps be rendered as "this for the case under investigation".
[4] O: "the argument appropriate for this purpose".

### Commentary to Chapter 14

We assume as known, for a given moment, the epicyclic anomaly $\alpha$ of a planet (cf. fig. 13 p. 31). Also known are the anomalies $\alpha_s$ and $\alpha_t$ from Tab. 27 to 56, col. 6. In order to find the time interval which separates the planet from a station, we are told to consider the proper difference $\alpha_s - \alpha$, or $\alpha - \alpha_s$, etc. The text does not tell us what should be done with these differences but obviously one can obtain time intervals from them by division by the appropriate mean motion of anomaly.

This is implied in the final paragraph in which we are referred, for the outer planets, to the auxiliary tables which are found with Tab. 9 to 14, where the increase in anomaly for days and hours is tabulated. For the inner planets the main tables (15 to 18) serve the same purpose.

Strictly speaking this procedure is not correct since the anomalies $\alpha_s$ and $\alpha_t$ are functions of the longitudinal distance $\bar{\kappa}$ from the apogee and thus functions of time.

The reference to the auxiliary tables mentioned before shows, however, that the problem is only considered for a few days' and hours' distance from the stations, such that for all practical purposes $\alpha_s$ and $\alpha_t$ can be considered constant.

## Chapter 15. Chapter on the Knowledge of the Declination of the Sun[1]

The following is the method of finding the distance which the sun deviates from the region of the equinoctial circle. One enters the table of the solar declination[2] with the previously corrected place of the sun.[3] The amount found there is how much (the sun) deviates from the equator.

[Version O:] What is found between the first degree of Aries and the third sign is called *shemeli*, that is, on this (northerly) side, and increasing; if, however, between the third and sixth, *shemeli* and decreasing. If between the sixth and ninth (sign) it is called *genubi*, that is on the other side (i.e. southerly), and decreasing; if it is between the ninth and the twelfth (sign) it is called on the right side[4] and increasing.

[Version C and M:] Beginning the reckoning at the head of Aries, if it is found within the first three signs, it will be *elmeil*, that is, the declination[5] is to the north and sloping up; if within the second three signs again northern but sloping down. What is found within the third three signs is southern[6] and sloping down; if within the remaining three (signs) it will be south but sloping up.

[1] Heading omitted in CC. Version O: "On the deviation of the sun from the equinoctial circle".
[2] Tab. 21 to 26, column 5.
[3] That is the true longitude of the sun.
[4] Of the equator in the direction of the daily rotation.
[5] Suter indicates a lacuna here, but there is nothing missing.
[6] Expressed by two terms: *australis* and *meridionalis*; in the lacuna between these words was perhaps given the Arabic term; cf. version O.

## Chapter 16. On the Latitude of the Moon[1]

The knowledge of the latitude of the moon is obtained as follows: after the place of the moon has been found, the mean position of the ascending node[2] is added to it. This total is the argument with which is entered (the table of) the latitude of the moon,[3] and one takes what is found in (the column for) the latitude of the moon. And one should know that the same that has been said[4] about (the declination of) the sun also holds for the location of the moon with respect to the ascending node, that is with respect to its *elmeil*.[5]

[1] Heading omitted in CC.
[2] The longitude of the ascending node, as given in Tab. 19 and 20, is the distance of the node from $\Upsilon$ 0°, counted in negative direction (cf. fig. 3 p. 21). Thus the above mentioned total is the distance $\omega$ of the mean moon from the ascending node.
[3] Tab. 21 to 26, column 6.
[4] In chapter 15.
[5] I.e. being to the north of the ecliptic, etc.

## Chapter 17. On the Latitude of the Three Superior Planets[1]

Section on the latitude of the three superior planets. One enters with the *elheca examinatum*[2] of any one of them the table of the first latitude[3] and one keeps the latitude which is found there. Then the number of its ascending node is to be added to the previously corrected place of the planet and this total is called the argument. With it one enters (the table of) the second latitude.[4] Then the second latitude is to be divided by the first latitude. The result is what you ask (namely, the latitude of the planet).

For Venus and Mercury, however, as follows. One enters with its *argumentum examinatum*[5] the (column of the) first latitude.[3] Then the number of the ascending node of either one of the two planets is added to the mean position of the sun, together with the *argumentum examinatum* of the planet; this total will be the *elheca*.[6] With this one now enters (the table of) the second latitude.[4] Then the second latitude is to be divided by the first latitude. The result is what you ask for[7] (namely, the latitude of the planet).

[1] This heading is omitted in CC. Actually it is incorrect insofar as the latitudes for all five planets are discussed in this chapter.
[2] Or "equated anomaly"; cf. the commentary (p. 40).
[3] Tab. 27 to 56, column 7.
[4] Tab. 27 to 56, column 8.
[5] Or "equated anomaly"; cf. the commentary (p. 40)
[6] I.e. argument.
[7] Here ends the text in CC.

### Commentary to Chapter 17

A simple model for the description of the variable latitudes of the outer planets would be the following one: the deferent is inclined at a fixed angle $i$ toward the ecliptic, while the plane of the epicycle remains parallel to the ecliptic during the motion of the center of the epicycle along the deferent. For the inner planets we may assume that the plane of the deferent coincides with the ecliptic while the plane of the epicycle is inclined toward it at a fixed angle $i$, moving always parallel to itself. These models could be expected to explain at least the main features of the tables for planetary latitudes. The following discussion will show, however, that serious numerical discrepancies exist, which cast doubt on the soundness of our basic assumptions.

#### 1. The Epicyclic Model

#### The Outer Planets

If $R$ is the radius of the deferent, C the center of the epicycle, $\omega$ the argument of latitude counted from the ascending node of the deferent plane, we find for the distance CE of the epicycle from the ecliptic (cf. fig. 14)

$$\mathrm{CE} = R \sin \gamma = R \sin i \sin \omega .$$

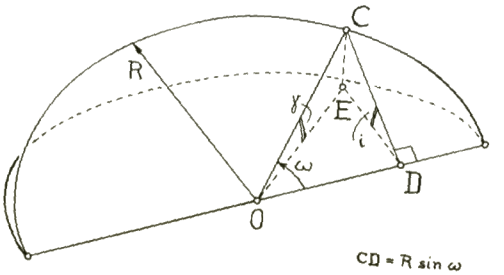


Fig. 14.

If $\rho$ is the distance from O of the planet on the epicycle, then its latitude $\beta$ must satisfy

or
$$\rho \sin \beta = CE$$

$$\sin \beta = \frac{R}{\varrho} \sin i \sin \omega. \tag{1}$$

Since both $\beta$ and $i$ are small angles we have approximately

$$\beta = \frac{R}{\varrho} i \sin \omega. \tag{2}$$

We can now compare this formula with the rule of our text for finding $\beta$ from

$$\beta = \beta_2/\beta_1 \tag{3}$$

where $\beta_2$ is a function of strictly sinusoidal character $c \sin \Theta$. If we interpret (correctly, as can hardly be doubted) $\Theta$ as argument of latitude and $c$ as inclination $i$ of the deferent, then we would obtain from (3) and (2)

or
$$\beta = Ri \sin \omega/\rho = i \sin \omega/\beta_1 \tag{4a}$$

$$\beta_1 = \rho/R. \tag{4b}$$

In other words, our interpretation leads to the conclusion that the "first latitude" $\beta_1$ is simply the normed distance of the planet from the observer.

For the values of $i$ one finds from the tables

$\text{♄}$: $i = 5°$ $\quad$ $\text{☿}$: $i = 2;30$ $\quad$ $\text{♂}$: $i = 3;45$.

There is no reason to doubt that these values represent the porper inclinations.

Unfortunately, however, all numerical data for $\beta_1$ obtainable from column 7 of Tab. 27 to 44 lead to serious contradictions with the relation (4).

Let us, e.g., consider the case $\omega = 90$ or

$$\beta = i R/\rho.$$

According to our model, $\rho$ is a function of the epicyclic anomaly $\alpha$. Since we know the values $r$ for the radii of the epicycles[1] we can compute for each planet the anomaly $\alpha_0$ at which $\rho = R$. Indeed (cf. fig. 15) we have

$$\alpha_0 = 90 + \frac{\delta}{2} \qquad \text{and} \qquad \sin\frac{\delta}{2} = \frac{r}{2R}.$$

In this way one finds (cf. the graph of fig. 30 p. 102):

| | $r$ | $\alpha_0$ | tables: $\beta_1 = 1$ for $\alpha_0$ |
|---|---|---|---|
| ♄ | 6 | 90 + 2;52 | – |
| ♃ | 11;20 | + 5;25 | 90 + 75 to 78 |
| ♂ | 39 | + 18;58 | + 41 |

One could try to explain this result by the assumption that the center of the epicycle is not located at the distance $R$ from O (at C in fig. 15) but farther removed from it

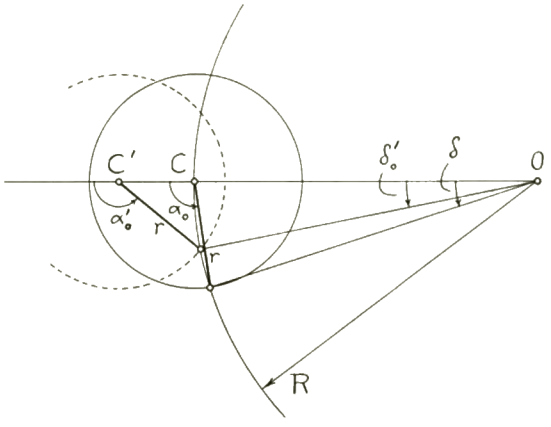


Fig. 15.

(at C′) in order to take into account the eccentricity of the orbit. Such a hypothesis seems to be supported by the fact that in the case of Saturn the eccentricity $e = 8;36$ is greater than the radius $r = 6$ of the epicycle. Consequently even the perigee of the epicycle would be at a distance $\rho = R + e - r = 1,2;36 > R$ away from O and the condition $\rho = R$ for $\beta_1 = 1$ could never be satisfied. And indeed Tab. 32 shows that the minimum value $m$ of $\beta_1$ is $> 1$, namely $m = 1;10$. If, however, we compute for the remaining planets the distances CC′ which would correspond to the increase from $\alpha_0$ to $\alpha_0'$ we find distances much greater than the eccentricities:

$$♃ : CC' = 10;54 \quad e = 5;6 \qquad ♂ : CC' = 20;23 \quad e = 11;13.$$

[1] Cf. p. 100.

This shows that it is not the eccentricity which causes us to find the value $\beta_1 = 1$ much farther away from $\alpha = 90$ than expected.

It is easy to derive additional discrepancies between the parameters of the epicyclic model and the tables for $\beta_1$. For example one would expect that the minimum $m$ of $\beta_1$ and the maximum $M$ satisfy because of (4b):

$$r = RM - R = R - Rm. \tag{5}$$

In fact, however, one finds for $R = 1;0$:

| | $M$ | $m$ | $RM-R$ | $r$ | $R-Rm$ |
|---|---|---|---|---|---|
| ♄ | 1;33 | 1;10 | 33 | 6 | −10 |
| ♃ | 1;36 | 0;57 | 36 | 11;20 | 3 |
| ♂ | 2;14 | 0;27 | 1,14 | 39 | 33 |

Again it can be shown that the eccentricities do not suffice to remedy the situation.

### Inner Planets

If the argument of latitude $\omega$ measures the angular distance of the center C of the epicycle from the radius of the deferent which is parallel to the nodal line of the

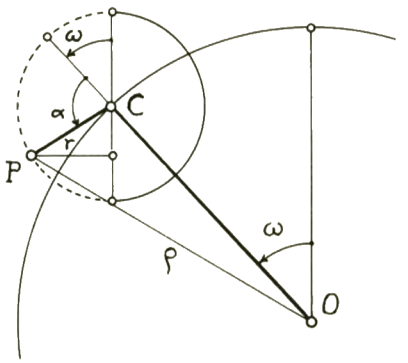


Fig. 15 a.

epicycle then it is easy to see (cf. fig. 15a) that the latitude $\beta$ of a planet P of epicyclic anomaly $\alpha$ is given by

$$\beta = \frac{ri \sin(\omega + \alpha)}{\rho}. \tag{6}$$

If we again write $\beta$ in the form

$$\beta = \beta_2/\beta_1 \tag{7}$$

with

$$\beta_2 = i \sin(\omega + \alpha) \tag{8a}$$

we have for $\beta_1$

$$\beta_1 = \rho/r. \tag{8b}$$

The maxima of the tabulated values $\beta_2$ show that

$$\text{for } ♀ : i = 5;0 \qquad \text{for } ☿ : i = 6;15. \tag{9}$$

The tables for $\beta_1$, however, lead only for Venus to agreement with our model, whereas flagrant contradictions exist in the case of Mercury.

If again $m$ and $M$ denote the minimum, respectively the maximum value of $\beta_1$ given in the tables we should find from (8b)

$$r = \frac{R}{M-1} = \frac{R}{m+1}.$$

In fact, however, on has

| | $M$ | $m$ | $\frac{R}{M-1}$ | $r$ | $\frac{R}{m+1}$ |
|---|---|---|---|---|---|
| ♀ | 2;24 | 0;22 | 42;51 | 44 | 43;53 |
| ☿ | 1;44 | 0;48 | 1:22 | 22 | 33;20 |

Obviously only the values for Venus are in principle acceptible while the numbers for Mercury make no sense at all.

Similar is the situation for the point at which $\beta_1 = 1$. According to the tables this is the case at

$$90 + 43 \text{ for } ♀ \qquad 90 + 43 \text{ to } 45 \text{ for } ☿.$$

For Mercury we have $r = 22$ but $\rho \geq 38$ and it is therefore clear that $\rho = r$ has no solution. Thus (8b) can not be correct. For Venus, however, with $r = 44$ we find that $\rho = r$ for the anomaly $90 + 43$ in agreement with the tables.

The fact that it is only Venus for which our models explain the tabulated values make also this agreement suspect.

### 2. The Hindu Theory

In the following we compare al-Khwārizmī's tables with sections of Hindu treatises: Āryabhaṭīya I,6 and 7[2], Khaṇḍakhādyaka VI, 1 – 3[3], and Surya-Siddhānta I, 68 – 70[4]. These texts agree in the rules for the computation of planetary latitudes. If we denote 150 sin $\Theta$ by $\mathrm{Sin}_{150}\Theta$ one computes the distance $\rho'$ from the observer to the planet[5] from

[2] Trsl. W.E. Clark p. 16.
[3] Trsl. Sengupta p. 115ff.; also VIII,1,2 (p. 134).
[4] Trsl. Burgess p. 50 (Calcutta 1935) or p. 188f. of the ed. of 1860.
[5] Called "variable hypothenuse" (Sūr.-Siddh. II,41) or "*shīghra*-hypothenuse" because it depends on the epicyclic anomaly (*shīghra*; cf. above p. 24).

$$\rho' = \frac{\mathrm{Sin}_{150}\,\sigma_0 \cdot \mathrm{Sin}_{150}\,\alpha}{\mathrm{Sin}_{150}\,\sigma} \tag{10}$$

which means (cf. fig. 16) that

$$\rho' = \frac{150 \sin \sigma_0 \cdot \sin \alpha}{\sin \sigma} \cdot \frac{r'}{150} = \frac{r' \sin \alpha}{\sin \sigma}$$

as is obviously correct. Distances are here measured in units in which the deferent has a radius 150, as is often the case in Hindu trigonometry. We also know, e.g. from

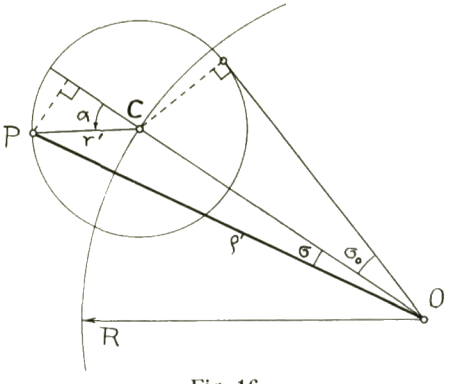


Fig. 16.

al-Bīrūnī, that al-Khwārizmī's tables of sines were based on this radius.[6] If $\omega$ is the argument of latitude, $i'$ a given angle, then $\beta$ is found from

$$\beta = \frac{i' \,\mathrm{Sin}_{150}\,\omega}{\rho'}. \tag{11}$$

The values of $i'$ from the Hindu treatises compare as follows with the values of the inclinations $i$ from al-Khwārizmī's tables:[7]

| | $i'$ | $i$ |
|---|---|---|
| ♄ | 2; 0° | 5; 0° |
| ♃ | 1; 0 | 2;30 |
| ♂ | 1;30 | 3;45 |
| ♀ | 2; 0 | 5; 0 |
| ☿ | 2;30 | 6;15 |

Thus we have for all five planets exactly the relation

[6] Cf. Lesley [1] p. 126 and below commentary to Tab. 58 and 58a (p. 104).

[7] Usually the value for Mercury is given as 2;0°. But the commentary of Āmarāja gives 2;30 while Brahmagupta has 2;32, according to Sengupta Kh.–Kh. p. 116. The valus 2;30 is now confirmed by Hugo Sanctallensis' Latin version of a commentary to al-Khwārizmī's tables, since this text gives exactly the values $i'$ of our list (Se fol. 40r, 21–23).

$$i = 2;30\ i'. \tag{12}$$

It seems tempting to equate al-Khwārizmī's

$$\beta = \frac{Ri \sin \omega}{\rho} = \frac{\frac{Ri}{150} \mathrm{Sin}_{150}\, \omega}{\rho} \qquad R = 60$$

the Hindu formula

$$\beta = \frac{i' \mathrm{Sin}_{150}\, \omega}{\rho'}.$$

If it were correct to say that $\rho = \rho'$ then we would have the relation (12):

$$i = \frac{150}{60} i' = 2;30\ i'.$$

Unfortunately, however, $\rho$ is measured in units of a deferent of radius 60, $\rho'$ in units of a deferent of radius 150. This again points to a basic misinterpretation in our analysis of al-Khwārizmī's procedures.

### 3. The Rules of Chapter 17

For both outer and inner planets it is said that the "first latitude" $\beta_1$ should be taken (from column 7 of Tab. 27 to 56) as function of the "equated anomaly". This term occurred in previous chapters in two meanings: either, in the simple eccentric epicyclic model of chapter 13, as the true epicyclic anomaly ($\alpha$ in fig. 13 p. 31), or as the true epicyclic anomaly ($\alpha_2$ in fig. 9 p. 27) obtained by the procedure described in ch. 10 and 11, that leads to the true longitudes of the planets. Which one of these two possibilities is to be chosen here is not certain, but it seems to me that the simpler interpretation (fig. 13) is preferable since it makes little sense to use great accuracy for the anomaly for the computation of the slowly variable latitudes. Whatever the case may be, however, this much seems to follow: that the term "equated" anomaly implies a correction for eccentricity. In the preceding discussion of the tables for $\beta_1$ we did not succeed in bringing the eccentricities into a plausible relation to the tabulated values of $\beta_1$. The formulation of chapter 17 only underlines what one has to expect a priori, namely, that the eccentricity must play a role for the computation of $\beta_1$.

For the "second latitude" $\beta_2$, where we have no serious theoretical difficulties, the text is more explicit. For the outer planets we are told that the argument (for column 8 in Tab. 27 to 56) is the sum of the number (or longitude) of the ascending node (given in chapter 19) and the corrected longitude of the planet. It is obvious that not the sum but the difference of the two longitudes has to be taken in order to obtain the argument of latitude $\omega$. The error probably is caused by the analogous problem for the moon (ch. 16, p. 33) where the retrograde motion of the lunar nodes leads to an addition of two motions. The use of the "corrected longitude" of

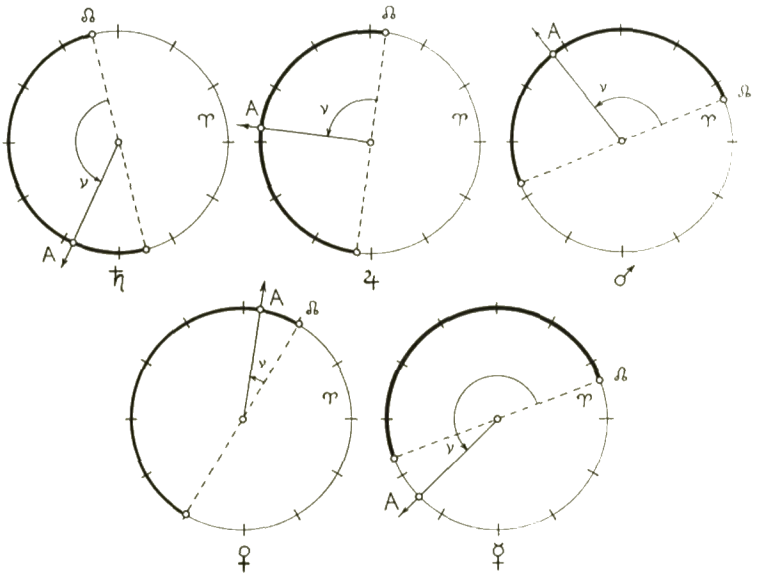


Fig. 17.

the planet shows again that the true longitude of the center of the epicycle has to be taken (corresponding to κ, not to $\bar{\kappa}$, fig. 13 p. 31), thus corrected for eccentricity.

In the case of the inner planets "sum" should again be replaced by difference, in order to find ω. The center of the epicycle would have the same mean longitude as the sun. The true longitude should be obtained by correcting the mean longitude by the amount μ, which depends on the distance κ from the apogee of the planet (given in ch. 18). In Tab. 45 to 56 the equation $\mu_1$ is tabulated in column 5 as "*examinatio centri*". Chapter 17 does not mention this quantity but refers instead to the "equated anomaly of the planet" as in the case of $\beta_1$. This could refer to the argument $\omega + \alpha$ in (8 a) p. 37.[8] Finally β is to be found as quotient $\beta_2/\beta_1$.

[8] The text in O and CC is somewhat different but apparently garbled (cf. Suter p. 14, note 20).

## Chapter 18. On the Apogee and Perigee of the Planets

The apogee of the planets and their perigee are as follows: The apogee of the sun is 2[s] 17°55′0″[1] (i.e. ♊ 17;55); of Saturn 8[s] 4°55′0″ (= ♐ 4;55); of Jupiter 5[s] 22°32′0″ (= ♍ 22;32); of Mars 4[s] 8°24′ (= ♌ 8;24); of Venus 2[s] 21°15′0″ (= ♊ 21;15); of Mercury 7[s] 14°54′ (= ♏ 14;54).

The apogee of the moon moves 0[s] 0;6,40,18,48° per day, 0[s] 0;0,16,42,27° per hour.

[1] Here and in the following I do not accurately reproduce all the different ways of expressing numbers and units (e.g. here "two signs, 17 degrees and 55 *dakaicae*, no *elthenia*"). – For a geometric representation of the data given here and in ch. 19 cf. fig. 17. The same parameters are used in Tab. 27 to 56 (cf. below p. 99).

## Chapter 19. On the Nodes of the Planets

The (ascending) nodes of the planets are: Saturn 3$^s$ 13°12′ (= ♋ 13;12); of Jupiter 2$^s$ 22°1′ (= ♊ 22;1); of Mars 0$^s$ 21°54′ (= ♈ 21;54). The (ascending) node of Venus (is at) 1$^s$ 29°27′ (= ♉ 29;27). The the *eliauzehar* of Mercury, i.e. the (ascending) node (is at) 0$^s$ 21°10′ (= ♈ 21;10).

Finally the node of the moon is movable, (moving) 0$^s$ 0;3,11,48° per day,[1] 0$^s$ 0;0,7,57° per hour.

[1] Sic, instead of 0;3,11 or 0;3,10,48. Cf. Tab. 19 and 20 (p. 95).

## Chapter 20. On the Daily Motion of the Moon

The daily motion of the moon is 0$^s$ 13;10,34,52,48°, per hour 0$^s$ 0;32,56,47,52°.[1]

[1] Error for 0;32,56,27,12° which is the exact result of division by 24 of the number quoted for the daily motion. Otherwise one would have a daily motion of 13;10,43, ... which is certainly wrong.

## Chapter 21. On the Daily Motion of the Sun

The daily motion of the sun is 0$^s$ 0;59,8, per hour 0$^s$ 0;2,27,50,25°.

[In C in a second hand:] According to Ptolemy in the Almagest the year consists of 365;14,48$^d$, the day[1] of about 0;59,8,17,13,12,31°, the hour of about 0;2,27,50,43, 3,1°, the month of 29;34,8,35,36,15,30°,[2] the Egyptian year of 359;45,24,45,21,8,35°; the accumulated year, which is 18 years, (consists) of 1350°[3] 37;25,36,20,34,30′. This number is the remainder of the accumulation of 18 years, after complete cycles have been disregarded.

[1] Actually: the motion per day; similarly in the following.
[2] Error for 29;34,8,36,36,15,30°.
[3] Error for 355°.

### Commentary to Chapters 20 and 21

The mean motion of the moon, given in ch. 20 as 13;10,34,52,48°/$^d$, does not agree exactly with the value derived from Tab. 6, which would lead to 46 in the last place (cf. p. 92).

The values quoted in a later addition to ch. 21 as Ptolemy's values in the Almagest indeed agree (except for two scribal errors) with this source.[1]

[1] Alm. III,1 p. 208,13 Heiberg for the length of the tropical year, III,2 for the mean motions of the sun.

## Chapter 22. On the Appearance of the Moon on the 29th Day in the Evening

So far the equations of the planets have been described, and what belongs to the subject. Now we have to write about (the questions) which concern the appearance of the moon. Since the problem is whether the moon will be visible or not on the

evening of the 29th day of the lunar month, one must find the place of sun and moon at noon on this day, adding also the correction for six hours; in this way one will find their place at evening on this day. Then one must carefully note down by how much the place of the moon exceeds the place of the sun. Then, if the moon was *shemeli*, that is to the north of this part, the latitude is to be added to the previously noted difference; if, however, *genubi*, that is, towards noon or to the south of this part, (the latitude) is to be subtracted from this (difference), and this is the definite distance.

Then we enter in the table given below[1] with the sign of the moon and its decan[2] and one notes down the corresponding angle. If the above-mentioned distance is greater than the number thus found, the expected crescent will appear on the same evening; if less, not; if, however, equal, it will be barely visible to a sharp eye and in clear air.

Concerning the problem (of last visibility) on the 27th day in the morning hour, one must note down the place of the sun and that of the moon at noontime of this day and subtract the amount (of motion) during six hours, and operate for the rest as said above.

One must also realize that the above mentioned hours are understood as referring to the present locality, similarly to what has been stated above when we dealt with the determination of the mean motion of the planets.[3] If, namely, our place is located to the east with respect to Arin, the amount of this distance must be added to our meridian, in order that by this addition the computation corresponds to the meridian (assumed for) this treatise. If we are located, however, toward the west, then the opposite (procedure is to be followed). The danger of an error thus being removed, the aforementioned correction of six hours is either to be added (in the case of first visibility) or subtracted (for last visibility).

[1] Tab. 57a.
[2] *signi facies.*
[3] Chapter 7 (p. 18).

### Commentary to Chapter 22

The visibility of the new crescent (and similarly the last visibility) is made dependent on three variables: The longitude $\lambda_{☾}$ of the moon, its elongation $\lambda_{☾} - \lambda_{\odot}$ from the sun, and its latitude $\beta$, all computed for 6 hours after noon of the 29th day of a lunar month (or 6 hours before noon of the 27th). Then one forms

$$\Delta\lambda = \lambda_{☾} - \lambda_{\odot} + \beta$$

and compares the result with a value $\Delta\lambda_0$ found in Tab. 57a for the argument $\lambda_{☾}$. For visibility it is necessary and sufficient that $\Delta\lambda \geq \Delta\lambda_0$.

How the function $\Delta\lambda_0$ was computed is unknown (cf. fig. 18). If one computes the corresponding right ascensional differences $\Delta\alpha_0$ for the latitude $\varphi = 36$ of southern Spain or northern Mesopotamia, then one finds a function which varies

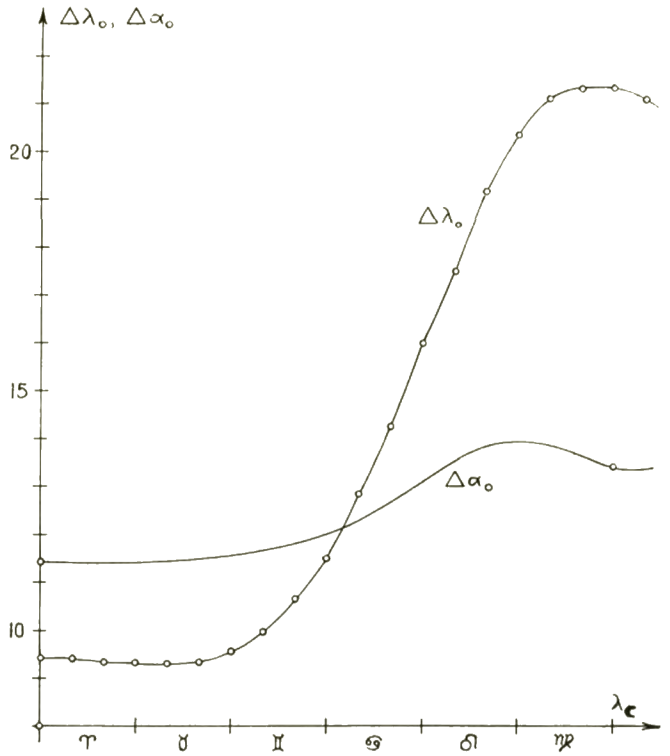


Fig. 18.

only between about 11 1/2° and 14° (cf. fig. 18).[1] This gives the impression that one had considered as the main criterion a right ascensional difference of sun and moon of about 13°.

It is, of course, a rather crude procedure to add the lunar latitude (positive if to the north, negative if to the south) to the elongation. This means in effect that one deals with the latitude as if it were a right ascensional arc.

[1] If $\rho(\lambda)$ denotes the oblique ascension of $\lambda$ for $\varphi = 36°$ then $\Delta\alpha_0 = \rho(\lambda_{☾} + 180) - \rho(\lambda_{\odot} + 180)$.

## Chapter 23. Determination of the Sine[1] from the Arc and Conversely[2]

It is now in order to know what the *elgeib* (sine) is, both plane and *diminutum*.[3] The plane sine for any place is to be found as follows: one enters[4] with the given argument in the column of sines[5] and notes down what is associated with it. If the

[1] Always called *elgeib*.
[2] Danish translation of this chapter in Bjørnbo [2] p. 4 ff.
[3] I.e. Sin $\alpha$ and Sin vers $\alpha$ = 60 – Cos $\alpha$. For this terminology cf. Suter p. 69.
[4] Tab. 58 and 58a.
[5] Sic; error for column of numbers.

argument contains also minutes one enters (the table) a second time, having added one degree to the degree of the given argument. Taking into consideration how the second sine is related to the first, it will be evident how much corresponds to these minutes. Thus, if the second sine is greater than the first, one has to add to the first the amount that corresponds to these minutes; if, however, less, the same amount has to be subtracted from it. In this way will be found the sine for which you ask.

If you wish to find the *elgeib diminutum* (i.e. the sinus versus) and if the argument for the entry is less than ninety degrees, it is to be subtracted from ninety and the sine of the remainder is to be taken and then subtracted from sixty. What remains is what you ask. But if the original argument is greater than ninety degrees, the appropriate plane sine is given for these ninety,[6] and also what belongs to the rest;[7] then this is to be added to sixty and the corresponding total is called *elgeib elmankuz* (sinus versus).[8]

In a similar fashion one goes from the sine to the arc by the inverse procedure, as is remarked in the following table.[9]

Whoever desires to obtain skill in the field of astronomy should grasp this treatise with the application of his whole mind. On this, indeed, depends rightly not only the addition and subtraction of the correction[10] but also the determination of the longitudes themselves. If anyone questions why to given arguments such sines are assigned and conversely, he should know that he can get the reason for it from the Almagest of Ptolemy.

[6] Namely Sin $90° = 60$.
[7] Namely Sin $\alpha'$ if $\alpha = 90 + \alpha'$.
[8] Thus Sin vers $\alpha = 60 + \text{Sin } \alpha'$. For the Arabic term cf. Suter p. 69.
[9] No such special table exists in our text; cf. Suter p. 71. A similar reference is found in ch. 25 (below p. 47).
[10] I.e. the determination of the positive or negative corrections (equation of center, etc.).

### Commentary to Chapter 23

The rules for finding the versed sine can be formulated as follows

$$\text{for } \alpha < 90: \quad \text{Sin vers } \alpha = 60 - \text{Sin } (90 - \alpha)$$
$$\text{for } \alpha > 90: \quad \text{Sin vers } \alpha = 60 + \text{Sin } (\alpha - 90)$$

or, in both cases

$$\text{Sin vers } \alpha = R - \text{Cos } \alpha \qquad R = 60.$$

The reference to the Almagest is reminiscent of the end of the introduction (cf. p. 10). The remark that the sine "for any place" (*cuiuslibet loci*) can be found according to the rules given here, probably refers to the sine of longitudes of celestial bodies. This is confirmed by the statemant at the end about the usefulness of these tables for the computation of longitudes. Such a statement seems to be much more appropriate for Hindu treatises than for the present work in which most corrections are tabulated for ready use.

At the end of this chapter the manuscript M gives a figure for the "*sinus plani*" and the "*sinus diminuti*" in a circle whose circumference is divided into 12 signs. The value given for Sin 60° agrees with Tab. 58. For the details cf. Suter p. 70f.

## Chapter 24. How to Find the Latitude of Any Locality

So much let it be written about arguments and the sines which are associated with them. Now it seems in order to discuss how the latitude of the single regions can be found, i.e. their distance from Arin.[1]

If the sun is located in the first degree of Aries or Libra, its *artifa*, i.e. altitude, which reaches a maximum at noon for the given place, must be subtracted from ninety degrees. The remainder is then necessarily the latitude of the given place.

If, however, the sun is in some other sign, its altitude at noon, minus the sun's declination[2] if it is toward north, but plus (the sun's declination) if it is southerly, must be subtracted from ninety. The result of this subtraction indicates the latitude of the place.

The same can furthermore be established by means of the *elmahwar*, i.e. the pole. That is, one observes with the astrolabe the amount of the altitude of the pole at the given locality, and the same is necessarily the latitude of this place.

And the altitude of the pole can (also) be obtained as follows. For any of the stars which never set, its *elmuhit*, i.e. circumference,[3] must be observed(?) by investigating its greatest and smallest altitude. Its midpoint, obtained in this way, is obviously the pole.

[1] This gloss is not found in O, which has, instead, the following marginal note (Suter p. 18 note 22) which is partially incomprehensible to me: "Note: the altitude and the latitude of the climate and the shadow being equal(?), while I determined(?) the longitude of the sun from(?) an eclipse ...; longitude, however, (is reckoned) from the eastern or western circumference to the place (in question), latitude to the place from Arin, i.e. from the center of the sphere of the earth".

[2] *obliquatio*.

[3] The orbit of its daily rotation.

## Chapter 25. The Horoscopes of the Signs at Sphaera Recta[1]

Here it is necessary to remind the reader that there exist several regions which have no latitude – but countless ones which do have latitude – and that we therefore must consider (also) the rising of the signs for a place which has no latitude. In those regions, namely those which do have latitude, their risings and settings are found to be different at different times. Those (signs) which rise faster set slower and conversely. In the regions, however, which have no latitude, as in the region of Arin, where the days are always equal, their rising is uniform at all times. But for every region they rise in the meridian[2] as in Arin.

[1] *Elfelek elmustakim*.

[2] We would say: their transits are the same as the risings in Arin.

If one wants to know the risings of the signs one must note down the sine of the sun's declination with respect to the equator – which amounts to 23 degrees 51 minutes – called first sine. Then one subtracts from 90 the *elmeil*, i.e. the declination, and one has to find with the remainder as argument the sine – called second sine. Thereafter one must take as argument the declination of the last degree of the sign under consideration and find its sine, which is called third sine. Finally one subtracts the declination of this last degree from 90 and finds with the remainder as argument its sine, called fourth sine. This being done, the third sine is to be multiplied by the second sine and the product divided by the first sine; the result must be multiplied by sixty. This result then is to be divided by the fourth sine. And the result thus obtained is to be reverted to its arc in the way mentioned above.[3] This arc gives the amount of the rising of the whole sign,[4] e.g. of Aries, to the equator.[5]

Similarly one has to proceed for the other (signs). This, however, under the strict precaution that, if one asks for an arc beyond (the beginning of) Taurus, the amount which belongs to Aries is to be subtracted from the latter arc, which was found according to the above-given rules; the remainder equals the rising of Taurus. Similarly, if one is dealing with the rising of Gemini, one must subtract what belongs to Aries and Taurus from the latter arc, which was found according to the above rules; the remainder represents the rising of Gemini.

After (the rising) for the three signs has been determined in this way also the rising for the remaining ones are obvious. Pisces corresponds to Aries, and Libra to Virgo, Leo to Taurus and Scorpius to Aquarius; finally Cancer to Gemini and Capricorn to Sagittarius. All these, however, are listed for easier use in the following tables for sphaera recta.[6]

These (tables) are to be entered as follows: beginning with Capricorn, enter the table for the rising of any degree of whatever sign that is given as heading for the equinoctial circle, i.e. *elmustakim*. The corresponding number shows immediately the rising point[7] of the zodiac. [In O: the place of the rising of the sign in the equator.]

[3] Chapter 23, though only mentioned in passing (p. 45 and note 9 there).
[4] *ipsius signi*.
[5] I.e. in equatorial degrees.
[6] Tab. 59 to 59b.
[7] *horoscopum*.

### Commentary to Chapter 25

Let $\alpha_1$ denote the rising time of the zodiacal sign ♈ for sphaera recta ($\varphi = 0°$), $\alpha_2$ for ♉, etc., all $\alpha$'s measured in degrees. Chapter 25 gives the rules for computing these "right ascensions", in particular for ♈, ♉, and ♊. The remaining values are then also known because of symmetries.

For the computation four "sines" are required:

"1st sine": Sin $\varepsilon$ $\varepsilon = 23;51$ $R = 60$
"2nd sine": Sin $(90 - \varepsilon)$

"3rd sine": Sin δ
"4th sine": Sin (90 – δ)

where ε denotes the obliquity of the ecliptic, δ the declination of the sun[1] at longitude λ. Then the formula of the text for the right ascension α, corresponding to the longitude λ is[2]

$$\text{Sin}\,\alpha = \frac{\text{Sin}\,\delta \cdot \text{Sin}\,(90-\varepsilon)}{\text{Sin}\,\varepsilon} \cdot 60 \cdot \frac{1}{\text{Sin}\,(90-\delta)} \qquad (1\,a)$$

or in modern notation

$$\text{Sin}\,\alpha = R\frac{\tan\delta}{\tan\varepsilon} \qquad (1\,b)$$

which is correct. Concerning the value ε = 23;51 cf. the commentary to the table for the solar declination (p. 96) and to ch. 26 and 26a (p. 54).

These rules for the computation of the right ascensions of the twelve zodiacal signs are followed at the end by rules for the use of Tables 59 to 59b, which, however, do not belong to the previous text but follow the principle of the tables for sphaera recta in the "Handy Tables".[3] Not only do we find there tables proceeding in steps of single degrees but what is tabulated is not the right ascension α (λ) which corresponds to an ecliptic point of longitude λ, but α′ (λ) = α (λ) + 90°. This procedure has the advantage that it is very easy to determine from these tables for any geographical latitude the rising point H of the ecliptic (the "ascendant" or "horoscope") if the longitude $\lambda_M$ of the culminating point M is known, because the oblique ascension ρ ($\lambda_H$) is always equal to

$$\rho(\lambda_H) = \alpha(\lambda_M) + 90 = \alpha'(\lambda_M). \qquad (2)$$

Consequently the tables 59 to 59b give for every culminating degree the corresponding ρ ($\lambda_H$) of the ascendant. With this value one has to enter the table of oblique ascension for the given geographical latitude and find which argument $\lambda_H$ is associated with ρ ($\lambda_H$). In the text of ch. 25 no mention is made of the necessity of going to the tables of oblique ascensions, probably because such tables are missing in the present work. Therefore the version O replaces the reference to the "rising point of the zodiac" with the correct, but trivial, reference to the rising point of the equator.

[1] Tabulated in Tab. 21 to 26, col. 5.
[2] The text considers only λ = 30°, 60°, 90°.
[3] Halma HT I, p. 148 to 155.

## Chapter 26. With how many Degrees any Sign rises in Whatever Region

From these (matters) so far dealt with, we may naturally go on to something else, namely, to the discussion of how to find the amount of the rising(-time) of the (zodiacal) signs for an arbitrary region; for example first for Aries.

When the sun is located in the first degree of Aries one has accurately to measure the noon shadow of a given object, how many digits it contains of the 12 equal digits into which the object is divided. The number of these digits (contained in the length of the shadow) is to be multiplied by 114 and the product divided by 115. The single term (i.e. the result) of this division will be the *argumentum* for Aries. This *argumentum* is to be subtracted from the number of rising(-time) of Aries for the equator (i.e. for sphaera recta), or it is to be added to it; the difference represents the rising(-time) of Aries and Pisces, the sum, however, the rising(-time) of Virgo and Libra.

Similarly for Taurus. The noon-shadow of the sun, when at the beginning of Aries,[1] is to be multiplied by 113 and the product divided by 116.[2] What results from these single steps[3] will be the *argumentum* for Taurus. This *argumentum* is first to be subtracted from the rising(-time) of Taurus for (sphaera) recta,[4] then added to it. The remainder will indicate the rising(-time) of Taurus and Aquarius, the sum the rising(-time) of Leo and Scorpius.

Similarly for Gemini. The above-mentioned shadow for Aries should be divided by three and this third of it will be the *argumentum* for Gemini. This *argumentum* is first to be subtracted from the right ascension of Gemini, then added to it. The remainder gives the rising(-time) of Gemini and Capricorn, the total the rising(-time) of Cancer and Sagittarius.

[1] *Race alhamel.*
[2] For these numbers cf. the commentary (p. 52).
[3] *Quot autem singulis excidet* must mean the final result. Cf. in the case of Aries: *Huius autem partitionis unum membrum.*
[4] *Bilmustakim.*

## Chapter 26a. On the Rising of the Signs, in Whatever Region, with Respect to the Equator[1]

In order that you know for any region with non-vanishing latitude the amount of the difference of rising(-time) of the (zodiacal) signs with respect to the equator a table for its determination is added below,[2] the use of which is as follows: first the (noon-)shadow (when the sun is at the beginning) of Aries has to be noted down for the given region. Then one has to enter with the given degree (of longitude) in the table of (ascensional) differences and then one has to multiply the corresponding number with the above-mentioned shadow (-length), and (finally) one has to find the arc of this product.[3] This will be the arc which represents the diminution for Aries with respect to the equator for the given degree (of longitude); similarly also for Pisces; for Libra and Virgo, however, (it represents) the increment (with respect to the case of sphaera recta). Taurus and Aquarius, however, follow Aries and

[1] Only in MS O and separated from ch. 26 by ch. 27; cf. Suter p. 76.
[2] This table is missing in the extant texts; cf. the commentary.
[3] I.e. arc sin $p$ if $p$ represents the product mentioned.

Pisces in diminution, but Scorpius and Leo, again differently, (follow) Libra and Virgo with increments. Gemini and Capricorn, on the contrary, follow Taurus and Aquarius in diminution, but Cancer and Sagittarius (follow) Scorpio and Leo, but differently, with increments. Just as Aries gives (of) its time to Libra, so the five following (signs) give to their opposite (signs) what they can not keep.

## Commentary to Chapter 26 and 26a

### 1. The "Ascensional Difference"

Before discussing al-Khwārizmī's text we shall first define the concept "ascensional differences" and indicate methods for their computation, first by following essentially the Sūrya-Siddhānta II,60 to 63,[1] then Ptolemy.

Let in fig. 19a denote V the vernal point, E the east point of the equator, H the rising point of the ecliptic. Then $VH = \lambda$ is the longitude of the rising point, $VF = \alpha(\lambda)$ its right ascension, and $VE = \rho(\lambda)$ its oblique ascension. Then

$$\gamma = \alpha - \rho \tag{1}$$

is called the "ascensional difference"[2] of the point of longitude $\lambda$, at a geographical latitude $\varphi$.

In order to compute $\gamma$ we consider the parallel circle HGM (fig. 19a, b, c) to the equator. It represents the parallel circle in which the sun moves during a day when its longitude is $\lambda$. The radius $r_d$ of this circle is called in Hindu astronomy the "day radius" and is given by

$$r_d = R \cos\delta = \mathrm{Cos}\,\delta \tag{2}$$

if we assign to the celestial sphere as well as to the unit-circle of the trigonometric functions the radius $R$. Looking at the circle of radius $r_d$ (fig. 19c) we express the ascensional difference also by the arc HG because HG appears from B under the same angle as EF appears from O in the plane of the equator. Consequently

$$\sin\gamma = \frac{e}{r_d} \qquad \text{or} \qquad \mathrm{Sin}\,\gamma = \frac{eR}{r_d} \tag{3}$$

and (fig. 19b)

$$e = \mathrm{OB}\tan\varphi = R\sin\delta\tan\varphi = \mathrm{Sin}\,\delta\tan\varphi. \tag{4}$$

At equinox the sun is at noon in K. Thus $s_0 = \mathrm{OL}$ is the equinoctial noon shadow of a gnomon of height $g = \mathrm{LK}$ and thus

$$\tan\varphi = \frac{s_0}{g} \tag{5}$$

and finally from (3)

$$\mathrm{Sin}\,\gamma = s_0 \frac{R}{g}\tan\delta. \tag{6}$$

[1] Cf. for this problem in Hindu astronomy Schmidt [1].
[2] Bīrūnī calls this quantity the "equation of daylight"; cf. Lesley [1] p. 123.

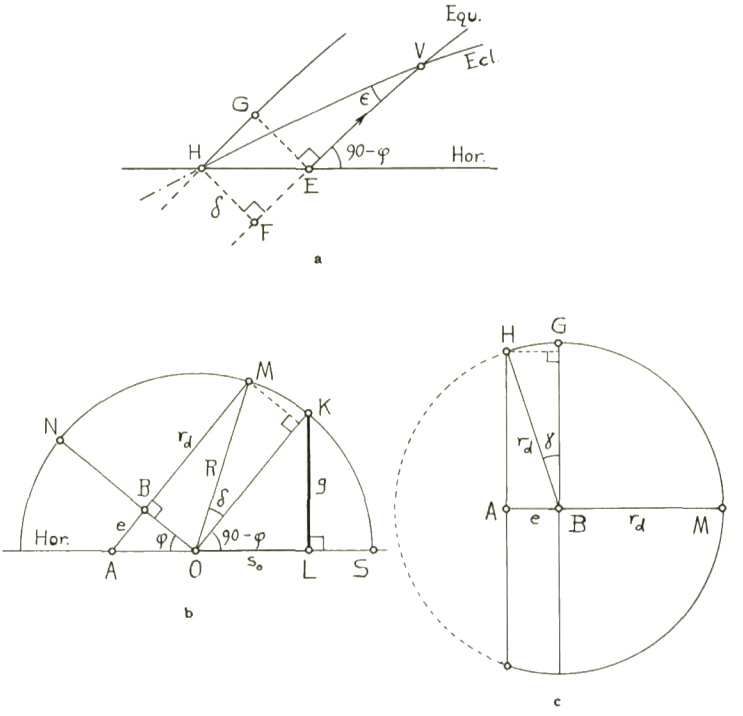


Fig. 19.

Here $s_0$ depends on the geographical latitude whereas the term

$$c = \frac{R}{g} \tan \delta \tag{7}$$

is a function only of $\lambda$ and can therefore be computed for all climata alike.[3] Multiplication by the equinoctial noon shadow then produces the ascensional differences, and thus the oblique ascensions, for a given clima.

Ptolemy, in the Almagest (II,7), finds the ascensional difference from spherical triangles which lead to a relation which is the equivalent of

$$\frac{\sin \gamma}{\cos \frac{m}{2}} = \frac{\tan \delta}{\tan \varepsilon} \tag{8a}$$

where $m$ denotes the length of the shortest day, expressed in degrees. Thus

$$\sin \gamma = c' \cos \frac{m}{2} \qquad c' = \frac{\tan \delta}{\tan \varepsilon} \tag{8b}$$

where $c'$ is independent of $\varphi$.

[3] Because $\sin \delta = \sin \varepsilon \cdot \sin \lambda$ such tables differ according to the value of $\varepsilon$ chosen; cf. Lesley [1] p. 127.

If we assume that Ptolemy used for the inclination of the ecliptic a value $\varepsilon'$ which is slightly different from the value used in (7) and that correspondingly $\delta'$ is the declination resulting from Ptolemy's $\varepsilon'$ as compared with $\delta$ in (7) for the same value of $\lambda$, then we find from (7) and (8) that

$$\frac{c}{c'} = \frac{R}{g} \tan \varepsilon' \frac{\tan \delta}{\tan \delta'}. \tag{9}$$

Within the accuracy of extant tables we have very nearly

$$\tan \varepsilon / \tan \varepsilon' \approx \tan \delta / \tan \delta'$$

and thus from (9):

$$\frac{c}{c'} \approx \frac{R}{g} \tan \varepsilon \tag{10}$$

for all values of $\lambda$.

**2. Al-Khwārizmī's Procedure from Ch. 26**

The rules given in ch. 26 can be formulated as follows: let $\sigma_1, \sigma_2, \ldots$ denote the rising times of sphaera recta for the single zodiacal signs, beginning with $\Upsilon$, and similarly $\tau_1, \tau_2, \ldots$ the corresponding rising times for a geographical latitude at which a gnomen of height $g = 12$ casts at noon of equinox a shadow of length $s_0$ ($= 12 \tan \varphi$), measured in the same units. The rising times $\sigma_1, \sigma_2, \sigma_3$ for sphaera recta are supposed to be known. Then one finds the rising times for the given locality from

$$\left.\begin{aligned} \tau_1 = \tau_{12} &= \sigma_1 - \kappa_1 s_0 & \qquad \tau_6 = \tau_7 &= \sigma_1 + \kappa_1 s_0 \\ \tau_2 = \tau_{11} &= \sigma_2 - \kappa_2 s_0 & \qquad \tau_5 = \tau_8 &= \sigma_2 + \kappa_2 s_0 \\ \tau_3 = \tau_{10} &= \sigma_3 - \kappa_3 s_0 & \qquad \tau_4 = \tau_9 &= \sigma_3 + \kappa_3 s_0 \end{aligned}\right\} \tag{11a}$$

with coefficients (called *argumenta*)

$$\kappa_1 = \frac{114}{115} \qquad \kappa_2 = \frac{113}{116} \qquad \kappa_3 = \frac{1}{3}. \tag{11b}$$

The symmetry relations of (11a) are correct. In order to explain the numerical values in (11b) one has only to remark that the rising times of the $i$th zodiacal sign can be expressed as difference $\Delta_i$ between the oblique ascension of its two end-points, thus, using (1) of p. 50

$$\tau_i = \Delta_i \rho = \Delta_i \alpha - \Delta_i \gamma = \sigma_i - \Delta_i \gamma \qquad (i = 1, 2, 3) \tag{12}$$

where $\sigma_i$ represents the rising time at sphaera recta of the $i$th sign. We know furthermore that

$$\operatorname{Sin} \gamma = s_0 \cdot \frac{R}{g} \tan \delta \qquad \operatorname{Sin} \gamma = R \sin \gamma. \tag{6}$$

If we assume that approximately

$$\gamma \approx \sin \gamma \tag{13}$$

we have for $\gamma$, measured in degrees,

$$\gamma^\circ \approx \frac{180}{\pi} \frac{\tan \delta}{g} s_0$$

and thus

$$\Delta_i \gamma = \frac{180}{\pi g} \cdot \Delta_i \tan \delta \cdot s_0. \tag{14}$$

Comparison of (12) and (14) with (11) shows that we should have

$$\kappa_i = \frac{180}{\pi g} \cdot \Delta_i \tan \delta. \tag{15}$$

To check these relations numerically, we take the values of the declination $\delta$ from Tab. 21 to 23, col. 5, which are based on the value $\varepsilon = 23;51$.[4] For $180/\pi$ we use the coefficient which is known from Hindu astronomy, namely $57;18$[5]; finally $g = 12$ gives

$$\kappa_i = 4;46,30 \cdot \Delta_i \tan \delta. \tag{16}$$

In this way we obtain the following table

| $\lambda$ | $\delta$ | $\tan \delta$[6] | $\Delta \tan \delta$ | $4;46,30 \cdot \Delta \tan \delta$ | $\kappa_i$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | | | |
| 30 | 11;39,50 | 0;12,23, 9 | 0;12,23, 9 | 0;59,13 | $114/115 \approx 0;59,29$ |
| 60 | 20;29,50 | 0;22,25,47 | 0;10, 2,38 | 0;47,58 | $113/116 \approx 0;58$(!) [$93/116 \approx 0;48,6$] |
| 90 | 23;51, 0 | 0;26,31,33 | 0; 4, 5,46 | 0;19,34 | $1/3 = 0;20$ |

The agreement for $\kappa_1$ and $\kappa_2$ is good. For $\kappa_2$ we must assume a scribal error, e.g. CXIII instead of XCIII.[7] It is clear that one can expect better agreement only for larger values of $\varepsilon$. Indeed, if we use the declinations $\delta$ from the Khaṇḍa-Khādyaka I,29 which are based on $\varepsilon = 24$ one obtains

$$\kappa_1 \approx 0;59,25 \qquad \kappa_2 \approx 0;48,28 \qquad \kappa_3 \approx 0;19,41$$

which come still closer to the values in the text. The main source of uncertainty lies, of course, in the values for $\tan \delta$ which were obtained by division from a table of sines and not directly.

### 3. Al-Khwārizmī's Procedure from Ch. 26a

The method of ch. 26 is based on the approximation $\gamma \approx \sin \gamma$, a procedure well attested in Hindu astronomy.[8] Ch. 26a, however, operates with the accurate relation

$$\gamma = \text{arc Sin}\,(s_0 \cdot c) \tag{6}$$

[4] Cf. p. 96.
[5] Sūrya-Siddh. II,22. The corresponding approximation of $\pi$ is 3;8,28,53, ... (instead of 3;8,29,44, ...).
[6] From Ulūgh Beg's Tables (Schoy TLB p. 92ff.).
[7] Suter p. 75 suggests $103/126$ ($\approx 0;49,3$).
[8] Cf. Schmidt [1] p. 206.

where $c = c(\lambda)$ is to be found in a table which is not extant in the present text of al-Khwārizmī. It still existed, however, in the time of al-Bīrūnī, who describes it under the title "Differences of the ascensions for the (whole) earth".[9]

It is assumed in ch. 26a that $\gamma$ is determined by means of (6). The rising times of the signs for sphaera obliqua are then found [10] from the rising times of sphaera recta by formulae of the same type as (11a):

$$\tau_i = \sigma_i \pm \Delta_i \gamma. \tag{17}$$

For us the problem consists in the reconstruction of the missing table for $c(\lambda)$. The type of such tables is known to us from a "Table of ascensional differences for the whole earth"[11] (because $c(\lambda)$ is independent of $\varphi$) written in Newminster in 1428. These tables were computed for the parameters $R = 150$ and $\varepsilon = 23;51$ as has been shown by M. Lesley.[12] Since both these values are known as being used in al-Khwārizmī's tables,[13] Lesley suggested that the Newminster table actually represents the table referred to in ch. 26a. The same table also occurs in the "Toledan Tables" cod. Vindob. 2385[14] fol. 4$^v$, preceded by tables of sines and declinations based on $R = 150$ $\varepsilon = 23;51$, whereas the Newminster tables use for the declinations $\varepsilon = 23;33,30$. A "revised" table of declinations, based on this parameter, is also given in the Toledan Tables, with references either to al-Zarqalī or Jahib (= "Almeon"), the son of Abū Ma'shar.[15] Without the complete publication of the main astronomical tables nothing certain can be said about the relationship between these works.

It should finally be remarked that Hugo Sanctallensis (12th cent.) in his translation of an anonymus commentary to al-Khwārizmī[16] refers to a table called by al-Khwārizmī *aut si quis alius huius tabulae iudicatur auctor* "or whoever else is considered to be its author"[17] *residua orientia ad quamlibet regionem* "rising-time-differences for whatever location". Unfortunately the tables themselves are not preserved, but the following three values are quoted[18]

[9] Lesley [1] p. 125, quoting from Bīrūnī's *Risā'il* 129,1 to 130,18.

[10] The formulation in al-Khwārizmī's text is very condensed, but it is clear from ch. 26 that the ascensional differences $\Delta_i \gamma$ must be formed as $\gamma(\lambda + 30) - \gamma(\lambda)$.

[11] *Tabula differencie ascensionis universe terre* in MS Ashmole 191 fol. 137$^v$, published by Neugebauer-Schmidt [1] p. 226. The value 5,30,30 given in this publication for $\lambda = 86$ is my scribal error and should be read 5,30,36.

[12] Lesley [1] p. 127, using the above relation (10) p. 52. In the original paper it was incorrectly assumed that $\varepsilon = 23;33,30$ of the table of declinations (fol. 138$^r$) was also valid for the computation of $c(\lambda)$.

[13] Cf. the remarks to Tab. 58 and 58a (p. 104) and to Tab. 21 to 26, cols. 3 to 5 (p. 95 f.).

[14] Unpublished but described in some detail by Zinner [1].

[15] Zinner [1] p. 767 and p. 762 (Tafel 136). Cf. also Delambre HAMA p. 176 or Curtze [1] p. 348.

[16] Cf. for this work Haskins, SHMS p. 73f. Haskins accepted Suter's erroneous view that we have here a commentary by al-Bīrūnī "On the reasons of the tables of al-Khwārizmī". Actually the original text is a commentary by al-Muṭannā (10th cent.) on a commentary, perhaps by al-Farghānī (9th cent.), on a *zīj* of al-Khwārizmī. Muṭannā's commentary was translated into Hebrew by Ibn Ezra (about 1160), extant in one complete copy in Oxford (Bodl., Mich. 400) and in one incomplete copy in Parma (Bibl. Palatina 2636, olim De Rossi 212). A Latin translation by Hugo Sanctallensis (12th cent.) is extant in three MSS (Bodl., Arch Seld. B 34 and Savile 15, Cambridge, Gonville and Caius 456). These relations were first clearly recogniced by Millás-Vallicrosa in 1936 (cf. Bibl. Cat. Tol. p. 192).

[17] fol. 35$^r$,20f.; 35$^v$,7.

[18] fol. 35$^r$,27; 30; 24 respectively.

| | | |
|---|---|---|
| $\lambda = 30$; | 2;33,49 | (Newminster: 2;34,49) |
| $\lambda = 60$: | 4;40,20 | (Newminster: 4;40,23) |
| $\lambda = 90$: | 5;31,50 | (Newminster: 5;31,34). |

The last value would lead to an $\varepsilon = 23;53$ which hardly seems significant. The two preceding values can be easily explained as scribal errors. Thus the Newminster tables seem indeed to represent al-Khwārizmī's tables.

## Chapter 27. The Length of the Hour for Whatever Day and Whatever Region

Now it must be explained in what way one has to investigate the length of an hour for any given day and any given locality. After the arc of the given day is divided by 12, this 12th is the answer to the question.

For the night as follows: After one has subtracted the arc of the day from 360° and divided the remainder by 12, this 12th will measure the amount of one nightly hour.

### Commentary to Chapter 27

We are dealing here with seasonal hours measured in degrees, i.e., with twelfths of the arc the sun is above, respectively below the horizon. The text does not say how this arc is to be found, namely as the oblique ascension of the semicircle from $\lambda_\odot$ to $\lambda_\odot + 180°$, where $\lambda_\odot$ is the solar longitude for the day in question. Apparently this chapter is out of place here.

## Chapter 28. How One Can Determine from the Altitude[1] of the Sun the Plane[2] Shadow of Any Object[3]

With the *artifa*, i.e. the altitude of the sun on the (given) day, one enters the (table of) sines,[4] then one subtracts the same altitude from 90 and enters with the remainder again the (table of) sines. This second sine is to be multiplied by 12 and the result[5] is to be divided by the first sine. In case there are minutes present they should be multiplied by 60 and the results from this division is said to be equal to the number of digits contained in the shadow of the object, whenever the object (itself) is divided in 12 digits.

[1] *artifa.*
[2] *elmustewia.*
[3] Danish translation of this chapter in Bjørnbo [2] p. 6ff.
[4] Tab. 58 and 58a.
[5] *tota summa.*

## Chapter 28a. How One Can Determine from the Shadow the Altitude[1] of the Sun[2]

If you look for the altitude (of the sun) from the shadow, the (length of the) shadow is to be multiplied by itself and to the product is to be added 144; of the total one has to note down the *elgidher*, i.e. the side of the square, and this *elgidher* will be the diameter of the shadow. Then the (length of the) shadow itself is to be multiplied by 60 and the product is to be divided by the above-mentioned diameter, and the arc to the quotient has to be found (from the table of sines); the arc found is to be subtracted from 90. The remainder indicates the altitude you wish to find.

[1] *artifa.*
[2] Danish translation of this chapter in Bjørnbo [2] p. 8f.

## Chapter 28b. How One Can Determine from the Altitude[1] of the Sun the Inverse[2] (Shadow)[3]

You enter with the altitude (of the sun) the column of sines.[4] Then you subtract the same altitude from 90 and with the remainder enter again the column of sines. Then the first sine is to be multiplied by 12, and the product is to be divided by the second sine. In case there are minutes present, they should be multiplied by 60 and the result divided as said before. Thus we will have in the quotient digits and minutes which represent the inverse[2] shadow.

The entry in the tables of shadows given below[5] is as follows: if you enter this table with any solar altitude, then the corresponding (number) indicates the shadow.

[1] *artifa.*
[2] *elmankuz.*
[3] Danish translation of this chapter in Bjørnbo [2] p. 9.
[4] Tab. 58 and 58a.
[5] Tab. 60.

### Commentary to Chapters 28, 28a, and 28b

If the height of a vertical gnomon is 12 digits, the altitude of the sun $h$ degrees, and the length of the shadow $s$ digits (cf. fig. 20), then the following relations are used:

in chapter 28:

$$s = 12\ \mathrm{Sin}(90 - h)/\mathrm{Sin}\ h = 12\ \cot h,$$

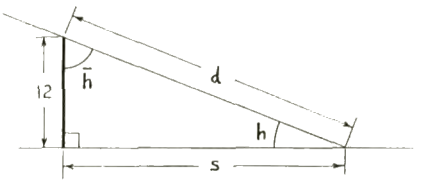


Fig. 20.

in chapter 28a:

$$d = \sqrt{s^2 + 12^2} \qquad \hbar = \text{arc Sin}\frac{s \cdot R}{d} \quad (R = 60) \qquad h = 90 - \hbar$$

in chapter 28b the "inverse shadow" $\bar{s}$ is introduced

$$\bar{s} = 12 \text{ Sin } h/\text{Sin}(90 - h) = 12 \text{ tan } h$$

which has no direct geometric significance[1] but is the cofunction of the "plane shadow" $s$.

No mention is made in ch. 28 that $s$ is tabulated as function of $h$ in Tab. 60. A reference to this table is made, however, in the final paragraph of ch. 28b, which is probably a later addition (together with Tab. 60) since the rules of computation are usually not given when tables are extant.

[1] If one, however, replaces the solar altitude $h$ by the geographical latitude $\varphi$, then 12 tan $\varphi = s_0$ is the equinoctial noon shadow (cf. ch. 26).

## Chapter 29. On the *elbuht* (i. e. the True Motion) and How to Find it

We now must talk about the *elbuht*, which, in literal translation, may be called *stupor*.[1] *Elbuht* is called the segment of the circle which a given planet traverses in a given time. This can be found in the following manner. Take the true place[2] of the given planet at the present day and again at the following day, and when you determine the difference between these two places it will give you the amount of the *elbuht* of the planet corresponding to the time of one day. And similarly the velocity[3] for one hour or for one minute can be found.

[1] *stupor*, i.e. dulness, stupidity, etc. I do not know what the basis is for this (incorrect) etymology. Suter's explanation (p. 80) as "hesitation, to stop in the movement" for a term representing velocity is not convincing. In fact the term *elbuht* comes from Sanskrit *bhukti*, lit. enjoyment. Cf. Burgess note to Sūrya-Siddh. I,27 and in particular Bīrūnī, India II p. 195 trsl. Sachau.

[2] *locus examinatus*, i.e. the "equated", not the mean position.

[3] *stupor*.

## Chapter 30. On the Size of the Solar Disk

We have now to examine the size of the solar disk. The velocity[1] for one day must be multiplied by 33 minutes; the result is the answer to your question. If the question, however, concerns one hour, the hourly motion of the sun is to be multiplied by 13 and one fifth. In this way you will find what you wish.

[1] *stupor*; similarly in the following. Cf. ch. 29 note 1.

### Commentary to Chapter 30

If $d_\odot$ denotes the apparent diameter of the solar disk, $v_\odot$ the solar velocity then we can express the rule given in ch. 30 in the form

$$d_\odot = 0;33\ v_\odot{}^{\circ/\mathrm{d}} = 13;12\ v_\odot{}^{\circ/\mathrm{h}} \qquad (1)$$

the latter equation being correct because $0;33 \cdot 24 = 13;12$.

From Tab. 61 to 66 we find[1] as mean motion of the sun $0;59,12^{\circ/\mathrm{d}}$ which would lead on the basis of (1) to a mean diameter of $0;32,33,36^\circ$ in agreement with the value $0;32,34^\circ$ obtainable from Tab. 61 to 66 and Tab. 78.[2]

The relation (1) also occurs in Hugo Sanctallensis' commentary[3] as well as in the Khaṇḍa-Khādyaka I,31 in the form

$$d_\odot = \frac{11}{20}\, v_\odot{}^{\circ/\mathrm{d}}.$$

Cf. also the commentary to ch. 30a.

[1] Cf. p. 105.
[2] Cf. p. 105 and p. 127.
[3] Se fol. 44r, 24f.

## Chapter 30a. On the Size of the Lunar Disk

The size of the lunar disk can be found as follows. By multiplying the daily velocity (of the moon) by 2 minutes 16[1] seconds, you will find what you are asking; and you will get what you wish (also) by multiplying the hourly velocity by 58 minutes and 10 seconds.

The entry in the table given below[2] is the following one: in order to know how much of the zodiac sun or moon traverse, at a given position, you enter with the anomaly[3] of either one of them the column of numbers[4]; the adjoining number[5] indicates the hourly motion. In the same way one ascertains the half (diameter) of the disk of sun or moon,[6] and also of the shadow.[7]

[1] sic! error for 26; cf. the commentary.
[2] Tab. 61 to 66.
[3] *argumentum*, as in ch. 8 (cf. p. 19).
[4] Cols. 1 and 2.
[5] Col. 3 for the sun, col. 4 for the moon.
[6] Cols. 5 and 6 respectively.
[7] *draconis*, i.e. the shadow at the ascending node; Tab. 61 to 66 cols. 7 and 8.

### Commentary to Chapter 30a

As in the case of the apparent diameter of the sun, also the lunar diameter can be related to the velocity because an increase of geocentric distance means a decrease in velocity as well as in apparent diameter. In the case of the moon the rule of the text is

$$d_{☾} = 0;2,16 \cdot v_{☾}^{o/d} = 0;58,10 \cdot v_{☾}^{o/h}.$$

These numbers are, however, incompatible with each other because $0;2,16 \cdot 24 = 0;54,24$ or $0;58,10/24 = 0;2,25,25$. That at least the coefficient 0;2,16 is incorrect follows from the fact that a mean lunar velocity of $13;10,35^{o/d}$ would lead to a mean diameter of $0;29,52°$ which is too small. Thus one has to read either 0;2,26 or 0;2,25[1] though one then obtains for the second coefficient either 0;58,0 or 0;58,24 but not 0;58,10. On the other hand 0;58,10 is supported by the tabulated values in Tab. 61 to 66, cols. 4 and 6 (cf. below p. 106).

The commentary of Hugo Sanctallensis[2] and the Khaṇḍa-Khādyaka I,31 which both agreed exactly with al-Khwārizmī's rule in the case of the solar diameter give here

$$d_{☾} = \frac{10}{247} v_{☾}^{o/d} \quad \text{or} \quad d_{☾} \approx 0;2,25,44,56, \ldots v_{☾}^{o/d} = 0;58,17,36, \ldots v_{☾}^{o/h}.$$

This speaks in favor of 0;2,26 for the first coefficient.

The rule for the computation of the lunar diameter is followed by a reference to Tab. 61 to 66 where one finds tabulated, as function of the anomaly, not only the hourly velocities but also the apparent radii of the sun, the moon, and the shadow. Such a detailed table makes the rules of computation in ch. 30 and 30a superfluous; perhaps this is an indication of a later addition of the tables, probably similar to the situation in ch. 28, 28a, 28b where the reference to the tables is only added at the end (cf. p. 56). It is perhaps also significant that no explicit rule is given as to how the diameter of the shadow should be computed from the two corresponding columns (7 and 8) in the tables.[3]

[1] The latter is suggested by Suter p. 79.
[2] Se fol. 44r,27 with fol. 45v,10ff.
[3] Cf. below p. 106 f.

## Chapter 31. On *elistima* and *elistikbel*, i. e. on Conjunction and Opposition

If one wishes to know conjunction or opposition of sun and moon by means of the tables of conjunction and opposition given below,[1] then the procedure is as follows.

One has to enter the said table with the accumulated completed Arabic years[2] nearest to the present year, and one must carefully note down the corresponding days, hours, and parts.[3] Similarly should be noted down separately the mean (position) of the sun and of the moon[4] and the anomaly of the moon[5] and the argument of latitude.[6] Secondly one must enter (the table[7]) with the single years, and thirdly

[1] Tab. 69 to 72.
[2] Tab. 69 for conjunctions, Tab. 70 for oppositions.
[3] Column 3 of Tab. 69 or 70.
[4] Column 4.
[5] Column 5.
[6] Column 6.
[7] Tab. 71.

with the month[8] under consideration. After we have reduced (if necessary)[9] smaller (units) to larger ones, we subtract from the accumulated total one day. The hours are to be reckoned from noon of the following day. According to these hours are also to be reckoned the mean conjunction and opposition of sun and moon.

He who wishes to determine this thing should proceed as follows. From the mean position of sun and moon and from the anomaly of the moon, given in the above mentioned table,[10] their true places can be found, namely, by subtracting the apogee from the mean (position)[11] and adding or subtracting the equation which corresponds to this anomaly, as has been shown near the beginning.[12] Here it must be remarked that when the anomaly of the moon requires the *tadil*, i.e. its equation, to be added to the mean position, then the same amount is also to be added to the argument of latitude – respectively subtracted if (the equation is) subtractive.

Then one has to return to the (thus found) places of sun and moon. In case their position is exactly the same,[13] you have what you want; if not, one has to find their distance by subtracting the smaller (longitude) from the greater and one-half of one-sixth of this interval has to be carefully retained in order to be added later on to either one of them.[14] Then, according to which (body) is ahead (of the other), the interval is called either "of the preceding sun" or "of the (preceding) moon". If it is of the sun, the above total[15] is to be added to the place of the moon, the half of the sixth, however, to the place of the sun[16]; if it is of the moon, these (quantities) are to be subtracted from these (places). In similar fashion the same total[17] is to be added to, or subtracted from, the argument of latitude. This, carried out, results either in exactly the same place of sun and moon in conjunction, or, if you are dealing with opposition, it produces for you the place of the sun, and the opposite place – namely adding 6 signs – will be (the place) of the moon. And also the definitive argument of latitude will be found in this way, which is useful for the determination of eclipses.

After the place (of the syzygy) has been found in this way, one must enter with the place of the sun[18] the table of velocity[19] and obtain from it the hourly motion of the moon. Comparing then the above-mentioned total[20] with the hourly velocity of the moon one must carefully compute how many hours correspond to it; if the inter-

[8] Tab. 72.

[9] Depending on Tab. 71 or 72, col. 3. Literally "whatever happens for the day to be subtracted", namely 0 or 1.

[10] Tab. 69 to 72, col. 5.

[11] This part of the rule concerns only the sun; the anomaly for the moon is found from the tables.

[12] Meaning ch. 8 for the sun, ch. 9 for the moon.

[13] Which would mean that the equations of sun and moon happened to be equal.

[14] Cf. the commentary (p. 63).

[15] Of the elongation $\Delta$ and $\Delta/12$.

[16] If $\lambda_1$ is the true longitude of the sun, found previously, $\lambda_2$ of the moon, and $\Delta = \lambda_1 - \lambda_2$, then one should form $\lambda_2 + \Delta + \Delta/12$ and $\lambda_1 + \Delta/12$ (both being equal to the longitude of the true conjunction).

[17] Namely $\Delta + \Delta/12$.

[18] These words are missing in O and enclosed in parentheses in C, obviously because they are an error for "with the anomaly of the moon".

[19] Tab. 61 to 66, col. 4.

[20] $\Delta + \Delta/12$.

val is (one) of the (preceding) sun, the result has to be added to the above amount of hours[21]; if (the interval is one) of the (preceding) moon, one has to subtract the result from it. In this way one will ascertain the hour of conjunction and opposition according (to the assumption) that(?) night and day taken together make equal hours.[22]

If you wish to find the anomaly of the moon after the addition of these hours (between mean and true syzygy), then these additional hours and their fractions must be multiplied by 32 minutes 40 thirds, and the product must be added to, or subtracted from, the first anomaly[23] as it was done with the hours. This, then, will be the definitive anomaly[24] of the moon at (the true) conjunction or opposition.

All this is said for the location of Cordoba. Therefore, if you observe the stars elsewhere, you must consider the longitudinal difference between your place and Cordoba, reckoning one hour for every 15 degrees of it. If, then, your place is to the east, the above-mentioned hours[25] must be subtracted[26] from the hours (for the syzygy) at Cordoba; if to the west, they must be added.[27]

In order to be able to reduce equal hours to unequal ones and conversely, we have given previously this table for the equation of time.[28] Its rule is as follows: for whatever degree of the sun in a given sign you enter, the corresponding amount (found in the table) must be subtracted from the given unequal days; thus one obtains the days of equal hours. But conversely from the equal ones the unequal ones are found by addition.

[Manuscript N, second hand:][29]

The true natural mean day is 24 hours (long), i.e., one rotation of 360 degrees and 59 minutes 5 seconds.[30] Thus one hour is 15° and 2′ and 28″.[31] But since one degree of the equator, according to which these (degrees) are defined, and (one degree) of the zodiac, in which the sun moves, do not cross the meridian in the same time, and since furthermore the sun moves sometimes faster, sometimes slower, with respect to the sky, it is necessary sometimes to add to, sometimes to subtract from, the natural mean day. Yet this table[32] is so constructed that one has always to subtract from the mean day, though sometimes more for the following day, sometimes less, than for the preceding day. But when more (is subtracted), then the corrected day will be less, from meridian to meridian, than the mean day; if, however, less (is subtracted), then (the interval from meridian to meridian) will be of necessity greater (than the mean day); because a larger subtraction detracts from the mean,

[21] I.e. to the time originally found for the mean syzygy.
[22] Probably meaning that the hours both of night and of daytime are hours not yet corrected for the equation of time; cf. below.
[23] At mean syzygy.
[24] *argumentum ratum*, lit. "calculated, established" etc.; parallel: *argumentum latitudinis definitum*.
[25] Corresponding to the difference in longitude.
[26] Error for added.
[27] Error for subtracted.
[28] Tables 67 and 68.
[29] Cf. Suter p. XIX,10.
[30] Error for 8 seconds; cf. also below, near the end of this scholion.
[31] Indeed 360;59,8/24 = 15;2,27,54.
[32] Tab. 67 and 68 for the equation of time.

while a smaller subtraction from the mean and equal (day) leaves a greater remainder than a larger subtraction would have done.

For example: if today 2′ and 8″ are subtracted from one rotation of the sphere and 59′ and 8″, but on the following day 3′ and 8″, then the corrected day would be one rotation of the sphere and 58′,[33] which is less than the mean day; similarly if on the previous day 3′ and 8″ are subtracted and on the following day only 2′ and 8″, then the corrected day will be one rotation of the sphere and 60′,[34] which is more than the mean day.

[33] Sic, instead of 58′ and 8″.
[34] Sic, instead of 60′ and 8″.

## Commentary to Chapter 31

### 1. The True Syzygies

We are dealing here with the problem of determining the longitude of the true syzygies from the mean syzygies which are obtainable from Tab. 69 to 72.[1] One wants furthermore to find the moment of the true syzygy and the corresponding lunar anomaly and latitude, i.e. all the elements required for an eclipse computation.

In the Arabic lunar calendar the dates of the syzygies must have the form

$$\text{year (of the hijra) } n, \text{ month M, day } \left\{ \begin{array}{l} 29 \text{ for conjunction} \\ 14 \text{ for opposition} \end{array} \right\} \qquad (1)$$

Thus we consider $n$, M, and day number as given. Then we have to proceed in the following fashion, using Tab. 69 (for conjunction) or Tab. 70 (for opposition): we write $n$ in the form

$$n = (30k + 1) + r \qquad k, r \text{ integers}, \qquad 0 \leq r < 30$$

and enter Tab. 69 or 70 with $30k + 1$, Tab. 71 with $r$, Tab. 72 with M. By adding the corresponding entries one finds the following four elements for the mean syzygy:

$$\left. \begin{array}{l} \text{date and hour } t_0 \\ \text{mean solar longitude } \bar{\lambda} \\ \text{anomaly of the moon } \alpha_0 \\ \text{argument of latitude for the moon } \omega_0. \end{array} \right\} \qquad (2)$$

[1] Suter, p. 82 to 85, has computed two examples, following the rules of ch. 31. Though his procedure is generally correct he comitted some small numerical errors which vitiate all subsequent steps; p. 82 read 4;16,35 (not 4;15,45) for the lunar equation; p. 85 line 4 read 4;33,50 (not 4;36,36). On p. 84, line 7 from the end, Suter used the value 67;12,57 for the argument of latitude, but this is an error of C and O whereas N correctly gives 67;52,57, a value which Suter himself adopted as the correct one in Tab. 70 (p. 184). The final results would be:

| | conjunction A. H. 288 I 29 | opposition A. H. 146 IV 14 |
|---|---|---|
| $t$ | 29 18;46,2$^h$ | 13 14;5,8$^h$ |
| $\lambda_\odot$ | 303;19,57° | 97;12,59° |
| $\omega$ | 169;7,48 | 359;52,7 |
| $\alpha$ | 125;8,0 | 285;47,48 |

The hours are counted from noon; for certain corrections concerning the day number cf. the commentary to Tab. 71 col. 3 (p. 109).

With the data (2) at our disposal we now can find, following the procedure of chapters 8 and 9 (p. 20), the equation $\Theta_1$ for the sun from Tab. 21 to 26 col. 3 (p. 95) and the equation $\Theta_2$ for the moon from col. 4 (p. 96). This gives for the moment $t_0$ of the mean syzygy the true longitudes

$$\lambda_1 = \bar{\lambda} + \Theta_1 \qquad \lambda_2 = \bar{\lambda} + \Theta_2 \tag{3}$$

for sun and moon respectively.[2]

Thus at $t_0$ we have an elongation

$$\Delta = \Theta_1 - \Theta_2 \qquad \text{called} \begin{cases} \text{"of the sun"} & \text{if } \Delta > 0 \\ \text{"of the moon"} & \text{if } \Delta < 0. \end{cases} \tag{4}$$

With the lunar anomaly $\alpha_0$ as argument we can find in Tab. 61 to 66 col. 4 (p. 106) the lunar velocity $v$, expressed in degrees per hour. Thus the time $\tau$ required for the moon to reach the sun is given by

$$\tau = \frac{1}{v}\left(\Delta + \frac{1}{12}\Delta\right) \tag{5}$$

where $\Delta/12$ represents the average increase of the elongation due to the solar motion.

Thus we have now found the time $t$ of the true syzygy as

$$t = t_0 + \tau \tag{6}$$

to which a solar longitude

$$\lambda = \lambda_1 + \Delta/12 = \lambda_2 + \Delta + \Delta/12 \tag{7}$$

belongs, which is also the true longitude of the moon at conjunction, or 180 plus the lunar longitude at opposition.

For the argument of latitude at the moment $t$ one has

$$\omega = \omega_0 + \Theta_2 + \Delta + \Delta/12 \tag{8}$$

and for the lunar anomaly

$$\alpha = \alpha_0 + \tau \cdot 0;32,40 \tag{9}$$

because the hourly increase of the lunar anomaly is according to Tab. 8 col. 3 given by $0;32,40°$.

Since the tables are computed for Cordoba a proper correction has to be made for other geographical longitudes.

### 2. The Equation of Time

The equation of time $E$, according to modern terminology, is the difference between true solar time and mean solar time, i.e., the difference between the hour angle[3] of the true and of the mean sun, or

[2] For oppositions the longitude of the moon is $\lambda_2 + 180$.

[3] This is the angle, counted on the equator (either in degrees or in time), since the crossing of the meridian.

$$E = \bar{\lambda} - \alpha = \lambda - \alpha - \Theta \tag{10}$$

where $\bar{\lambda}$ denotes the mean, $\lambda$ the true longitude of the sun, $\alpha$ its right ascension, $\Theta$ the equation of center. The term $\lambda - \alpha$ gives the reduction to the equator and is exclusively dependent on spherical geometry, whereas $\Theta$ is a function of the solar anomaly. The term $\lambda - \alpha$ shows a secular variation because the obliquity of the ecliptic is slowly changing, and also the second term varies slowly because of the displacement of the solar apogee relative to the vernal point. Thus both phase and amplitude of the function $E$ did slightly change since antiquity.

In the Almagest[4] the problem of the equation of time appears in a slightly different form. Let $d$ denote the "true solar day" i.e. the time between two consecutive transits of the sun, $\bar{d}$ the "mean solar day", i.e. the time of transit of an equatorial arc of $360° + \bar{v} \approx 360;59,8°$ where $\bar{v}$ denotes the mean daily velocity of the sun. Let $\Delta t = t - t_0$ denote the number of true solar days elapsed since epoch $(t_0)$. The problem arises to find the corresponding interval $\Delta \bar{t}$ counted in mean solar days. This problem is of importance because $\Delta t$ and $\Delta \bar{t}$ can differ by up to about 1/2 hour. The mean motion of the moon is given by the product of its mean velocity by $\Delta \bar{t}$ and not by $\Delta t$. Since the moon moves in 1/2 hour about 1/2 of its diameter the change from $\Delta t$ to $\Delta \bar{t}$ is of importance for the investigation of eclipses and lunar positions relative to planets and fixed stars.

Ptolemy's answer to the problem is given by the formula

$$\Delta \bar{t} = \Delta t + (\Delta \alpha - \Delta \bar{\lambda}) \tag{11a}$$

where

$$\Delta \alpha = \alpha(t) - \alpha(t_0) \qquad \Delta \bar{\lambda} = \bar{\lambda}(t) - \bar{\lambda}(t_0) \tag{11b}$$

represent the corresponding changes of right ascension $\alpha$ and mean solar longitude $\bar{\lambda}$.

The correction $E = \Delta \alpha - \Delta \bar{\lambda}$ from true to mean time intervals can again be represented as function of one variable, e.g. the solar longitude $\lambda$, if we consider the epoch $t_0$ as fixed. Then we have

$$E = e - e_0 \qquad e = \alpha(t) - \bar{\lambda}(t) \qquad e_0 = \alpha(t_0) - \bar{\lambda}(t_0) \tag{12}$$

and $e_0$ is a constant. As function of $\lambda$ the term $e$ has a minimum near ♏ 0°, a maximum in ♒, a secondary minimum at the beginning of ♊ and a secondary maximum in ♌.

In the Handy Tables the epoch $t_0$ is the noontime of Thoth 1 of the year 1 of the era Philip (−323 Nov. 12). For this moment the solar longitude is ♏ 17, thus very near the point where $e$ has its minimum. Thus for this choice of $t_0$ we can say that $e \geq e_0$ or that $E$ is a non-negative correction. This has the great practical advantage that the transformation of $\Delta t$ to mean time intervals $\Delta \bar{t}$ always operates with the same sign (cf. fig. 32 p. 108).[5]

Al-Battānī in his table of the equation of time[6] uses as epoch the Seleucid era,

[4] IV,9.
[5] Cf. for this whole discussion Rome [1] and Rome, Comm. vol. III p. CXXXIII to CXXXVI.
[6] Nallino II p. 61 to 64.

i.e., in his norm, −311 March 1.[7] The solar longitude for $t_0$ is therefore about )( 4°[8] or near to the point where $e$ as function of $\lambda$ has its maximum. Consequently for this norm we have $e \leq e_0$ or again a fixed sign for $E$, namely, always negative.

Al-Khwārizmī's tables[9] share with Battānī's the rule that the "*examinatio dierum*" which transforms true to mean solar days is always negative. Al-Khwārizmī's epoch, however, is the era hijra (622 July 15) such that for $t_0$ the sun was in about ♋ 24°,[10] thus nowhere near the maximum or minimum of $e$. If one nevertheless wishes to obtain an always negative correction, one can proceed as follows: We write (11) and (12) in the form

$$\Delta\bar{t} = \Delta t + e - e_0 = \Delta t + (e_{\max} - e_0) + (e - e_{\max}).$$

Thus, if $\bar{v}$ is the mean velocity of a celestial body, $\bar{\lambda}_0$ its mean longitude at epoch $(t_0)$, $\bar{\lambda}$ its mean longitude $\Delta t = t - t_0$ later, then we have

$$\begin{aligned}\bar{\lambda} &= \bar{\lambda}_0 + \Delta\bar{t}\cdot\bar{v} \\ &= \bar{\lambda}_0 + (e_{\max} - e_0)\,\bar{v} + (\Delta t + (e - e_{\max}))\bar{v}.\end{aligned}$$

Here the last term contains the interval $\Delta t$ of true solar days, modified by a correction $e - e_{\max} \leq 0$ as before. The epoch value of the mean longitude, however, has to be modified from $\bar{\lambda}_0$ to $\bar{\lambda}_0 + (e_{\max} - e_0)\,\bar{v}$. But $e_{\max} - e_0$ is a constant for a fixed epoch date. From al-Khwārizmī's tables we find $e_{\max} = 0;34,28^h$ and for $\bar{\lambda} = ♋\,24$ a value $e_0 = 0;15,24^h$. Thus $e_{\max} - e_0 \approx 0;20^h$ to which corresponds a mean motion for the sun of only about 0;0,49°, for the moon of about 0;11°.[11] If one adds these corrections to the actual epoch values for sun and moon, then one has again an always negative equation of time as it is assumed in ch. 31 and in Tab. 67 and 68.

The contents of the scholion at the end of ch. 31[12] can be rendered as follows. If one true solar day ends $0;2,8^h$ before the mean solar day, the next solar day $0;3,8^h$ before the mean solar day, then this second true solar day was $0;1^h$ shorter than a mean solar day. Similarly with interchanged numbers.[13]

[7] Nallino I p. 246.
[8] Rome, Comm. III p. XXXV.
[9] Tab. 67 and 68.
[10] Tab. 4, Section 1, gives for the beginning of A.H. 1 $\bar{\lambda}$ = ♋ 23;25,48, Battānī (Nallino II p. 19): 23;58,4.
[11] Al-Khwārizmī's Tab. 5 and 8 respectively.
[12] Cf. above p. 61.
[13] Actually, as Tab. 68 shows, corrections 0;2,8 and 0;3,8 cannot occur on two consecutive days but are about 4 days apart.

## Chapter 32

See between ch. 35 and 36 (below p. 76).

## Chapter 33. [First Version[1]]. How an Eclipse of the Sun or of the Moon Has to Be Determined

Since from the above discussion[2] one can readily find when and where a conjunction or opposition of sun and moon takes place, it seems (now) proper to teach what concerns the determination of their eclipses. And, because it is nearer, I think one should first deal with the moon. After the conjunction and opposition of the luminaries has been noted down, and, as said before, also in each case[3] for the same time the argument of latitude has been established, then one (can) conclude from this argument whether or not the eclipse will take place. If it is appropriate[4] to enter (with the argument of latitude) the columns of eclipse(s),[5] one should do so; what corresponds to this entry then shows for the eclipse in question the quality[6] and the duration; if not, then neither this nor that, and no eclipse will take place.

For a possible eclipse one thus enters the eclipse table;[7] the first corresponding number[8] indicates the eclipse and its magnitude; the next[9] the amount (such that) the diameter of the shadow is divided into 20 digits,[10] one digit into 60 minutes; the diameter of the moon, however, is divided into 12 digits. And the second (next) number[11] determines the quality and duration.

Every eclipse is either partial or total; a partial eclipse during the time it enters (the shadow) is called *elzukut* by the Arabs, but *casus* (immersion) with us,[12] while it is leaving the shadow *elingile*, which we may call *respiratio* (recovery). Totality is called *elmukdh* by them, i.e. duration, that is to say the interval during which (the moon) is completely obscured. The remaining (phase of the eclipse) is to be called (again) either *casus* or *respiratio*. In the table given below[13] the time for immersion as well as for duration is given in minutes and seconds.[14] You must understand, however, that you should use the tables (headed) "for farther distance"[15] when the moon is far distant, and the table (headed) "for near distance"[16] when the moon is near. This is of importance because the diameter of the shadow at far distance is divided into 20 digits, but the diameter of the shadow at near distance into 21 or

[1] In O, C, and N.
[2] Chapter 31.
[3] Lit.: singly in their order.
[4] I. e. in case the given argument of latitude is found within the tabulated range.
[5] In Tab. 73 to 76, columns 1 and 2.
[6] Partial or total.
[7] Tab. 73 to 76.
[8] In column 3.
[9] Column 4.
[10] This is not meant accurately since magnitudes greater than 20 are possible (20;46 in Tab. 73, 21;31 in Tab. 75; cf. also the next paragraph). Furthermore it is not the diameter of the shadow which is measured by this figure but the greatest possible immersion, measured in digits of the moon; cf. (7) p. 119.
[11] In column 5.
[12] Cf. for all these terms Suter p. 87f.
[13] Tab. 73 to 76, columns 4 and 5.
[14] O incorrectly: "of hours". In fact: minutes and seconds of arc.
[15] Tab. 73 and 74.
[16] Tab. 75 and 76.

even more,[17] the reason for this being that the orbit of each planet is found to be eccentric with respect to the earth. If the moon is not located in either one of the before-mentioned places,[18] then one has to enter with the argument of latitude in both of these tables and note down the corresponding numbers. Then one subtracts the smaller value from the greater one and puts down the remainder. Thereafter one enters with the lunar anomaly the table of proportional parts[19] and notes down the corresponding value.[20] With this number the above mentioned difference has to be multiplied and the result must be added to the number which has been found for the (given) argument of latitude at great distance. In this way you will find for any lunar eclipse a definite amount in digits.

## [Chapter 33. Second Version[21]]. Procedure[22] for the Investigation of Lunar Eclipses

If you wish to know the lunar eclipses, establish an opposition such that the moon reaches the ascending or descending node. After having carefully determined the argument of latitude which corresponds to the accurate hour of opposition, you have to investigate whether the moon is at far distance. In case it is, you enter with the argument of latitude the table of lunar eclipses for far distance[23] and take (the number) of digits which you find[24] and the minutes (and seconds) of the *casus*[25] (immersion) and of the *elmuht*,[26] i.e. duration (of totality), if there is any *elmuht* (different from zero listed for the given argument). If, however, the moon is at near distance, then with the argument of latitude you enter the eclipse table for near distance[27] and take what you find opposite it for the digits, for the minutes of *elciicuth* (the partial phase), and for the *elmuht* (totality), if there is any *elmuht*.

If the moon is located in neither one (of the extremal distances), enter with the argument of latitude in both tables.[28] Then subtract the first value (for the apogee) from the second (for the perigee), for each of its kind separately, and keep the amount of their difference in mind. Thereafter you enter with the corrected anomaly the column of numbers in the table of proportional parts[29] and take the corresponding number as minutes of the remainder and multiply by these minutes the re-

[17] Cf. note 10 p. 66.
[18] I.e. neither in apogee nor in perigee.
[19] Tab. 73 to 75, columns 6 or 7.
[20] From col. 8.
[21] This version is only found in C and N, not in O. Millás-Vallicrosa, Azarquiel p. 144 note 3 has shown that it is a Latin version of chapter 23 of Azarqiel's "Almanac". Cf. also below note 33, and ch. 37, second version (p. 79 note 10).
[22] *scientia*.
[23] Tab. 73 and 74.
[24] In column 3.
[25] Column 4.
[26] Column 5.
[27] Tab. 75 and 76.
[28] Tab. 73 or 74 and 75 or 76.
[29] The last two columns in Tab. 73 to 75.

mainder which you have kept in mind, namely of the digits, of the minutes of the partial phase, and of the totality. The result you add to the values which you have found for the distance, for each of its kind separately, and these will be the corrected digits of the eclipse with respect to the diameter of the moon,[30] and the corrected minutes of the partial phase and of the totality (to) the middle of the eclipse.[31]

Thereafter add to the minutes of the partial phase one half of its sixth and divide the result by the hourly motion of the moon. The resulting hours represent (the time) from the beginning of the eclipse to the beginning of totality, if (the eclipse) becomes total. Then you add to the minutes of totality[32] one half of its sixth and divide (the result) by the hourly motion of the moon; the result is half the totality of the lunar (eclipse). This you double and add to it twice the hours of the partial phase and you obtain (the time) from the first beginning of the eclipse to the completion of the recovery.[33]

[Marginal note in C:] Here it should be said that sun and moon are at far distance when they are located in the last degree of Gemini or in the first degree of Cancer, but at near distance when it (the eclipse) occupies the last degree of Sagittarius or the first of Capricorn.

[Marginal note in N:] Here it should be said that the sun is at far distance when (it is) in the 18(th) degree of Gemini, but at near distance when it occupies (the) 18(th degree) of Sagittarius. The distance of the moon is far(thest) when its anomaly is 0 0, near(est) when it is 6 signs.

[30] Not area digits.
[31] *liwacat elcuzuf*; cf. Suter p. 244.
[32] *mora*, i.e. the duration from the beginning of totality to its midpoint.
[33] Azarqiel's version continues beyond this point by giving similar rules for the moments and longitudes of the beginning and end of the different phases. Cf. Millás-Vallicrosa, Azarquiel p. 144 lines 13 to 24 and the end of note 3 to p. 144.

### Commentary to Chapter 33

For the computation of lunar eclipses the moment of the true opposition is supposed to be known (from ch. 31) and also the corresponding argument of latitude $\omega$. Then Tab. 73 to 76 col. 3 give the magnitude $m$ of the eclipse for the moment of greatest immersion, i.e. at the middle of the eclipse, expressed in linear digits, i.e. twelfths of the apparent lunar diameter. Tab. 73 and 74 concern the case when the moon is at the apogee (anomaly $\alpha = 0$), Tab. 75 and 76 when at perigee ($\alpha = 180$). The tables which are appended as columns 6 to 8 to Tab. 73 to 75 give a coefficient $c$ as function of $\alpha$ ($c = 0$ for $\alpha = 0$, $c = 1$ for $\alpha = 180$; cf. fig. 34 p. 116) such that

$$f(\alpha) = f_a + c(\alpha) \cdot (f_p - f_a)$$

where $f_a$ and $f_p$ are functions of $\omega$, tabulated for apogee and perigee respectively.

Column 4 serves for the determination of the partial phase ("immersion") by giving the corresponding motion in degrees with respect to the shadow, again as

function of ω. Column 5 in the same fashion concerns totality from inner contact to eclipse middle (cf. fig. 36 p. 117).

The text which actually belongs to our set of tables ends with this general description of the use of the tables. Suter added to this chapter similar rules, found in two of his manuscripts, which belong, as Millás-Vallicrosa has seen, to Azarqiel's "Almanac". They describe essentially the same procedure – which, of course, goes back at least to the Almagest – but tell us in addition how one finds from the tabulated angular distances (columns 4 and 5) the corresponding durations for the phases. Because the tabulated values concern only the motion relative to the shadow one has to increase them in the ratio 13 to 12 in order to obtain the absolute distances travelled by the moon during the partial and the total phase respectively. Division by the hourly velocity of the moon (obtainable from Tab. 61 to 66, col. 4) gives the corresponding time intervals.[1]

A marginal note of MS C incorrectly makes the lunar apogee dependent on the longitude instead of on the anomaly.[2] In MS N a correct formulation is given and the solar apogee is placed at ♊ 18, in fair agreement with ch. 18 (cf. above p. 41).

[1] Suter, p. 86 to 89, computed with the rules of ch. 31 the details of the lunar eclipse A.H. 146 IV 14 (= A.D. 763 June 30; Oppolzer, Canon No. 3050). His computation is affected by the small errors mentioned above p. 62 note 1. The corrected results are:

| | | |
|---|---|---|
| magnitude: | 20;47 | Oppolzer: 22.3 (≈ 22;20) |
| duration of totality: | 1;34,44$^h$ | 1;44$^h$ |
| total duration: | 3;40,34$^h$ | 3;44$^h$. |

[2] This error has also vitiated Suter's discussion (p. 86) but has no effect on his computation.

## Chapter 34. On the Parallax[1] of the Moon for Longitude in Hours and for Latitude

Before we are ready to turn to solar eclipses, we must discuss the moon. Thus if you wish to know its parallax in longitude and the (corresponding) hours of distance, the ascendant at the hour of conjunction must be noted accurately and carefully. From it the middle point[2] is found by subtracting (from the longitude of the ascendant) three signs. After this has been determined one has to enter the table of the equinoctial circle[3] with (the longitude of) the middle point which belongs to this ascendant, and then one has again to enter the (table of the) equinoctial circle with the degree (of longitude) of the moon, and the corresponding numbers should be taken. Then, if the moon is between ascendant and middle point, the (number found for the) middle point has to be subtracted from the (number found for the) place of the moon. If, however, the moon is between the setting point and the middle point, then the (number for the) place of the moon must be subtracted from the (number found for the) middle point. The remainder gives the interval (in right

[1] Here and in the following I render by "parallax" expressions like *diversitas aspiciendi* or *diversitas aspectuum* etc.

[2] *medium caelum*, which is, however, not the culminating point of the ecliptic but its highest point, as is clear from the rule given here. Cf. below p. 71 note 3.

[3] This is the table of right ascensions, Tab. 59 to 59b (p. 104).

ascension) between the moon and the middle point (which belongs to) this ascendant. With this (value) one has to enter the table for the parallax given below[4] and note down the corresponding hours and their parts.[5] This corresponding number (of hours, etc.) is to be multiplied by the variable hourly motion of the moon[6] and what results in this way represents the parallax in longitude of the moon. If the moon is between the ascendant and the middle point, then this product is to be added to the hours[7] of the conjunction; if, however, (the moon) is between setting and middle point, then it is to be subtracted from it. Whatever results in this way gives the hour[8] of the eclipse for the place of investigation, corrected for parallax.

To know the parallax in latitude: If you wish to know the parallax in latitude the ascendant for this hour must be securely known. For it the middle point is to be taken as above. Then the amount of the declination must be carefully determined[9] for this middle point. If it is northerly and less (than the geographical latitude) it has to be subtracted from the (geographical) latitude of the given locality; if it is (northerly and) greater (than the geographical latitude), the (geographical) latitude has to be subtracted from it. If, however, (the declination is) southerly it (always) has to be added to it (to the geographical latitude). The result is the distance between the middle point, belonging to the given ascendant, and the zenith.[10] Then the latitude of the moon has to be known; if it is northerly, this latitude has to be subtracted from the before mentioned (zenith-)distance; if southerly it has to be added to it. The result thus obtained represents the distance between the moon and the zenith.

With this number one has to enter the table of parallax[11] and take the corresponding number in the latitude (column[12]). It represents the parallax in latitude. It is obvious that also the parallax is northerly if the middle point is north of the zenith and southerly if it (the middle point) is to the south (of the zenith).

[Addition in N by a second hand:][13] This (holds) near the meridian, but elsewhere (proceed as follows): after the true latitude of the moon has been found . . .[14] put the degree of the conjunction in the astrolabe over the hour of the conjunction and see how many degrees the degree of the conjunction is distant from the center of the smallest *almucantarat* (circle of constant altitude) (drawn on the astrolabe); and, if the true latitude of the moon is northerly it should be subtracted from these degrees, if southerly, added. And with this (number) one should enter the tables of parallax in latitude.

[4] Tab. 77 and 77a.
[5] From column 2.
[6] From Tab. 61 to 66, col. 4.
[7] sic, instead of to the place.
[8] Actually again the longitude and not the time.
[9] From Tab. 21 to 26, col. 5.
[10] Literally: (the point) above our heads.
[11] Tab. 77 col. 1 and 4, Tab. 77a col. 1.
[12] In cols. 3 and 6 of Tab. 77, col. 3 of Tab. 77a.
[13] Cf. Suter p. XX remark 12).
[14] The text has the word *nadair*, to which Suter remarks "wahrscheinlich nadir", but this makes no sense. The word should be deleted.

## Commentary to Chapter 34

### 1. Parallax in Longitude

If H denotes the rising point of the ecliptic, the ascendant, and V the highest point of the ecliptic (fig. 40 p. 122), then we have for the longitudes of these points the relation

$$\lambda_V = \lambda_H - 90°. \tag{1}$$

If P is the place of the moon in the ecliptic we can find from Tab. 59 to 59b the right-ascensional difference between P and V

$$\left.\begin{array}{lll} \Delta\alpha = \alpha_P - \alpha_V & \text{if} & \lambda_P > \lambda_V \\ \Delta\alpha = \alpha_V - \alpha_P & \text{if} & \lambda_P \leq \lambda_V. \end{array}\right\} \tag{2}$$

The tables 77, 77a are so constructed[1] that one finds there directly, for $\Delta\alpha$ as argument, the longitudinal component of the parallax. The units in these tables are, however, not degrees but hours, corresponding to the time needed for the moon to travel the ecliptic arc which is the longitudinal component of the parallactic displacement. Obviously it is assumed that the moon moves with mean velocity,[2] i.e., that it is located at mean distance from the earth. In order to account for the fact that the parallax decreases with increasing distance the text gives the rule that the longitudinal component $\pi_\lambda$ of the parallax, measured in degrees, is obtained from the tabulated value $p_\lambda$, measured in hours, by multiplying $p_\lambda$ with the hourly lunar velocity $v$:

$$\pi_\lambda^{\circ} = p_\lambda^{h} \cdot v^{\circ/h}. \tag{3}$$

This formula underlies the essentially correct assumption that parallax and lunar velocity vary in the same way (namely linearly) with geocentric distance.

Because the longitudinal component of the parallax always points away from the highest point V of the ecliptic, one has the rule that the longitudinal parallax is to be added to $\lambda_P$ when P is located between H and V, otherwise subtracted.

### 2. Parallax in Latitude

The first step consists in determining what is supposedly the zenith distance $\bar{a}$ of the middle point V of the ecliptic.[3] If $\varphi$ is the geographical latitude and $\delta$ the declination of V[4] then the text gives the rule

$$\left.\begin{array}{lll} \bar{a} = \varphi - \delta & \text{if} & \left.\begin{array}{l}\delta \leq \varphi\\ \delta \geq \varphi\end{array}\right\} \text{and } \delta \text{ northerly} \\ \bar{a} = \delta - \varphi & & \\ \bar{a} = \varphi + \delta & \text{if } \delta \text{ southerly.} & \end{array}\right\} \tag{4}$$

[1] Cf. the commentary to these tables, p. 123.

[2] According to Tab. 8 col. 2 or Tab. 63 col. 4: 0;32,56o/h.

[3] In Kepler's terminology the *nonagesimus gradus* or simply *nonagesimus* (Kepler, Werke 2 p. 275,30f. et passim; Werke 3 p. 123,39 etc.).

[4] Found in Tab. 21 to 26, col. 5.

Since $\varphi$ is the zenith distance of the culminating point of the equator the formula (4) would be correct if V were the culminating point of the ecliptic, which, however, in general does not coincide with the middle point of the ecliptic. Thus (4) is only an approximation of the correct procedure; it is characterized as such, e.g., in the Sūrya-Siddhānta V,7.

With $\bar{a}$ from (4) and with the given latitude $\beta$ of the moon one forms the argument $\bar{a}' = \bar{a} - \beta$ for the entry in Tab. 77 or 77a.[5] Since parallax is of interest mainly for eclipses, i.e. for small values of $\beta$, the zenith distance $\bar{a}$ of V will be only little different from $\bar{a}'$ which gives the latitudinal parallax $\pi_\beta$ of the moon from the tables. But the moon is generally not located in V. Nevertheless the procedure of the text is correct because it is possible to show (cf. below p. 123) that $\pi_\beta$ can be considered constant along the ecliptic for a fixed position of the latter, i.e. for a given value of $\bar{a}$.[6] The same will hold, of course, for points close to the ecliptic, e.g. for the point of a circle parallel to the ecliptic with zenith distance $\bar{a}' = \bar{a} - \beta$ for small values of $\beta$.

For the latitudinal component of the parallax no correction is introduced depending on the variable distance of the moon. I see no good reason for a different treatment of longitudinal and latitudinal component, except, perhaps, that one was more interested in an accurate determination of the moments of the phases of an eclipse than in the determination of the magnitudes.

### 3. Zenith Distance from the Astrolabe

A later addition to the MS N remarks that the rule (4) p. 71 holds only for the culminating point M, not for the middle point V of the ecliptic. Therefore it is suggested that the zenith distance $\bar{h}$ of the moon be found from the astrolabe by reading off the distance from the point which represents longitude and moment of the conjunction to the center of the smallest parallel circle of constant altitude. Finally $\bar{h} - \beta$ should be used as argument for finding $\pi_\beta$ in the tables.

The center of the smallest parallel of constant altitude drawn on the instrument is here taken as approximately representing the zenith. In fact it would be much better to read (or estimate) the altitude $h$ of the point that represents the conjunction directly from the nearest circle of constant altitude. Then $\bar{h} = 90 - h$ is the zenith distance. It is again only a crude approximation to consider $\bar{h}' = \bar{h} - \beta$ as zenith distance of the moon with latitude $\beta$ since only at V is the ecliptic perpendicular to the circles of altitude. Finally it is wrong to use $\bar{h}'$ as argument in the tables instead of $\bar{a}$ because it follows from (2) and (3) p. 71 that

$$\pi_\beta = \pi_0 \sin \gamma \sin \bar{h}$$

where $\gamma$ is not known.

[5] Tab. 77, col. 3 or 6, Tab. 77a, col. 3.

[6] The zenith distance $\bar{a}$ of V is called in Islamic astronomy the "Latitude of visible climate"; cf. Kennedy [6] p. 37.

## Chapter 35. On the Variation[1] of Solar Eclipses

If you ask for the variation[2] of solar eclipses, in order not to be deficient in this problem, you can find it as follows. First one has to establish that the conjunction happens in daytime, when the sun is located near the ascending or descending node. Under this condition the occurrence of an eclipse is possible, otherwise, however, not. Here we deal (only) with the possible case.

Now one must note down carefully the argument of latitude (of the moon) and the (longitude of the) middle point (of the ecliptic)[3] for the moment of the conjunction and also the hourly parallax, as well as the longitudinal parallax.[4] If then the conjunction takes place between the ascendant and the middle point (of the ecliptic) one has to subtract the longitudinal parallax from the place of conjunction; similarly the hourly parallax is to be subtracted from the amount of the conjunction. If the conjunction takes place between setting point and middle point (of the ecliptic) one has to add the parallax to the place of conjunction and similarly the hourly parallax to the moment of conjunction. The result of this addition or subtraction represents the moon's place in the zodiac at the (moment of the) eclipse as you wanted to know. And the result obtained from the moment of conjunction by this addition or subtraction gives the (actual) moment for the conjunction under investigation.

[Addition by second hand:][5] The entry for these tables[6] is as follows: first one must look at the distance of the conjunction of sun and moon from the ascending or descending node and if the moon is at greater longitude of its small circle (i.e. of the epicycle) you should enter the first tables,[7] otherwise the second ones,[8] and take so many digits as correspond[9] to the before mentioned distance (from the node). Then one has to take the lunar parallax and see how many digits are its equivalent. And if then the moon was to the north of the sun, these digits must be added to the digits found in the table; if to the south they must be subtracted, if possible; otherwise there will be no eclipse. In this way one finds the magnitude of an eclipse.

And then, also found with it (the magnitude corrected for parallax) in the table[10] the minutes of the immersion[11], (given) in its line,[12] and this will be the duration (from the first contact) to the middle of the eclipse.

And the equivalent in digits of the parallax is found as follows. From the tables

[1] *De diversitatibus.* Or should one translate "On the parallaxes of solar eclipses"? Cf. also note 2.
[2] *varietatem quaerens*; cf. note 1.
[3] Cf. p. 69 note 2 and p. 71 note 3.
[4] In fact both cases concern the longitudinal parallax, but in one case measured in hours, in the other in degrees.
[5] From N; cf. Suter p. XX, No. 13.
[6] Tab. 78.
[7] Tab. 78 left section (for the moon at the apogee).
[8] Tab. 78 right section (for the moon at perigee).
[9] In column 3 of either section of Tab. 78.
[10] In column 4.
[11] Cf. ch. 33 (p. 66).
[12] *in eius directo.* The same terminology, e. g., in Thâbit, *De Motu*, versio A and B, § 2 (Carmody p. 87 f.).

of (true) motion[13] one should find the size of the solar body and divide it in 12 equal parts because the 12th part of the solar diameter is always called one digit. Then one sees how many times its (the sun's diameter's) twelfths are contained in the parallax and the result gives the corresponding digits.

Thus, if it (the parallax) has been found to amount to more than 12;48 digits in the case of nearer longitude, or more than 10;48[14] in the case of greater (longitude), and if the moon was to the north of the sun, then the before mentioned digits[15] should be subtracted from the obtained (total of tabulated magnitude and parallax) and the remainder (indicates how much) of the sun is left unobscured. If, however, the moon was to the south (of the sun), the remainder[16] found (indicates the amount) that hides itself from the sun.[17] This[18] (holds) for those for whom the sun is always to the south;[19] if it were (always) to the north, one would have to proceed in the opposite fashion.

[Addition by third hand:] But also at a greater elongation before the descending node or after the ascending node, because a solar eclipse occurs up to 23 degrees (distance from the nodes). And then the true latitude of the moon should be known and its latitudinal parallax; and one must subtract the smaller (number) from the larger one and from the remainder will be known the magnitude of the eclipse, and its digits must be determined from the amount of (the diameter of) the sun and of the remainder, as said before, and through them the duration in the table. One must add the radii[20] of sun and moon and if the before mentioned remainder is equal to it, or even greater, then no eclipse will occur; if the remainder is zero, it will be total; but if the remainder is smaller (than the sum of the radii) then a partial eclipse will take place.

[13] Tab. 61 to 66, column 5.
[14] Error for 10;44.
[15] Namely 12;48 or 10;44 respectively.
[16] I emend *inventum secundum* to *inventum residuum*.
[17] This is the case when no eclipse occurs and the result gives the distance of the moon from the sun; cf. the commentary (p. 75).
[18] Here begins a new sentence and Suter's punctuation has to be changed.
[19] These are the inhabitants of the northern hemisphere.
[20] Literally: the half diameters (*medietas diametrorum*).

### Commentary to Chapter 35

The main part of chapter 35 is absolutely trivial. It only repeats rules for subtraction or addition of the longitudinal parallax, expressed either in degrees or in hours, in order to find the place or the moment of the conjunction. It is only in the remarks by later hands that the tables for solar eclipses (Tab. 78) are mentioned, but there is no chapter extant which would give specific rules for the use of Tab. 78.

The first addition to the original ch. 35 deals with the effect of the latitudinal parallax on the eclipse magnitudes found in column 3 of Tab. 78. To this end both pararallax and lunar radius must be measured in units of solar digits, i.e. twelfths of the solar diameter. We then denote these quantities by $\pi''_\beta$ and $r''_\zeta$ respectively; *m*

stands for the eclipse magnitude found from column 3 of Tab. 78, measured in digits. Obviously one has

$$\pi''_\beta = 12\,\frac{\pi_\beta}{d_\odot} \qquad r''_{\mathbb{C}} = 12\,\frac{r_{\mathbb{C}}}{d_\odot}$$

where $d_\odot$ is the apparent solar diameter for the given anomaly, found in Tab. 61 to 66, col. 5.

Because parallax reduces the altitude of the moon, we have for the moon north of the sun a corrected magnitude $m + \pi''_\beta$ but for the moon south of the sun $m - \pi''_\beta$. The latter case shows that parallax can make a solar eclipse invisible which would have been visible otherwise.

Using this magnitude corrected for parallax as argument in column 3 of Tab. 78

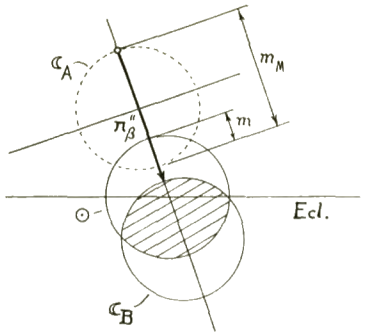

Fig. 21.

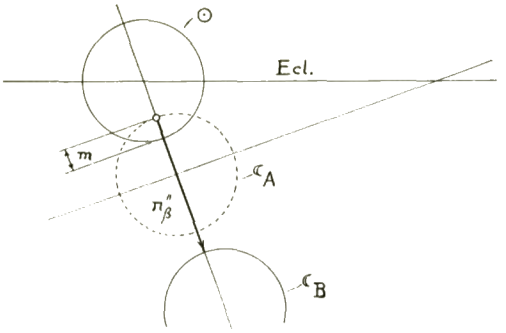

Fig. 22.

one finds in column 4 the corresponding corrected half duration of the partial phase (*casus*).

If the latitudinal parallax exceeds the greatest possible magnitude $m_M$ (10;44 at apogee, 12;48 at perigee) then

$$\pi''_\beta + m - m_M$$

gives the unobscured part of the solar diameter for an eclipse of magnitude $m$ without parallax, assuming that the moon is to the north of the ecliptic. The proof follows from fig. 21. The moon without parallax would be in a position A. Its diameter, measured in digits, represents the maximal possible magnitude $m_M$; its immersion $m$ is the magnitude found in the table. The latitudinal parallax $\pi''_\beta > m_M$ moves the moon to the position B. Thus the unobscured part of the solar diameter is given by $\pi''_\beta - m_M + m$ q.e.d. Similarly fig. 22 shows for a southern position of the moon that the northern rim of the moon is separated from the southern rim of the sun by the difference $\pi''_\beta - m$.

A second addition to ch. 35 by another hand states that solar eclipses are possible within 23° distance of the (center of the mean) moon from the nodes. I do not know on what basis this number was computed; Ptolemy finds under the condition stated

in the Almagest VI, 5 limits of 20;41°.[1] Then follows a shortened version of the rules given in the previous addition. At the end is given a criterion which probably means

$$\text{if } m + \pi''_{\beta} - (r''_{\odot} + r''_{\leftmoon}) \begin{cases} > r''_{\odot} + r''_{\leftmoon} \\ = 0 \\ < r''_{\odot} + r''_{\leftmoon} \end{cases} \quad \text{then} \begin{cases} \text{no eclipse} \\ \text{total eclipse} \\ \text{partial eclipse,} \end{cases}$$

assuming a northern latitude of the moon as in fig. 21.

[1] Heiberg p. 483,13; cf. Manitius p. 356,9 ff.

## Chapter 32.[1] Determination of the Culminating Degree and of the Distribution of the 12 Houses. This Distribution is called *tezwiet elbuiut*

Consider the (given) degree of the ascendant[2] and enter with it the column (headed) "equal degrees"[3] (of the table) of "ascensions at sphaera recta"[4] and take the corresponding (number of) ascensions and add to it the time of two nocturnal hours corresponding to the time of the degree of the ascendant in your locality. The total thus obtained find (again) in (the column) of "ascensions at sphaera recta" and from it return to the corresponding "equal degrees". The resulting number of these degrees is the beginning of the second house.

Thereafter add the time of two nocturnal hours to the ascensional degrees by means of which you have determined (the beginning of) the second house and you will find in the previously described fashion the beginning of the third house. Thereafter add (again) the same amount (of two nocturnal hours) to the number by means of which you have found the third house and from this total you will find, as before, the beginning of the fourth house. The beginnings of the fifth and of the following (houses) you will obtain by a similar procedure, with the only (easily) remembered difference that for the first four (houses) two nocturnal hours had to be taken, but for the remaining (houses) the addition of two daylight hours of the time of your locality is required. Notice also that you have correctly computed (only) when the seventh house agrees with (the degree of) the ascendant.[5]

If you wish to find the time(-degrees) (corresponding to) two hours for your given ascendant,[6] enter with the ascending degree the column of numbers (in the table) of ascensions for your locality[7] under the same sign (as the sign) of the sun and take

[1] This chapter is obviously out of place if counted as No. 32, intruding between the discussion of syzygies (ch. 31) and the theory of eclipses (ch. 33). It is missing in O, but in N it is given at its proper place before ch. 36. Cf. Suter p. 86 note 3 and p. 96.

[2] The *horoscopus*.

[3] *gradus aequales*, in Tab. 59 to 59b called *gradus regulares*.

[4] *horoscopi felek elmustakim*; cf. p. 104 for Tab. 59 to 59b.

[5] But six signs apart.

[6] I.e. the length of two seasonal hours corresponding to a solar longitude which is equal to the longitude of the given ascendant.

[7] Such tables for rising times at sphaera obliqua are not extant in our text.

the corresponding number of time(-degrees)[8] and double it: so you will get the time (-degrees) of two daylight hours; then subtract this doubled (number) from 60, and the remainder will be the time(-degrees) of two nocturnal hours, with which you have to operate as explained above.

[8] *de temporibus horarum*, probably the column title.

### Commentary to Chapter 32

The astrological "houses" (τόποι, *loci*) are here defined as sections of the ecliptic, beginning with the ascendant H (cf. fig. 23) and counted in the direction of increasing longitudes, of equal ascensional differences for the six daylight houses and

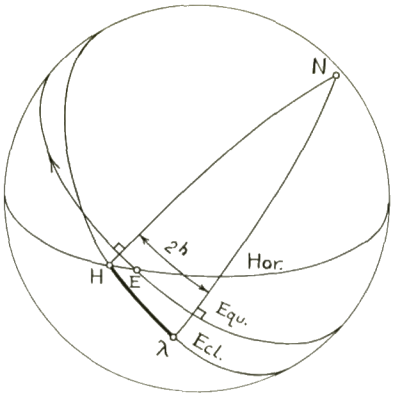


Fig. 23.

similarly for the six nocturnal houses.[1] Consequently the first house reaches from H to a point of longitude λ such that, for the right ascension α,

$$\alpha(\lambda) = \alpha(H) + 2 \text{ hours of night,}$$

these hours computed for a solar longitude $\lambda_\odot = \lambda(H)$ and the given geographical latitude.

The tables for the determination of the right ascension α(H) and then conversly of λ from α(λ) are our tables 59 to 59b but the tables for oblique ascensions and corresponding seasonal hours are not contained in the extant tables. Actually they are not needed since the results of these operations are tabulated here for all zodiacal signs and integer degrees in Tab. 79 to 90 (cf. p. 128).

[1] This procedure is known from antiquity, e.g. with Rhetorius (5th cent. A.D.); cf. Neugebauer-Van Hoesen, Greek Hor. p. 139.

## Chapter 36. On the Equation of the 12 Houses

Because among the (subjects) discussed so far[1] the ascendant has not only the greatest power in all astronomical predictions but also imparts it to the subsequent (houses) in their proper order, a table of the equation of the 12 houses is given below,[2] which shows their beginnings. The entry into it is as follows. Beginning with Aries, one has to enter the table[3] with the degree at which the ascendant is located; then the corresponding numbers represent the beginnings of the second, third, fourth, fifth, and sixth house. One has to observe that the (beginning of the) seventh (house) corresponds to the (beginning of the) first, the eighth to the second, and the remaining (houses) to the remaining ones in their (respective) order. Therefore the other six signs were ignored (in the table).

[1] Referring to ch. 32 which should directly precede the present chapter; cf. p. 76 note 1.
[2] Tab. 79 to 90.
[3] For Aries: Tab. 79, column 1.

## Chapter 37. How to Find the Aspect[1] of the Stars[2], in Sextile or Quartile or Trine

The aspect of the stars is to be found as follows. Whatever aspect is to be found, sextile or quartile or trine, one has first to investigate where the ascendant is located in its sign, that is to say, whether it is the fifth (degree) or the tenth or the fifteenth or the twentieth or the twenty-fifth or even the thirtieth.[3] Then one must note down the degree of the particular planet for which you wish to find the aspect. With the corresponding degree of the planet, or with the nearest larger number (contained in the table), one enters the number of degrees,[4] using the first sub-table[5] in the case of five degrees (as location of the ascendant), the second in case of ten, the third for fifteen, the fourth for twenty, the fifth for twenty-five, the sixth for thirty. From there one has to proceed to the table of the horoscopes[6] to the number[7] (of degrees) under the sign of the given planet and use the corresponding number. To it one has to add the number which is written at the top of the table for the aspect in question and the sum thus obtained has to be found in the same table, or at least (the number) nearest to it. One of the two signs given in the corresponding heading and the degree number indicate the aspect in question.

If the table does not contain either the number (of the sum) itself or one near to it, subtract from it the nearest smaller number (given in the table) and find the re-

[1] Here and in the following I translate *radiatio* by "aspect"; cf. Bouché-Leclercq AG p. 246.
[2] I.e. planets, including sun and moon.
[3] These are the entries found in column 1 of Tab. 91 to 114.
[4] In Tab. 91 to 114, column 1: *gradus aequales*; cf. above Tab. 59 to 59b, p. 104 n. 2.
[5] Literally: area.
[6] The "horoscopes" are here not the ascendants but refer to the numbers called *horoscopus* in the columns for the longitudes of the planet and its aspects; cf. the commentary, p. 81.
[7] Instead of Suter's *usque*, or *usque ad*, *sub signum* I read *usque ad* [*numerum*] *sub signum*, assuming a lacuna.

mainder in the table. This shows you sign and degree of the aspect in the above mentioned fashion, with the only exception that the directly found (number), or the (number) next to it, goes with the upper sign (given at the heading), whereas the number found by subtraction often goes with the lower sign.[8]

If the ascendant is located between (the points indicated by) the tenth (degree), or the 15th, or the 20th, or the 25th, or the 30th, then two aspects have to be used, namely from the two sub-tables which concern the numbers between which the ascendant is contained. For example, if it falls between (5 and) 10, the first aspect is to be established from the sub-table for five – that is the first one –, the second (aspect) from the sub-table for ten, that is the second one; and in similar fashion for the rest. After this has been done, one has to note down the difference of these aspects and to divide it by five; the result is to be multiplied by the increment[9] of the ascendant. If the first aspect is the greater one (of the two) then this product has to be subtracted from it; if it is, however, the smaller one, it has to be added. In this way one will obtain the correct aspect for the case in question.

All this is said for the left aspect; but there are also others, that are called right aspects: namely opposite the left hexagon is located the right trine; but opposite the left quartile is the right quartile and opposite the left trine is the right hexagon.

[Second version:][10] If you wish to know the casting of rays[11] of the planets in the sextile, quartile, and trine, then you should know (the sign of) the ascendant and (at) how many (degrees within the sign) its rising occurs, whether at five degrees or 10 or 15 or 20 or 25 or 30.[12] You also should know the degree of the planet whose aspect you wish to find, and the sign in which it (the planet) is located. Thereafter you enter with the degree of the planet in the column of degrees[13] in the table for the horoscopic degree[14] and take what you find corresponding to it for the ascendant (and) for the sign in which the planet is located and keep it in your memory. If you now ask for the hexagon, add to it what is given by the heading of the table for the hexagon, or if you ask for the quartile add what is given by the heading for the quartile,[15] or if you ask for the trine add to it what is given by the heading for the trine. The (resulting) sum you try to find in a similar fashion in the same table, or a (number) which is nearest to it but smaller; and when you find this smaller (number), return to the (column of) degrees[16] in the same way as one returns from the degree of the ascendant to the (corresponding) degree. But you should know that the degree at which

[8] Literally: with the only exception that the sound finding, or the (finding) nearest to a sound one, selects the upper sign, while the diminished (finding) often (selects) the lower (sign).

[9] Literally: addition. What is meant is the excess over a multiple of 5°, the nearest below the longitude of the ascendant.

[10] The preceding version is common to C, N, and O. In C and N (but not in O) a second version of the same chapter follows. This is the same situation as with chapter 37 (cf. above p. 67 note 21) but the present version seems not to be found in Azarqiel.

[11] *projectio lucis*, corresponding to the Greek ἀκτινοβολία; in Tab. 91: *projectio radii*.

[12] One has to change Suter's punctuation in order to obtain the proper parallelism with the first version.

[13] Cf. above p. 78 note 4.

[14] This refers to the heading *horoscopus* in the sub-tables; cf. above p. 78 n. 6.

[15] Text: *trigoni* (misprint for *tetragoni*?).

[16] *gradus equales*; cf. above p. 78 n. 4.

you end up in the (column of) degrees is the (degree) into which falls the left aspect – in the order of the signs – of the planet from the sign which is given by the heading of the table.[17]

You should (also) know (the following): when it happens that a star falls toward the end of the signs which are[18] (listed) in the first line[19] (of the headings) then you will obtain a total – from which as horoscope[20] you have to go backwards (from the interior of the table to the corresponding arguments, sign and degree) – which is greater than (the number) at the end of the sub-table, then subtract from it the number at the end of the sub-table and find (only) the remainder in the same sub-table and take the sign which is given for this number by the heading of the table in the second line.[21]

If the ascendant (is located) between 5 and 10 (degrees of its sign), or between 10 and 15, or between 15 and 20, etc., then determine the aspect for those five (degrees) which are before it, i.e. less, thereafter for the five (degrees) which are after it, i.e. greater, and find the difference. Then find how much the degree of the ascendant exceeds the five (degrees) which are before it and determine the ratio of this difference to five and take the same ratio of the difference between the two numbers (for the aspects), and remember (the result). Then look back at the first aspect, and if it exceeds the second one subtract from it (the number) that I have told you to remember; if (the first aspect is) less (than the second), add it (the remembered number) to it (the first aspect). The result of this addition or subtraction will be the aspect of the planet.

[Addition in C:] Concerning rumors: the ascendant is the ruler and one must investigate the sign of the moon and its (the sign's) ruler. These, depending on (their) being in a strong or in a weak sign, confirm or weaken rumors.

[17] Of the two signs in the heading of the table, the one which is nearest in the sense of the signs has to be chosen.

[18] I emend Suter's *quod est*, or *quae est*, to *quae sunt*.

[19] *semita* usually means column, not line. Probably the two zodiacal signs, listed in each heading, were originally written beside each other and not in two lines as shown in Suter's printed tables.

[20] Again in the sense explained p. 78 n. 6.

[21] *semita* as above in note 19.

### Commentary to Chapter 37

Chapter 37 is preserved in C and N in two consecutive versions. In O, however, only the first version is given.[1] Both versions concern the use of the tables 91 to 114.[2]

The problem consists in determining the "aspects", i.e., sextile, quartile, and trine, to a given point P of longitude $\lambda_P$, at a given moment which is characterized by the rising of a given ascendant H of longitude $\lambda_H$. The "aspects", however, are not simply defined as the points of longitude $\lambda_P + 60°$, $\lambda_P + 90°$, and $\lambda_P + 120°$ respectively, but are in a certain way modified depending on the specific time of the day, defined

[1] Cf. p. 79 n. 10.

[2] Cf. p. 129.

by H. Thus the oblique ascension of the arcs under consideration must play a role, the details of which I have not investigated. There is also a distinction made between "left" and "right" aspects, illustrated by fig. 24.

Finally there is a remark added only in C[3] which connects the ascendant with the confirmation or weakening of rumors. I do not know the source of this doctrine.

The rules for using the tables can be formulated as follows. One enters the main tables with the zodiacal sign of $\lambda_H$ and the corresponding sub-table within the sign

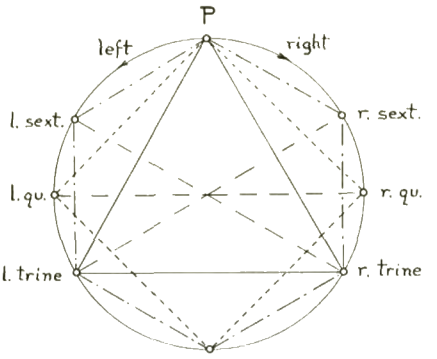


Fig. 24.

for the proper degree (using linear interpolation for degree numbers which are not multiples of 5). This sub-table is now entered with $\lambda_P$, finding (if necessary by interpolation) a number $n$, called for some strange reason *horoscopus* although it is associated with $\lambda_P$ (or later on with the aspect of $\lambda_P$) and not with $\lambda_H$. The heading of the sub-table gives three numbers, one ($s$) for sextile, one ($q$) for quartile, one ($t$) for trine. According to the aspect under investigation one adds now $n$ to $s$, $q$, or $t$. If the result is a number contained[4] in the sub-table just used, then the corresponding longitude, determined by column heading (for the sign) and line (for the degrees) gives the longitudes of the aspect. Otherwise one has to subtract from the sum $n+s$ (or $n+q$, or $n+t$) the last (i.e. greatest) number contained in the sub-table and use the remainder, as before, to find sign and degree of the aspect. – For examples cf. Suter p. 100 f.

[3] Cf. p. 80.
[4] Of course, if necessary, by means of linear interpolation.

## Commentaries to the Tables

For the tables themselves cf. Suter, p. 109 to 231.

In the counting of columns, integers and fractions of the same quantity are always taken as one. Alternative arguments, however, e.g. from 1 to 180 and from 359 to 180, are counted as two columns. This is in agreement with the usual practice in Greek tables (e.g. in the Almagest IV,10; V,8; XI,11 etc.).

### Table 1

"Accumulated Roman, i.e., solar years, divisible by 28". This title actually belongs only to the left section (Section 1) where it appears once more as a subtitle.

**Section 1:** "Accumulated Roman years divisible by 28". This is simply a table of all multiples of 28 from 28 to $75 \cdot 28 = 2100$, intended to simplify the reduction modulo 28 of julian year numbers.[1]

**Section 2:** "Time intervals between the reigns of different kings; years, months, days". The earliest epoch quoted in this table occurs in the last line, the time since Adam. All other epochs are truly historical eras, allowing us to check the given intervals.[2] The following list gives all epochs mentioned in their proper chronological order, together with their julian dates:

($\alpha$) "Adam": – 5380

($\beta$) "diluvium": actually Kaliyuga 1, Chaitra 1 = – 3101 Febr. 17 = julian day 588,465

($\gamma$) "Nabochodonosor": Nabonassar 1, Thoth 1 = – 746 Febr. 26 = 1448,638

($\delta$) "Philippus": Philip Arhid. 1, Thoth 1 = – 323 Nov. 12 = 1603,398

($\varepsilon_2$) "Alexander": actually Seleucid era 1, Syrian norm, Tishri 1 = – 311 Oct. 1 = 1607,739

($\zeta$) "Aszophar": Spanish era[3] 1 = – 37 Jan. 1 = 1707,544

($\eta$) "Christus": A.D. 1, Jan. 1 = 1721,424

[1] Cf. ch. 5 (p. 16). A similar table but of smaller extent, namely from $50 \cdot 28 = 1400$ to $72 \cdot 28 = 2016$, is preserved in manuscript O at the end of Tab. 3, but belonging actually to Tab. 3a; cf. p. 89.

[2] Cf. also the version of CC, fol. 114r, right lower table (below p. 137 and p. 143 ff.).

[3] Cf. for this era Ideler, Chron. II p. 423 ff. or Ginzel, Hdb. III p. 175 ff.; also Kubitschek, Zeitr. p. 77, or Poole, Chron. p. 4. For the name cf. Suter p. 241.

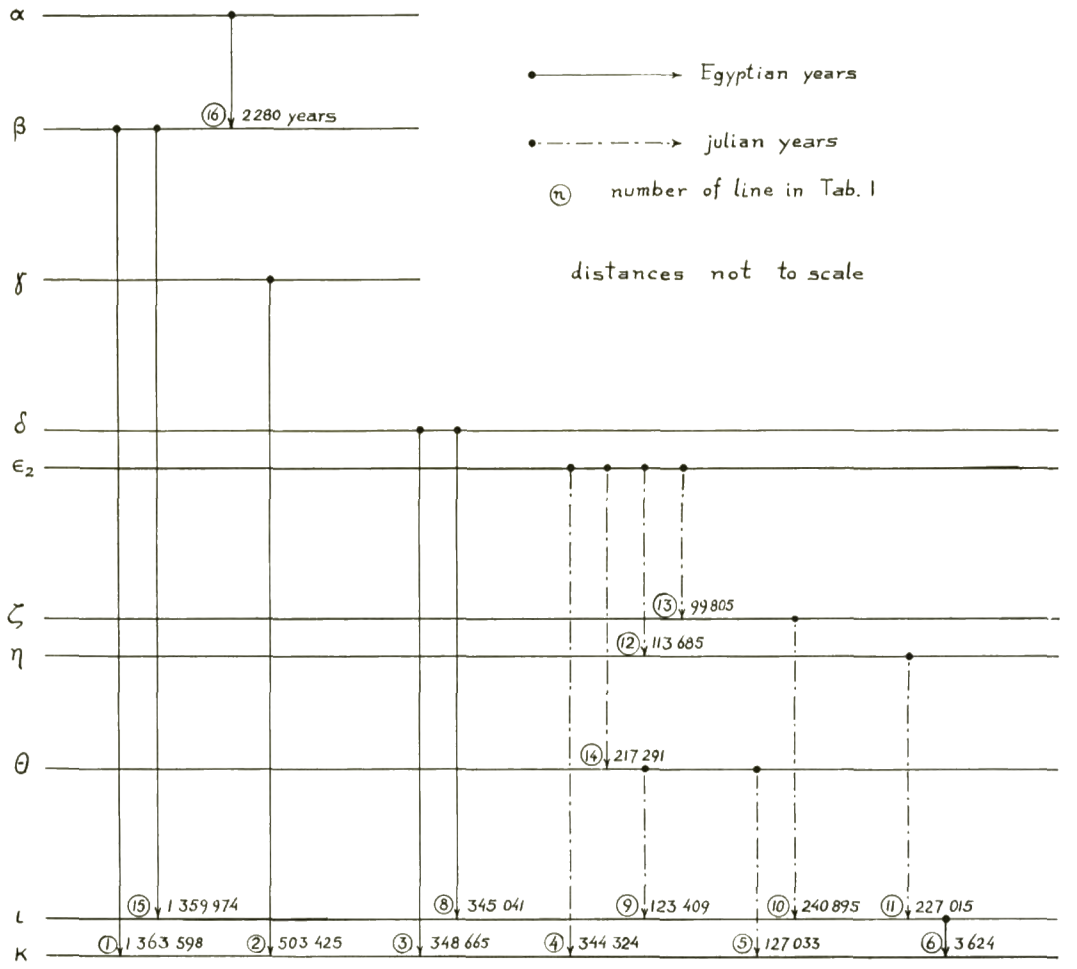


Fig. 25.

(θ) "Diocletian": Diocl. 1, Alexandrian Thoth 1 = 284 Aug. 29 = 1825,030
(ι) "Arabs": A.H. 1, Muḥar. 1 = 622 July 15 = 1948,439
(κ) "Iazdeiardus": Yazd. 1, Farw. 1 = 632 June 16 = 1952,063.

The intervals which correspond to this list agree with the years, months, and days given in Tab. 1, under the assumption that "years", counted from epochs before "Alexander", mean Egyptian years, from then on, however, julian years (cf. fig. 25), except for the interval from (ι) to (κ) where Persian years (of 365 days) are meant.[4] Months are always reckoned as 30 days.[5] In the numbers of the single days one has to make the following adjustments:

[4] Cf. Tab. 3, Section 1a (below p. 88).
[5] This is standard chronological practice; cf., e.g., ch. 5 (above p. 16).

| | | | |
|---|---|---|---|
| text line | 4: | $17\,^1/_2$ days | read: $18\,^1/_2$ days[6] |
| | 10: | 17 | 15 |
| | 12 and 13: | 0 | 2. |

An era "from Adam" is also mentioned by al-Bīrūnī[7] in the following words: "For some (of the Christians) count the interval between Adam and Alexander as 5069 years, others as 5180 years; the majority uses the latter number".

Bīrūnī means here julian years. With these units we find (cf. also fig. 25 p. 83):

| | | |
|---|---|---|
| from (β) to (κ) | $1363{,}598^d$ | line 1 in Tab. 1 |
| from ($\varepsilon_2$) to (κ) | $344{,}324^d$ | line 4 in Tab. 1 |
| thus from (β) to ($\varepsilon_2$) | $1019{,}274^d = 2790$ jul. y. $+ 226\,^1/_2{}^d$. | |

In the last line of Tab. 1 (line 16) we are told that between (α) and (β) there are 2280 (Egyptian) years,[8] i.e. $832{,}200^d = 2278$ jul.y. $+ 160\,^1/_2{}^d$. Consequently we find for the total from Adam (α) to Alexander ($\varepsilon_2$)

$$(2278 \text{ j.y.} + 160\,^1/_2{}^d) + (2790 \text{ j.y.} + 226\,^1/_2{}^d) = 5069 \text{ j.y.} + 22^d$$

which is Bīrūnī's first figure, except for the 22 days.

The Christian versions for the epoch of "Adam" seem to be irreconcilable with the Islamic tradition followed by our text. The "*Liber generationis*", e.g., gives as the interval from Adam to Deluge 2242 years[9] instead of our difference (β) − (α) = 2279 years. All the schems in the Christian versions seem to be connected with a Byzantine world era of an epoch near 5503 B.C.[10]

For the "Era of the Flood" (β) cf. Abū'l Ḥasan Kūshyār ibn Labban (about A.D. 1000) in Ideler, Chron. II p. 625 ff. and Bīrūnī, Chronol. p. 27 ff. and p. 133.

[6] The manuscript N has 28 1/2. The correct number occurs, e.g., in Bīrūnī, Astrol. § 282. N also contains an addition by a later hand: "From Christus to Yazdigerd 631 (years), 5 (months), 14 (days)" which should read however 631 years, 5 months, 19 days.

[7] Chronol. p. 300,6.

[8] This leads to −5380 Sept. 9 = jul. day 9,756,265 as the date of (α), assuming that no days are missing.

[9] Grumel, Chron. p. 10.

[10] Grumel, Chron. p. 11, p. 223.

## Table 2[1]

"Table for finding the day (of the week) in which any Arabic lunar month begins". Cf. ch. 4 (above p. 15) and Appendix p. 135 and p. 221.

Addition in MS C: "one has to enter this lunar table and the other table on the conjunction of sun and moon (Tab. 69 to 72) with the number of the current year, the other ones with the number of the past (year)". This is indeed correct: the entries in Tab. 4 to 19 mean completed years, months, etc., whereas Tab. 2 and Tab. 69 to 72 operate with current dates (cf. p. 108). Essentially the same remark is found in the MS CC fol. 142$^r$ (cf. below p. 207).

**Section 1:** Column 1: "Accumulated Arabic years", i.e. Arabic years in 30-year steps, from A.H. 30 to originally A.H. 570[2] (in MS O), then extended to A.H. 720.[3]

[1] Cf. also CC fol. 114$^r$ left table (below p. 135) and fol. 142$^r$ (below p. 206).

[2] From A.D. 650/651 to 1174/1175.

[3] A.D. 1320/1321.

Column 2: "Weekdays" (*notae*) i.e. day-numbers ($w$) in the week, counting from Sunday as $w = 1$. The tabulated values show a constant difference of 5 (mod. 7) because[4]

$$30 \text{ Arabic years} = 30 \cdot 354\ 11/30 \text{ days} = 10631 \text{ days} \equiv 5 \text{ mod. } 7.$$

**Section 2:** Column 1: "Single (Arabic) years" from 1 to 30. The following eleven years are intercalary, i.e. 355 days long instead of 354 days of an ordinary year:

2 5 7 10 13 16 18 21 24 26 29.

The word *kebice*[5] in column 3 of this section should refer to these years but five of the entries are misplaced, at least as printed, (concerning the years 5, 18, 21, 24, and 29) and one (year 26) is omitted.

Column 2: "Weekdays" with differences 4 (mod. 7) for ordinary years, because $354 \equiv 4$ mod. 7, and differences 5 (mod. 7) for intercalary years.

Column 3: cf. column 1 of this section.

**Section 3:** "Arabic lunar (months)". Column 1: "Months of the Arabs" i.e. names of the Arabic months and their length, alternately $30^d$ (beginning with Muḥarram) and $29^d$.

Column 2: "Weekdays" with alternating differences 2 and 1 (mod. 7) because $30 \equiv 2$, $29 \equiv 1$ (mod. 7).

**Use of Tab. 2.**

Extending the scheme of Section 1 one line backwards we obtain for the 30-year intervals the following beginnings:

| year | | |
|---|---|---|
| | 0 | $w = 0$ |
| | 30 | 5 |
| | 60 | 3. |

The first entry for single years in Section 2 is

$$1 \qquad w = 4$$

while Section 3 concerns the first day of each month, beginning with

$$\text{Muḥarram } 1 \qquad w = 1.$$

For the first day of the era Hijra we have 0 as accumulated 30-year period, year 1 for the current year A.H. 1, and Muḥarram 1 as current day. This gives for the day of the week

$$w = 0 + 4 + 1 = 5 \qquad \text{i.e. Thursday}$$

which is correct for A.H. 1 Muḥ. 1 = A.D. 622 July 15 (cf. fig. 1 p. 10 and p. 83 epoch 1). This gives the reason for the initial parameters in Tab. 2.

[4] Cf. ch. 1, above p. 11.
[5] Cf. ch. 1, above p. 11.

Example: find the weekday of A.H. 31 Safar 1:

| | |
|---|---|
| 30 years: | $w = 5$ |
| 1 year: | 4 |
| Safar 1: | 3 |
| | $w = 5$ |

which is correct (cf. p. 109 fig. 33).

## Table 2a[1]

"Table for finding the day (of the week) in which any Persian month begins". The same table is found in Azarqiel's Almanac.[2] Cf. also Appendix p. 136 and p. 222.

**Column 1:** "Number of the Persian (months)" i. e. current number $m$ of the Persian months, from $m = 1$ to 12.

**Column 2:** "Names of the months", beginning with "Affordime", i.e. Farwardin.

**Column 3:** "Days", giving the length of the months, i.e. $30^d$ each, except for the 8th month with $35^d$.

**Column 4:** "Signs or weekdays (*notae*)" arranged in seven columns, representing the years $n = i + 7 \cdot k$ for $i = 1, 2, \ldots, 7$ and $k = 0, 1, 2, \ldots$ of the era Yazdigerd. If $n \equiv i$ mod. 7 then the number in column $i$ and line $m$ gives the weekday $w$ of year $n$, month $m$, day 1.

The entry for Yazd. 1 Farw. 1 is $w = 3$ (Tuesday), which is correct for A.D. 632 June 16 (cf. p. 83, epoch κ). From this initial date all subsequent entries are obtainable by adding 1 for each Persian year, because $365 \equiv 1$ mod. 7. For each month one has to add 2, because $30 \equiv 2$ mod. 7, except for $m = 9$ which has the same entry as $m = 8$ because $35 \equiv 0$ mod. 7.

Marginal note in MS M: "Note for the translation of terms: *elwacat* is the mean motion, *elheza* is anomaly, *kamar* is the moon, i.e., lunar month in Arabic, *arin* is the earth,[3] *elkebice* is the year".[4]

[1] Cf. CC fol. 114r right upper table (below p. 136).
[2] Millás-Vallicrosa, Azarquiel, p. 155.
[3] For this use of *Arin* cf. also Price, Equatorie, p. 169 s.v. *centre aryn*. I owe this reference to Prof. A. Sachs.
[4] Sic, instead of intercalary year.

## Table 2b

"Table for finding the day in which any Egyptian month begins" i.e. month of the Alexandrian calendar.[1] The same table is found in Azarqiel's Almanac.[2]

**Column 1:** "Remaining Egyptian years" i.e. residues $i$ mod. 28 of julian years of the Diocletian era for $i = 1, 2, \ldots, 28$. Leap-years are the years Diocl. $n + i$ for

[1] In the MS CC fol. 115r (below p. 142), the title of this table is "Table to know on what weekday any month of the Egyptian years begins, which are called *Kepthie* (i.e. Coptic) in Arabic".
[2] Millás-Vallicrosa, Azarquiel, p. 155.

$i = 2, 6, 10, 14, 18, 22, 26$

denoted as *bissextus* in the last column.[3]

**Column 2:** first line of heading: numbers 1 to 12; second line: number of days in each month, i.e. 30 except for the last one which contains 35 1/4 days; third line: the late Egyptian names from *tut* (= Thoth) to *muzre* (= Mesore). If $n = i$ mod. 28 then the number in the column of months $m$ and in line $i$ gives the weekday $w$ for year $n$, month $m$, day $i$.

The entry for Diocl. 1, Thoth 1, is $w = 6$ (Friday) which is correct for A.D. 284 Aug. 29 (cf. p. 83, epoch Θ). From this initial date all subsequent entries are obtainable by adding 1 for each ordinary year (because $365 \equiv 1$ mod. 7) and 2 for each leap-year. For each month one has to add 2, except for the last month which contributes 0 in ordinary years, 1 in leap-years (because $35 \equiv 0$, $36 \equiv 1$ mod. 7).

Marginal note in N: "The year of the Egyptians without intercalary day begins on August 29, (a year) with intercalary day on the before-mentioned August 30, according to Isidorus". These are the correct dates for Thoth 1 of the Alexandrian calendar. The quotation comes from Isidorus, *De natura rerum*, ch. VI, 7 (Migne, PL 83 col. 974 B). The beginning of this passage is found with Tab. 3 (cf. p. 89).

[3] The intercalary days fall on August 29 of A.D. 287, 291, 295, ...

## Table 3

"Table for reducing Alexandrian years to (years of) the Hijra". Actually, however, one enters with (completed) Arabic years and months and finds (completed) years etc. of the Seleucid era.

**Section 1:** Column 1: "Accumulated Arabic years" i.e. Arabic years in 30-year steps from A.H. 0 (*radix*) to A.H. 600,[1] in MS C extended to A.H. 690.[2]

Column 2: "Accumulated Alexandrian years with intercalations", giving "Years, months, days, fractions (of days)", i.e. julian years of the Seleucid era.

The initial correspondence is:

A.H. 0 = 932 jul.y. 9 months 17;0 days from beg. of Sel. era or 340,700$^d$ between the epoch days of the Seleucid era and the Hijra in agreement with the difference $(\iota) - (\varepsilon_2)$ in the list on p. 82 f.[3]

All subsequent lines can be found by addition of the constant difference of 29 jul. y. 1 month 8 3/4 days, the equivalent of 30 Arabic years.[4] The later extension of the table in MS C is, however, 5 1/4 days short of one year.[5] Thus one should have for the number of days:

[1] A.D. 1203/4.
[2] A.D. 1291.
[3] The above equation can be read: A.H. 0 completed = A.H. 1 Muḥ. 1 = 932 complete years of the Seleucid era + 9·30 + 17 days = Sel. era 933 Oct. 1 + 287 days = A.D. 622 July 15.
[4] Cf. chapter 5, in particular p. 17 n. 13.
[5] This is expressly stated in ch. 5 (p. 16).

A.H. 630: 5 0 instead of 9 1/4
660: 13 3/4 18 0
690: 22 1/2 26 3/4.

**Section 2:** Column 1: "Single Arabic years" from 1 to 30.

Column 2: "Single Alexandrian years" giving "Years, months, days, fractions (of days)". For an ordinary Arabic year the difference is $354^d$, for a leap-year[6] $355^d$. Thus the difference in the table is 11 months 18 $3/4^d$ or 11 months 19 $3/4^d$ respectively while the number of julian years increases by 1. The total for 30 Arabic years is correctly 29 jul. y. 1 month 8 $3/4^d$ (cf. sect. 1, col. 2).

**Section 3:** Column 1: "Arabic months" i.e. list of the names of the months in their order.

Column 2: "Total of months" giving "months" of 30 days length and "days". For the final column cf. Tab. 3a.

**Additions to Tab. 3 in MS N.**

Before Section 2, two more sections are inserted (Suter p. XV f.):

**Section 1 a** (Suter A): "Years of Yazdigerd after the era of the Arabs" comparing Persian (= Egyptian) years with steps of 30 Arabic years. The difference is correctly 29 Eg. years, 1 month, 16 days, or, after the accumulation of 12 months, only 11 days, taking into account the 5 epagomenal days. The entry for the *radix* (A.H. 0) is the same as the interval given in Tab. 1, namely 9 years 11 months 4 days. This means here that the era Hijra (ι) precedes the era Yazdigerd (κ) by 9 Persian years, 11 months, 4 days + 5 epagomenal days. In this way one obtains the correct interval of 3624 days for (κ) − (ι). This also explains the apparent discrepancy, noticed by Suter (p. XV, A), between 4 and 9 days in our table and a corresponding table in al-Battānī.

**Section 1 b** (Suter C): "Completed years of Diocletian". The years of the era Hijra are here paralleled, in steps of 30 Arabic years, with julian years of the era Diocletian. As the *radix* A.H. 0 we find here 337 years, 10 months, 19 3/4 days, which is 1/2 day less than given in Tab. 1 line 9. The difference corresponding to 30 Arabic years is correctly 29 julian years, 1 month, 8 3/4 days (or 5 1/4 days less after the accumulation of 12 months).

Errors: year 90: 2 days instead of 1 day; year 210: 7 days instead of 6 days. From year 600 to year 690 the months are counted in excess of a completed year, obviously representing a later addition by an incompetent scribe.

After Section 2 is inserted:

**Section 2 a** (Suter B): "Years of Yazdigerd". The difference is 11 months 24 days = 354 days = 365 − 11 days in ordinary Arabic years, 11 months 25 days in intercalary years.

Error: year 27: read 18 days instead of 19 days.

There follows, in a late hand, a table for a comparison of accumulated Arabic

[6] For the list of them in a 30-year cycle cf. Tab. 2, sect. 2, col. 1 (p. 85).

years and years of the Christian era (cf. for the details Suter p. XVIII,4). Finally a somewhat garbled version of Isidorus, *De natura rerum* VI,7 is given, except for the final sentence which is found with Tab. 2b (cf. above p. 87).

## Table 3a

"Table for finding on what weekday any Roman solar month begins". The same table is found in Azarqiel's Almanac.[1]

Column 1: "Single years Roman" i.e. julian years, from 1 to 28. The manuscripts C, M, and N have attached to Tab. 1 a multiplication table for 28 while manuscript O has a shorter table of the type associated with Tab. 3. Actually these tables belong to Tab. 3a. The table in O would cover the reduction modulo 28 for the year numbers from 1400 to 2016. Since julian years are used here only from the Seleucid era on (cf. fig. 25 p. 83), this interval corresponds to the range from A.D. 1117 to 1705. The table in Tab. 1 Sect. 1 would extend from −311 to 1789.

Main table: heading line 1: Syriac month names from "First Tishri" to "Elul". Line 2: "Number of days of months". Line 3 gives these numbers for each month, in particular 31 1/4 days for month III ("First Kanūn" = December). Line 4: Roman month names from October to September. Between III (December) and IV (January) a column "Leap-years" is inserted, declaring the years

3, 7, 11, 15, 19, 23, 27

as leap-years. In these years the First Kanūn = December has 32 days.[2] According to this table year 1 Oct. 1 is a Monday, $w = 2$. This is correct for the epoch of the Seleucid era −311 Oct. 1 (cf. Tab. 1, epoch $\varepsilon_2$, p. 82).[3] From this initial day number one obtains all the others by adding 3 for each month of 31 days ($31 \equiv 3 \bmod 7$), 4 for a December in a leap-year, 2 for each month with 30 days, 0 for February.

MS C contains a marginal note beset with grammatical difficulties which make a translation almost impossible; but, whatever the correct rendering would be, it seems clear that the procedure under discussion can not be an adequate rule for the determination of weekdays by means of Tab. 3a. The following can only be considered as a hypothetical rendering of the text: "Note: through Arabic years and months to Alexandrian years and months, after 5 days and a quarter were subtracted from each year, you add those which have been subtracted from the number of days found, (but) you subtract the excess over 30 of the days of the months, beginning with October to the (given) present (month); then you subtract 28 as many times as possible from the total of years and then you enter (the table) with the remainder and you will find (what you were asking) (in the table) beside the year(-number with which you entered) if the day(?) amounts to (?) the (?) 10 months".

[1] Millás-Vallicrosa, Azarquiel, p. 154. Cf. also the version of CC fol. 114$^v$ (below p. 140).

[2] Cf. chapter 2 (p. 11). According to CC fol. 144$^v$ this is the norm of the "Chotenian Christians" (cf. below p. 224).

[3] Thus the following years are leap-years: −309, −305, −301, ...

## Tables 4 and 5

"Mean (position) of the sun". Cf. chapter 7 (p. 17); also CC fol. 115$^{v}$, 116$^{r}$ (p. 146 ff.).

**Tab. 4, Section 1:** Column 1: "Accumulated years" i.e. Arabic years in 30-year intervals, from A.H. 0 (*radix*) originally to A.H. 570, with later extension to A.H.720.[1]

Column 2: "Mean (longitude) of the sun in the accumulated years" i.e. at the years listed in column 1; giving "signs, degrees, minutes, seconds".

According to this table the mean sun had the longitude ♋ 23;25,48 at the completion of A.H. 0, i.e. at A.H. 1 Muḥ. 1 (cf. also fig. 33 p. 109 and p. 94). After 570 Arabic years the mean sun was, according to the table, in ♋ 24;29,52. The total solar motion during this interval is therefore an integer number of rotations plus 1;4,4°. The number of complete rotations follows immediately from the fact that 570 Arabic years correspond approximately to 553 julian years. Thus the sun travelled a total of 9,13 ·6,0 +1;4,4 = 55,18,1;4,4° during 570 Arabic years of 5,54;22 days each, i.e. during 56,6,29 days. In this way we find that our tables are based on a mean solar motion of

$$55,18,1;4,4 : 56,6,29 = 0;59,8,10,21, \ldots {}^{o/d}.$$

**Tab. 4, Section 2:** Column 1: "Single (Arabic) years" from 1 to 30.

Column 2: "Mean (motion) of the sun in single years". In ordinary years 11$^{s}$ 18;54,13°, in intercalary years 11$^{s}$ 19;53,21°.[2]

Error: for year 14 read 6$^{s}$ 29;34,44 instead of 29;34,45.

**Tab. 4, Section 3 upper half, and Tab. 5:** Column 1, Tab. 4: "Arabic months"; Tab. 5: "number of days", "number of hours", "parts of hours".

Column 2, Tab. 4: "mean (motion) of the sun in Arabic months"; Tab. 5: ". . . in days", ". . . in hours", ". . . in parts of hours".

The "parts of hours" always progress in steps of 2 minutes. The table for months is based on a daily motion of 0;59,8,10° which leads to alternating differences of 29;34,5° in full months and 28;34,57° in hollow months.

Errors in Tab. 5, section 3, column 2: at entry 4: should be 0;0,10 instead of 0;0,8; at entry 34: should be 0;1,24 instead of 0;1,25. Both values are correctly given in CC.[3]

**Tab. 4, Section 3 lower half:** Column 1: "Names of signs", from Aries to Pisces.

Column 2: "Mean (longitudes) of the sun at the beginning of each sign", i.e. longitudes $\bar{\lambda}$ of the mean sun as function of the true longitude $\lambda$ for ♈ 0°, ♉ 0°, . . . , ♓ 0°.

Error: for Scorpius read 7$^{s}$ 1;36,55 (following MS O) instead of 1;26,55 (as Suter, following MS C).

This auxiliary table permits us to compute the equation of center $\Theta = \lambda - \bar{\lambda}$ for the true sun at $\lambda$ = 0, 30, . . .

[1] From A.D. 621/2 to 1174/5 and 1320/21 respectively.

[2] Or 0;0,1° more since the accurate values would be 18;54,13,3,54, . . . and 19;53,21,14,15, . . . respectively.

[3] Cf. below p. 147 n. 3.

| λ | Θ | λ | Θ |
|---|---|---|---|
| ♈ 0° | +2;11,48 | ♎ 0° | −2; 9,37 |
| ♉ | +1;41,22 | ♏ | −1;36,55 |
| ♊ | +0;42,27 | ♐ | −0;40,40 |
| ♋ | −0;28,20 | ♑ | +0;26,18 |
| ♌ | −1;29,58 | ♒ | +1;24,32 |
| ♍ | −2; 9, 7 | ♓ | +2; 5, 0 |

The extremal values (cf. fig. 26) are approximately 2;15°. For details cf. the discussion of Tab. 21 to 26, column 3 (below p. 95), where Θ is tabulated as function of the mean sun's distance $\bar{\kappa} = \bar{\lambda} - \bar{\lambda}_A$ from the apogee.

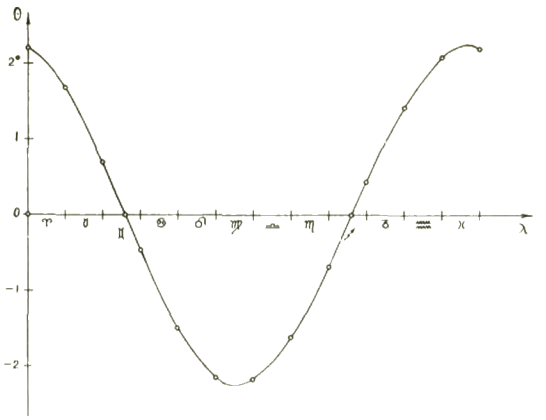


Fig. 26.

The values in Tab. 4 agree with the values from Tab. 21 to 26. We find, e.g., in Tab. 4 that $\bar{\lambda} = 10^s\ 27;55°$ should correspond to λ = ♓ 0°. Since $\lambda_A = 2^s\ 17;55$ we have to enter Tab. 24 with $\kappa = 8^s\ 10°$ and find $\Theta = +2;5,28° \approx +2;5°$. Thus $\lambda = \bar{\lambda} + \Theta = 11^s\ 0°$ as expected.

MS C contains the following additions: "Note that at the entry in Libra the mean motion of the sun should rather be 6 2;10,37,11,[4] at the entry in Capricorn $9^{(?)}$ 8 29;35,42".[5] This, however, is not correct, since the values of the original table agree exactly with the results obtainable from Tab. 24 and 26.

[4] As compared with $6^s\ 2;9,37°$.
[5] As compared with $8^s\ 29;33,42°$.

## Tables 6 to 8

"Mean (position) of the moon and its anomaly". Cf. also CC fol. 116v, 117 (p. 155 ff.).

**Tab. 6, Section 1:** Column 1: "Accumulated Arabic years" in 30-year steps, from A.H. 0 (*radix*) to A.H. 570, with later extension to A.H. 720. The MS CC, however, goes only to A.H. 510.[1]

[1] For later additions cf. below p. 157.

Column 2: "Mean (longitude) of the moon in the accumulated (years)".

Column 3: "Anomaly of the moon in the accumulated (years) ". No values entered for A.H. 600 to 720.

**Tab. 6, Section 2:** Column 1: "single (Arabic) years".

Columns 2 and 3: as in Section 1. Error: for year 26 read as mean longitude $2^s$ 26;56,20 as correctly given in O and C.

**Tab. 7 and 8:** Corresponding tables for "Arabic months", for "days", "hours", "parts of hours", the latter proceeding in steps of 2 minutes.

Errors:

Tab. 7, Sect. 1, col. 2: month XI: read $10^s$ 22;18,56 (instead of 22;14,56)
col. 3: month III: read $2^s$ 22;47,4 (instead of 22;46,4)
month V: read $4^s$ 13;37,8 (instead of 13;36,9);
in all three cases O and CC are correct.
Sect. 2, col. 3: day 1: read 13;3,54 instead of 13;3,53 (correct in CC)
Tab. 8, Sect. 2, col. 2: 26 min.: read $0^s$ 0;14,16 (instead of 0;14,12)
col. 3: 42 min.: read $0^s$ 0;22,52 (instead of 0;23,52)
44 min.: read $0^s$ 0;23,57 (instead of 0;24,57);
in all three cases CC is correct.

The entries in Tab. 6 for 570 as well as for 720 years lead to a mean motion in longitude of 13;10,34,52,46, . . .$^{o/d}$ whereas chapter 20 (cf. p. 42) gives 13;10,34,52, 48$^{o/d}$. From Tab. 7, entry for 30 days, one would find 13;10,34,52$^{o/d}$.

For the mean anomaly one finds in Tab. 6 from the entry for 540 years: 13;3,53, 58,49, . . .$^{o/d}$ and from Tab. 7, entry for 30 days: 13;3,53,58$^{o/d}$.

The epoch values for A.H. 1 Muḥ. 1 are:

$$\bar{\lambda} = ♋\,27;45,17 \qquad \alpha = 3^s\,14;36,1.$$

In the MS C is found the following note on the lower margin of Tab. 6[2]: "Multiply the lunar days by 13 and add this product to the place in which the moon follows the sun and you will find the place of the moon in signs and degrees. Subtract the days of the moon from the days of the sun. Observe that the epicycle[3] of the moon imitates the motion of the firmament, so that it (the moon) may look at the sun". I do not understand what is meant by these sentences.

MS O contains a marginal note above Tab. 7,[4] which is a parallel version of the end of the note in C. After some Arabic words which apparently seemed unintelligible to Suter, we read: "Observe that the moon follows in the epicycle[5] the motion of the firmament so that it (the moon) may look at the sun".

The lower margin of Tab. 7 in MS O contains the following remark:[6] "According to Aumonius (= Ammonius) Saturn, Jupiter, and Mars will be at their first station

[2] Suter p. 117.
[3] *felek elthedewir* = circle of rotation = epicycle; cf. Suter p. 246 s.v. *Tedewir*.
[4] Suter p. 118.
[5] Cf. note 3.
[6] Suter p. 118 and p. 236 n. 5.

if the interval from the beginning of their anomaly and the sun[7] to their center amounts to 120 degrees. And again if there are another 120 (degrees) between their center and the sun, then they will be in second station. One also has to know that Venus does not become more than 47 (degrees) distant from the sun, Mercury not more than 7 (sic) (degrees)".

[7] The apogee of the epicycle, which has the same longitude as the (mean) sun.

## Tables 9 to 14

"Mean motion of Saturn", ". . . of Jupiter", ". . . of Mars" respectively. Cf also CC fol. 118 to 120 (p. 159).

**Main Tables:** "Mean (motion in longitude) of Saturn", ". . . of Jupiter", ". . . of Mars" i.e. $\bar{\lambda}$ in fig. 4 p. 23. Tabulated for the customary intervals: "Accumulated (Arabic) years" i.e. 30-year steps from A.H. 0 (*radix*) to A.H. 570, "single years", "Arabic months", "days", "hours", and "parts of hours" i.e. multiples of 2 minutes.

**Auxiliary Tables** (right lower half of Tab. 9, 11, 13):

Column 1: "Days and their parts" or "days and hours".

Column 2: "The above-mentioned[1] anomaly of Saturn"[2], ". . . of Jupiter", ". . . of Mars". Here is tabulated the increase of the mean anomaly $\alpha = \bar{\lambda}_{\odot} - \bar{\lambda}$ for multiples of days and hours;[3] cf. the commentary to chapter 14 (above p. 32).

Epoch values for A.H. 1 Muḥ. 1:

♄: ♋ 27;58,37 ♃: ♓ 0;16,49 ♂: ♏ 0;25,15.

Mean motion in longitude from entry for 540 years:

♄: 0;2,0,22,57, . . .$^{o/d}$ ♃: 0;4,59,9,8, . . .$^{o/d}$
♂: 0;31,26,28,6, . . .$^{o/d}$.

Cf. for these mean values also Burckhardt [1] who relates them to Brahmagupta's parameters.

Marginal notes to Tab. 10 in MS M, written by first hand: "*zohal* is Saturn, *el-muster* is Jupiter, *almareh* is Mars, *eladel*(?) is the sun, *zohara* is Venus, *autarid* is Mercury, *kamar* is the moon, *elmukatil keiwen* is the equation of Saturn,[4] *meil elshemz* (is the) declination of the sun, *ard el* (is the) latitude".

[1] The "above-mentioned" (*supradictum* or *praedictum*) corresponds to the "above-mentioned anomaly" in the last paragraph of chapter 14 which concerns these auxiliary tables.

[2] In MS O is written above this title (Tab. 9, Suter p. 120): "Note that this represented(?) the anomaly of the epicycle; in(?) the epicycle .... which is the anomaly". MS C has beside the title: "Note that this (is the) anomaly (of the) epicycle".

[3] In Tab. 9 for 60 days read $1^8$ instead of $0^8$.
Tab. 11, 25 days: read 22;33,45 instead of 22;53,45 (correct in CC).
Tab. 14, 28 days: read 14;40,21 instead of 14;40,51.

[4] Actually *elmukatil* is only another name for Saturn (*keiwen*); cf. Suter p. 245 s.v. *mukatil*. For correction of spellings cf. Suter p. 121, apparatus.

## Tables 15 to 18

"Anomaly of Venus", ". . . of Mercury" i.e. mean motion of the epicyclic anomaly ($\alpha_1$ in fig. 12 p. 29). Tabulated for the same time intervals as in Tab. 9 to 14, main table.[1] Cf. also CC fol. 121 and 122 (p. 170 ff.).

Epoch values for A.H. 1 Muḥ. 1:

$$\left.\begin{array}{l} ♀\colon\ 1^s\ 16;\ 0,\ 3 \\ ☿\colon\ 2^s\ \ 8;53,10 \end{array}\right\} \bar{\lambda} = \bar{\lambda}_{\odot} = ♋\ 23;25,48.^2$$

Mean motions from entries for 540 years:

♀: 0;36,59,34,13, . . .$^{o/d}$ ☿: 3;6,24,8,6, . . .$^{o/d}$.

A marginal note in MS N which is described as "put in order by Robert of Chester"[3] contains with Tab. 15 the remark:[4] "The additions (*adjectiones*) for Venus and Mercury are (the same as) the addition for the sun; all these additions were computed in this book for the city of London and are added to the mean motions of the planets".

For the three outer planets and for the lunar nodes the corrections for geographical longitudes are found in marginal notes to the corresponding tables and listed by Suter as follows:

♄: 0;0,18° ♃: 0;0,45° ♂: 0;4,49° ☊: 0;0,30°.

Using the known mean motions for these bodies[5] one finds as corresponding time differences[6]

♄: 3;35$^h$ ♃: 3;37$^h$ ♂: 3;38$^h$ ☊: 3;46$^h$,

the small variations being the result of roundings in the given values of the corrections. Nevertheless it is clear that a time difference of about 3;40$^h$ must be close to the truth; thus we can say that the meridian of London is assumed to be about 55° to the west of the zero meridian.

If we assume that London is about 1° to the east of Winchester and if we give Winchester a longitude of about 19;30° from the Western Sea,[7] then we get for the zero meridian used by Robert of Chester a longitude of about 75;30, counted from the Western Sea, or about 14;30 west of Arin. This could be the meridian of Baghdad.[8]

[1] Error in Tab. 15 month V: read: 1;14,57 instead of 1;14,17.
[2] These are the same values as given in Tab. 4, Sect. 1 (cf. p. 90).
[3] Cf. above p. 9 n. 2.
[4] Cf. Suter p. XVIII No. 7 and Haskins SHMS p. 123.
[5] p. 93 and p. 94 respectively.
[6] The correction for the sun and the inner planets is omitted but would amount to 0;9,3°.
[7] Cf. p. 229.
[8] Cf. p. 111 n. 5.

## Tables 19 and 20

"Mean motion of the ascending node (of the moon)"; tabulated is the longitude $-\lambda_{☊}$ of the ascending node (cf. fig. 3 p. 21). Cf. also CC fol. 123 (p. 177 ff.). For variants from MS N cf. Suter p. XIX No. 9.

Epoch value for A.H. 1 Muḥ. 1: ♏ 23;47,38. Mean motion from entry for 540 years: – 0;3,10,48,22, . . .$^{o/d}$.

For a supplementary table for Cordoba noon and for the years A.H. 360 (A.D. 970) to 570 (A.D. 1174), presumably Muḥarram 1, cf. Suter p. XIX.

Error: year 18: read 8;5,53 instead of 8;5,13.

## Tables 21 to 26

"Equation of sun and moon" and, not mentioned in the title, the declination of the sun and the latitude of the moon. Cf. chapters 8 and 9 (p. 19 ff.) and chapters 15 and 16 (p. 33); also CC fol. 124 to 126 (p. 184 ff.).

**Columns 1 and 2:** "Columns of numbers", i.e. signs and degrees from 1° to 180° and, in inverse order, from 180° to 359°. Subtitles (omitted in CC): Column 1: *computatio minuenda et augmentum examinis et numeri*; Column 2: *computatio augenda et diminutio examinis et numeri.*

In order to find a correct interpretation of these headings one has to use the corresponding headings of the first two columns in Tab. 27 to 56 (cf. below p. 98). There it is clear that *augmentum* and *diminutio* refer to the increasing numbers in column 1 as compared with the decreasing sequence of column 2. Thus we have to translate *augmentum examinis et numeri* as "increasing (arguments) for the equation (of sun and moon) and of (the remaining) numbers (which concern solar declination and lunar latitude)"[1] – and similarly for the decreasing arguments. Thus we are left with *computatio minuenda* and *computatio augenda.* This must mean that the resulting equations are negative for arguments in column 1, positive for arguments in column 2. Thus we have to translate something like "in computing (the equations of sun and moon) to be used subtractively" and ". . . additively" in column 2.

One must admit that titles with such a formulation cannot have been of much use to an uninitiated computer.

**Column 3:** "Equation of the sun". In the first and fourth quadrant "slow motion", in the second and third "fast motion". The argument is $\bar{\kappa} = \bar{\lambda} - \lambda_A$ i.e. the distance of the mean sun from the apogee (cf. fig. 2 p. 20). The position of the solar apogee is given in ch. 8 (above p. 19) as $\lambda_A = 77;55°$.

As Kennedy and Muruwwa ([1] p. 118) have discovered, the method of computation of the solar equation is described by al-Bīrūnī as the "method of declination". This means that the equation $\Theta$ is found from

$$\Theta = \frac{\Theta_{max}}{\varepsilon} \cdot \delta$$

where $\Theta_{max} = 2;14$ is the maximum equation, $\varepsilon = 23;51$ the obliquity of the ecliptic, or maximum declination, and $\delta$ the declination of the sun, tabulated here in column

[1] Obviously it would be better if we had *augmentum examinationis et numerorum* but such variations in terminology occur everywhere in these texts.

5. Exactly the same value for the extremal equation is used in the Khaṇḍa-Khādyaka I,16, but the remaining values are simply given by

$$\Theta = \Theta_{max} \sin \alpha.$$

This is evident from the following comparison in which $0;5,37,6 = \frac{2;14}{23;51}$:

| | δ al-Khw. | δ·0;5,37,6 | Θ al-Khw. | Θ Kh.-Kh. | 2;14·sin α |
|---|---|---|---|---|---|
| 15 | 6; 0,25 | 0;33,44,56 | 0;33,45 | 0;35 | 0;34,41 |
| 30 | 11;39,50 | 1; 5,31,52 | 1; 5,32 | 1; 7 | 1; 7, 0 |
| 45 | 16;36,49 | 1;33,20,27 | 1;33,21 | 1;35 | 1;34,45 |
| 60 | 20;29,50 | 1;55, 9,37 | 1;55,10 | 1;56 | 1;56, 3 |
| 75 | 22;59,24 | 2; 9, 9,56 | 2; 9,10 | 2; 9 | 2; 9,26 |
| 90 | 23;51, 0 | 2;13,59,50 | 2;14, 0 | 2;14 | 2;14, 0 |

Error in Tab. 23: at $2^s\,20$ read 2;11,50 instead of 2;11,44 as all MSS have, including CC.

**Column 4:** "Equation of the moon". In the first and fourth quadrant "slow motion", in the second and third "fast motion".

Exactly as in the case of column 3 this table is computed by means of the "method of declinations" on the basis of column 5. In contrast to it the Khaṇḍa-Khādyaka uses (I,17) the "method of sines", though with the same extremal value $\Theta_{max} = 4;56$:

| | δ al-Khw. | δ·0;12,24,39 | Θ al-Khw. | Θ Kh.-Kh. | 4;56·sin α |
|---|---|---|---|---|---|
| 15 | 6; 0,25 | 1;14,33, 4 | 1;14,33 | 1;17 | 1;16,37 |
| 30 | 11;39,50 | 2;24,45,31 | 2;24,46 | 2;28 | 2;28, 0 |
| 45 | 16;36,49 | 3;26,11,20 | 3;26,11 | 3;29 | 3;29,18 |
| 60 | 20;29,50 | 4;14,23,15 | 4;14,23 | 4;16 | 4;16,21 |
| 75 | 22;59,24 | 4;45,19,30 | 4;45,20 | 4;46 | 4;46,15 |
| 90 | 23;51, 0 | 4;55,59,54 | 4;56, 0 | 4;56 | 4;56, 0 |

The coefficient of δ is given by $\frac{\Theta_{max}}{\varepsilon} = \frac{4;56}{23;51} = 0;12,24,39,14, \ldots$

**Column 5:** "Declination of the sun". In the first and fourth quadrant "increasing", in the second and third "decreasing".

Argument: the longitude λ of the sun in single degrees, counted from ♈ 0°; cf. chapter 15 (p. 33). Tabulated: the declination δ(λ) of ecliptic points of longitude λ, to seconds of arc. Maximum declination ε = 23;51,0.

Bīrūnī confirms[2] 23;51° as al-Khwārizmī's value for the obliquity of the ecliptic. Ṣāʿid al-Andalusī considers it as based on Ptolemy, in contrast to the round value of

[2] Cf. Kennedy-Muruwwa [1] p. 118.

24° in Hindu astronomy.[3] Actually we find in the Almagest and in the planetary Hypotheses[4] ε = 23;51,20, but ε = 23;51 is used in the Handy Tables.[5] The values found in the latter table are given in steps of 3° and to minutes only, but can be explained as roundings either of the values from the Almagest or from a table like al-Khwārizmī's with ε = 23;51,0 as maximum.[6] In the seconds of arc there is no agreement between the table in the Almagest and al-Khwārizmī's table. It is clear that al-Khwārizmī's table was computed by means of the same accurate trigonometric relations as used in the table in the Almagest,[7] but with ε = 23;51,0 instead of with ε = 23;51,20. The value 23;51 is also quoted in ch. 25 for the computation of the rising times at sphaera recta.[8]

Several errors in the extant version of al-Khwārizmī's tables are the result either of misreadings of Arabic alphabetic numerals or of scribal errors in Roman numerals.

| λ | Text[9] | read | CC |
|---|---|---|---|
| $0^s$ 13° | 5;13,50 | 5;13, 7 | 5;13,7 corrected to 5;13,50 |
| $1^s$ 8° | 14;24,34 | 14;24,54 | correct; written above it: 20 (for –20) |
| $1^s$ 18° | 17;29,54 | 17;29,14 | correct |
| $1^s$ 23° | 18;50,23 | 18;50,24 | correct, also in O and N |
| $1^s$ 28° | 20; 3,55 | 20; 3,15 | correct |
| $2^s$ 21° and $3^s$ 9° | 23;32,20 | 23;22, 9 | as in Suter |
| $3^s$ 19° | 22;28,19 | 22;28,39 | 22;28,39 with correction: – 20 |
| $3^s$ 20° | 22;19,18 | 22;19,53 | 22;19,58 with correction: – 40[10] |
| $4^s$ 2° | 20; 4,14 | 20; 3,15 | 20;8,14 |
| $4^s$ 8° | 18;35,49 | 18;34,49 | 18;34,49 with correction + 1 over 34 |
| $4^s$ 24° | 13;44,52 | 13;44,57 | as in Suter |
| $4^s$ 28° | 12;22,10 | 12;22,23 | 12;22,23 corrected to 12;22,10 |
| $5^s$ 6° | 9;27,17 | 9;27,57 | as in Suter |
| $5^s$ 12° | 7;10,50 | 7;10,40 | as in Suter |
| $5^s$ 17° | 5;13,50 | 5;13, 7 | as in Suter |
| $5^s$ 19° | 4;25,25 | 4;25,29 | as in Suter |

Deviations from symmetry with respect to 90° are disregarded in the above list when they amount only to 1″, in which case a decision between correct and incorrect value is practically impossible.

It is obvious from this list that CC was based on a better text than Suter's group but that it was compared with, and changed to, the version of the Suter family.

**Column 6:** "Latitude of the moon". In the first and fourth quadrant "increasing", in the second and third "decreasing". The argument is $\omega = \lambda - \lambda_{\text{☊}}$.

[3] Kennedy-Muruwwa [1] p. 120 and Blachère, Ṣā'id, p. 102/103.
[4] Ptolemaeus, Opera I,1 p. 81 and II p. 74,15, Heiberg.
[5] Halma I p. 144/145.
[6] Only in two instances are the tables in Almag. I,15 better than in al-Khwārizmī: λ = 15: H.T. = 6;1 al-Khw. = 6;0,25 Alm. = 6;0,31 and λ = 57: H.T. = 19;50 al-Khw. = 19;49,22 Alm. = 19;49,42.
[7] Cf. also the tables for right ascensions (below p. 104).
[8] The same table of declinations (ε = 23;51,0) is found in the Toledan Tables; cf. Zinner [1] p. 749 No. 25.
[9] As given by Suter.
[10] Written *lx* instead of *xl*.

The maximum latitude is 4;30° as in the Sūrya-Siddhānta[11] and in the Paulisa-Siddhānta.[12] Ptolemy always uses $\beta_{max} = 5;0°$.

Al-Khwārizmī's values are obtainable from the exact relation between argument of latitude $\omega$ and latitude $\beta$

$$\sin \beta = \sin i \cdot \sin \omega \qquad i = 4;30° \qquad (1)$$

as is shown from the following examples[13]

| $\omega$ | from (1) | al-Khw. |
|---|---|---|
| 15° | 1; 9,48 | 1; 9,52 |
| 30 | 2;14,53 | 2;15, 0 |
| 45 | 3;10,49 | 3;10,14 |
| 60 | 3;53,46 | 3;53,51 |
| 75 | 4;20,47 | 4;20,45 |

The same relation (1), but with $i = 5°$, was used for the computation of the tables of latitudes in the Almagest (V,8) and in the Handy Tables.[14]

[11] Sūrya-Siddhānta I,68. Cf. also Pança-Siddhāntika IX,6.
[12] Pança-Siddhāntika III, 31.
[13] The same values which we obtained here from (1) are also found in Tables published by Bailly AI p. 338 Tab. IV; cf. also p. 420f. Tab. IV.
[14] Halma I p. 144/14.

## Tables 27 to 56

"Equation of Saturn" (Tab. 27 to 32) etc. to "Equation of Mercury" (Tab. 51 to 56), including tables for stations and for latitudes. With each table is given the position of the ascending node; cf. below p. 103. Cf. also CC fol. 127 to 141 (p. 191ff).

**Columns 1 and 2:** "Columns of numbers"[1] i.e. signs and degrees from 1 to 180 and, in inverse order, from 180 to 359.

The subtitles *augmentum proprietatum omnium* (column 1) and *diminutio proprietatum omnium* (column 2) must express by *proprietates omnes* what Greek tables call ἀριθμοὶ κοινοί "common numbers", increasing in col. 1, decreasing in col. 2.

**Column 3:** *sublimatio examinata* (or *definita*) or *sublimatio ascensus examinata*, giving

$$s = \lambda_A - \tfrac{1}{2} |\sigma|$$

where $\lambda_A$ denotes the longitude of the apogee and $|\sigma|$ is taken from column 4. The term *examinata* means "equated", i.e. "modified" (by $\frac{1}{2} |\sigma|$); *ascensus* is probably "apogee" and the same meaning is established for *sublimatio* alone by the text of ch. 18.[2]

[1] In variable terminology: *pagina, semita, via,* etc.
[2] Suter p. 14.

For the astronomical meaning of the quantity $s$ cf. the commentary to ch. 10, step 2 (p. 24). For $\alpha = 180$ one has $\sigma = 0$ and thus $s = \lambda_A$. This leads to the following values for the planetary apogees used in the tables:

♄: ♐ 4;55 ♃: ♍ 22;32 ♂: ♌ 8;24

♀: ♊ 21;15 ☿: ♏ 14;54,

in complete agreement with ch. 18.[3] Nearest to this list comes, at least to my knowledge, the Siddhānta-Śiromaṇi of Bhāskara II (about A.D. 1150). The apogees given there have slightly smaller longitudes than al-Khwārizmī's but the greatest deviation is only 0;16,24° (for ♃), the smallest 0;5,46° (for ♂).[4]

Errors: Tab. 34 at $1^s\,26$: read $5^s\,18;31$
Tab. 35 at $2^s\ 4$: should be $5^s\,18;6$ (CC has 18;7, Suter 18;8)
Tab. 42 at $3^s\,28$: read $3^s\,18;37$ (as in CC; Suter: 18;39)
Tab. 44 at $5^s\,26$: read $4^s\,4;40$ (as in CC; Suter: 3;40)
Tab. 49 at $4^s\,24$: read $1^s\,27;57$ (as in CC; Suter: 27;55).

**Column 4:** *examinatio argumenti* (or *ipsius elheza*) i.e. "equation of anomaly". This column gives the absolute values $|\sigma|$ of the epicyclic equation $\sigma$ as function of

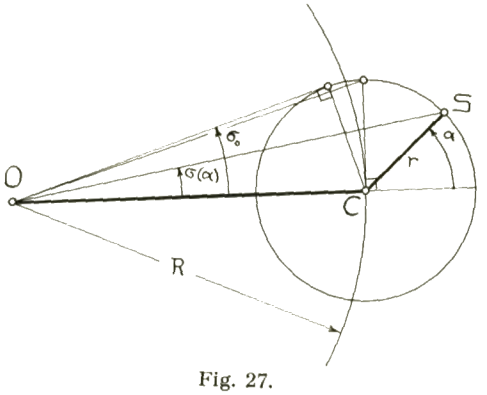


Fig. 27.

the anomaly $\alpha$. The underlying model is the simple epicyclic motion as represented in fig. 27. On the basis of this model it is easy to deduce from the tables for each planet the radius $r$ of the epicycle for a radius $R = 60$ of the deferent. One can use any one of three relations:

from the maximum equation $\sigma_0$: $\quad r = R \sin \sigma_0$

from $\sigma$ for $\alpha = 90°$: $\quad r = R \tan \sigma(90°)$

from general $\sigma$: $\quad r = \dfrac{R \tan \sigma}{\sin \alpha - \tan \sigma \cdot \cos \alpha}$.

[3] Cf. p. 41.

[4] Except for Saturn which deviates by about +16° from al-Khwārizmī's value. Cf. Burgess, S.S. p. 31.

All three relations lead, with variations of at most a few minutes, to the following values for $r$:

♄: 6 ☿: 11;20 ♂: 39 ♀: 44 ☌: 22.

The agreement between these different methods of computing the radius of the epicycle is proof of the use of the model represented by fig. 27.

The above values of the epicycle radii can also be found in Hindu treatises but I do not know of any single siddhānta which would display exactly the same values for all five planets. Al-Bīrūnī, however, quotes "Persian" values for the maximum epicyclic equations $\sigma_0$ which are, with the exception of Mars, identical with al-Khwārizmī's values[5] and which then should also give the same epicyclic radii.

**Column 5:** "Equation of center" i.e. the absolute values $|\mu|$ of the equation $\mu$

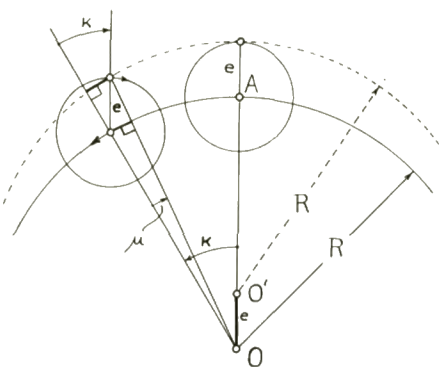

Fig. 28.

due to the eccentricity $e$ of the deferent, or, due to an epicyclic radius $e$. The independent variable is the distance $\kappa$ from the apogee.

Accurately $\mu$ should be computed from $\kappa$ by exactly the same formula as was used to find $\sigma$ as function of $\alpha$ in column 4. It is evident, however, that this was not the case since $\mu$ is symmetric with respect to $\kappa = 90°$. It is then easy to check that $\mu$ is simply computed by

$$\mu = e \sin \kappa$$

which is not too bad an approximation for comparatively small values of $\kappa$ (cf. fig. 28).

From the tables one reads for $\kappa = 90$ the values of the eccentricities:

♄: 8;36 ☿: 5;6 ♂: 11;13

♀: 2;14 ☌: 4;2.

[5] Cf. Kennedy, Survey p. 170 and p. 172 Tab. 2. For Mars, al-Bīrūnī gives $\sigma_0 = 41;30$ instead of 40;31. The corresponding value of $r$ would be 39;45 which is also used for odd quadrants by Āryabhaṭa (Aryabhaṭiya I,8).

Again, Bīrūnī's "Persian" values are almost identical with this set,[6] the deviations being at most 0;2°.

**Column 6:** "First station of Saturn" (. . . ". . . of Mercury"). Cf. also ch. 13 (p. 30). The "equated center" $\kappa_3$, found in step 3 of chapter 10 (above p. 25), serves as argument (columns 1 and 2). Tabulated is the corresponding epicyclic anomaly $\alpha_s$ at which the planet has its first station; the second station is then at $\alpha_t = 360 - \alpha_s$. The anomaly $\alpha_2$ found in step 4 of chapter 10 (p. 27), compared with $\alpha_s$ or $\alpha_t$, indicates whether the planet is at a station or in direct or in retrograde motion.

The values found in column 6 for the anomalies $\alpha_s$ of the first station are practically identical with the values given in Theon's "Handy Tables",[7] thus based on the theory presented by Ptolemy in the Almagest. This is particularly obvious in the case of Mercury where the smallest values occur at 120° (cf. fig. 29) and at 240° as

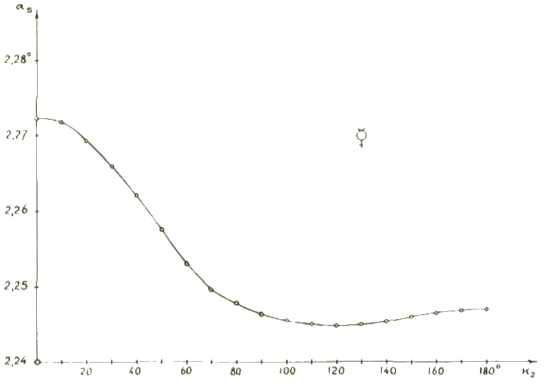


Fig. 29.

required by Ptolemy's model for the motion of Mercury, with its two perigees at 120° and 240° respectively. However, the rules for the computation of the longitudes, given by al-Khwārizmī in chapters 10 and 11, show no trace of Ptolemy's planetary theory. Thus it is evident that very different sources were used for the composition of the numerical tables, at least in the form in which they have reached us.

**Columns 7 and 8:** "Latitude of the ascending (resp. descending) Saturn" (. . . ". . . Mercury"); ascending for the first and last three signs, descending in the remaining six signs. Column 7: "first (latitude)" in degrees and minutes; column 8: "second (latitude)" in degrees, minutes and seconds.

In fact these headings are misleading. Instead of ascending and descending one should have positive (or northern) and negative (or southern) latitude; and the "first latitude" is a dimensionless coefficient.

The manuscript $M_1$ has in Tab. 28 (Saturn) a marginal note concerning the heading of columns 7 and 8: "One should note that for the determination of the above

[6] Kennedy, Survey, p. 172 Tab. 2; also Bīrūnī, Transits 30:17 and Kennedy's commentary p. 148.

[7] Halma HT III p. 11 to 15. The same tables, with very few deviations, are also found in al-Battānī (Nallino II p. 138, 139).

mentioned equation every line so far must be duplicated (i.e. written down twice?) and in the same way for the calculation. And so it can be done (also) for the remaining planets". What all this means I do not know.

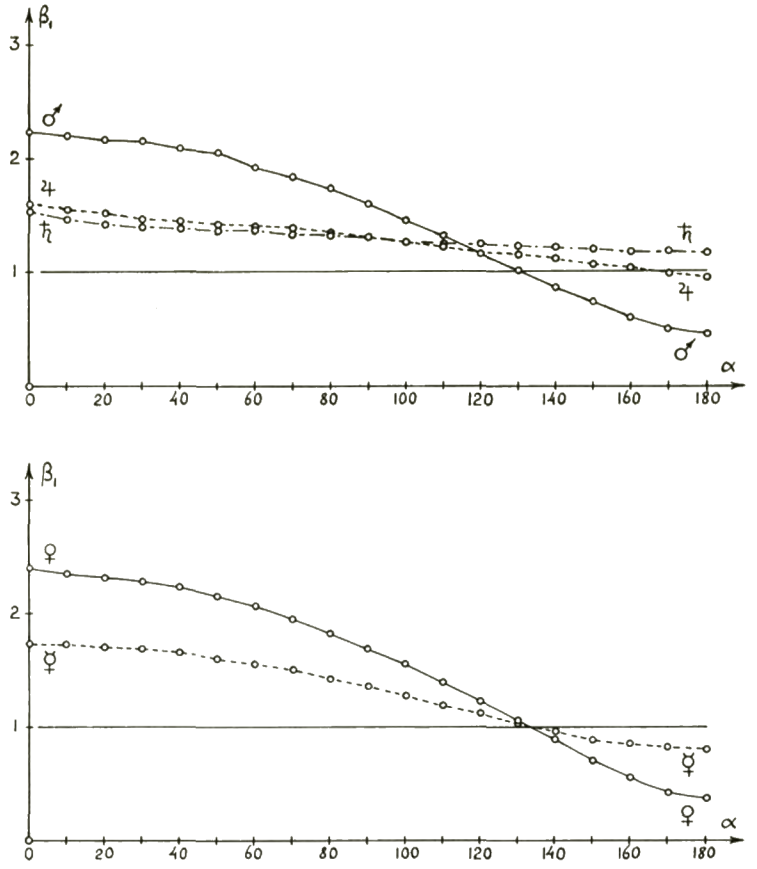


Fig. 30.

The first latituce ($\beta_1$) is monotonically decreasing from a maximum $M$ to a minimum $m$ as the argument $\alpha$ (the epicyclic anomaly) goes from 0° to 180°; cf. fig. 30. The subsequent little table gives the values for the extrema, their differences, and the value $\alpha_0$ of the argument for which $\beta_1 = 1$.

| | $M$ | $m$ | $M - m$ | $\alpha_0$ |
|---|---|---|---|---|
| ♄ | 1;32[8] | 1;10 | 0;22 | – |
| ♃ | 1;36 | 0;57 | 0;39 | 165 to 168 |
| ♂ | 2;14 | 0;27 | 1;47 | 131 |
| ♀ | 2;24 | 0;22 | 2; 2 | 133 |
| ☿ | 1;44 | 0;48 | 0;56 | 133 to 135 |

[8] From MS O; the other MSS give 1;33.

For the problem of computing the function $\beta_1$ cf. the commentary to chapter 17 (p. 34).

The second latitude $\beta_2$ is obtainable from

$$\beta_2 = i \sin \omega$$

where $\omega$ denotes the argument of latitude, i.e., the distance from the ascending node. The values of $i$ are

♄: 5;0° ♃: 2;30 ♂: 3;45 ♀: 5;0 ☿: 6;15.

One obtains excellent agreement between the values given in column 8 and the above formula if one uses the sine tables of al-Battānī.[9]

The values for the position of the ascending node, given with each table, are:

♄: 3$^s$ 13;12° ♃: 2$^s$ 22;1 or, "according to others", 2$^s$ 12;1[10]

♂: 0$^s$ 21;54 ♀: 1$^s$ 29;27 ☿: 0$^s$ 21;10

in agreement with chapter 19.[11]

[9] Nallino II p. 55f.
[10] Cf. Suter's apparatus to Tab. 34 to 38. Chapter 19 gives only 2$^s$ 22;1°.
[11] Above p. 42.

## Table 57a

No title. Double entries: "signs" from Aries to Pisces, and three "*facies*" i.e. decans of 10° length. Tabulated is the elongation $\Delta\lambda_0$ which must be transgressed in order to guarantee visibility of the first or last sickle of the moon (cf. ch. 22 p. 42).

The values of $\Delta\lambda_0$ are symmetric with respect to ♎ 0° (cf. fig. 18 p. 44). The maximum value is not 21;57° but only 21;17 as given in C and O.

## Table 57b

Trivial table (without title) for the order of magnitude of products $a \cdot b$ if both $a$ and $b$ range from degrees to sixths.

The table is preceded by sentences which seem, at least in part, out of place here: "Since this is such and since the order of discussion leads us to the eclipses of the luminaries, one must first note down certain things by means of which we . . . . And first of all one must understand what originates from the multiplication (?) of fractions either by themselves or with others. This can be found for whatever (fraction) one wants in the subjunct table of rectangular arrangement".[1] Finally: "What results from their multiplication (?)".[2]

[1] Version C.
[2] Version O.

## Tables 58 and 58a

"Table of Sines" (cf. ch. 23).

**Columns 1 to 4:** "Columns of numbers". Entries for single degrees for $\alpha$, $180 - \alpha$, $180 + \alpha$, $360 - \alpha$.

**Column 5:** "Sine". Given are the values of $|\operatorname{Sin}\alpha| = |R \cdot \sin\alpha|$ for $R = 60$, to seconds. The table as given by Suter in the main text is assimilated by Suter to Battānī's table as published by Nallino,[1] by using Battānī's value whenever the difference exceeded $1''$.[2] Thus one must use the apparatus, or Björnbo's original publication of this table,[3] if one wishes to compare actual texts.

It seems very doubtful, however, that the extant table of sines is al-Khwārizmī's original table, since we know from al-Bīrūnī's *Risā'il* that this table used $R = 150$ and not $R = 60$.[4] A sine table with the norm $R = 150$ is actually preserved in our MS CC in the form of an interlinear later addition, copyied from the Toledan Tables (cf. below p. 190).

[1] Nallino, Batt. II p. 55f., probably also corrected.
[2] Suter p. 69.
[3] Bjørnbo [2] p. 7.
[4] Lesley [1] p. 126. Also confirmed by the Latin commentary of Hugo Sanctallensis, where all trigonometric functions are normed for $R = 150$. – Cf. also the commentary to ch. 26a (p. 54).

## Tables 59 to 59b

"Horoscope (i.e. ascendant) of the signs for the location of Arin", actually a table of right ascensions with the norm of the "Handy Tables";[1] cf. the commentary to ch. 25 (p. 47) and ch. 32 (p. 77). In ch. 34 this table is quoted as "Table of the equinoctial circle" (p. 69).

**Column 1:** single degrees from 1 to 30, called *gradus regulares*.[2]

**Column 2:** for each zodiacal sign, beginning with ♑: "horoscope of Capricorn" etc. ". . . of Aquarius", then the corresponding Arabic version "ascension of Capricorn (etc.) for sphaera recta".

Tabulated: $\alpha(\lambda) + 90 = \alpha'(\lambda)$ (cf. p. 48). The values of $\alpha'$ are given to seconds of arc, i.e. one place more than in Theon's (or in al-Battānī's) tables. Usually, but not always, the values in Theon are roundings of the numbers shown in al-Khwārizmī's table. This is reminiscent of the situation which we found in the tables of the solar declination (cf. p. 96). As an example one may compare the totals which correspond to the rising times of the first three signs:

| | H.T. = Alm. | al-Khw. | S.-S. III, 44 |
|---|---|---|---|
| $\alpha'(30) - \alpha'(0) = \sigma_1 = \sigma_{12} = \sigma_6 = \sigma_7$ | 27;50 | 27;50, 9 | 27;50 |
| $\alpha'(60) - \alpha'(30) = \sigma_2 = \sigma_{11} = \sigma_5 = \sigma_8$ | 29;54 | 29;54, 3 | 29;55 |
| $\alpha'(90) - \alpha'(60) = \sigma_3 = \sigma_{10} = \sigma_4 = \sigma_9$ | 32;16 | 32;15,48 | 32;15 |

[1] Halma HT I p. 148 to 155.
[2] In Tab. 67, 68 the *gradus regulares* of MS O are called *gradus aequales* by C. Tab. 79 to 114 use only *gradus aequales*; likewise ch. 37 (Suter p. 30, 13/14 ff.).

That Greek tables existed which gave, at least in some cases, one more sexagesimal place than the extant Handy Tables is shown by the fragments of P. London 1278 which belongs to the time between Ptolemy and Theon.[3]

For some inconsistencies in al-Khwārizmī's tables cf. Suter p. 73.

[3] Cf. Neugebauer [1] p. 108.

## Table 60

"Table of shadows".

**Column 1:** "Number of the altitude (of the sun at noon)" in single degrees from $h = 1°$ to 90°.

**Column 2:** "Shadow" in "digits" and "minutes", i.e.

$$s = 12 \cdot \cot h$$

for a gnomon 12 digits long. Cf. fig. 20 p. 56 and the rules given in ch. 28 and in ch. 28b, last paragraph (p. 57).

## Tables 61 to 66

"Table of (the true) motion (of sun and moon)"[1] and, not mentioned in the title, the apparent radii of sun, moon, and shadow. Cf. also ch. 30 and 30a (p. 57).

**Columns 1 and 2:** "Columns of numbers", signs and degrees from 1° to 180° and, in inverse order, from 180° to 359°.

**Column 3:** "hourly motion of the sun". **Column 5:** "Half of the disc (i.e. radius) of the sun". The argument is the solar anomaly α. Tabulated is the solar velocity $v_\odot{}^{\circ/h}$ and the apparent radius $r_\odot = \frac{1}{2} d_\odot$. According to ch. 30 (p. 58) one should have

$$\frac{d_\odot}{v_\odot{}^{\circ/h}} = 13;12. \qquad (1)$$

Actually this is only approximately satisfied, as the subsequent selection of values for the endpoints of signs shows.

| α | $v_\odot{}^{\circ/h}$ | $r_\odot$ | $d_\odot / v_\odot{}^{\circ/h}$ | $v_\odot{}^{\circ/d}$ | $d_\odot$ |
|---|---|---|---|---|---|
| 0 | 0;2,22° | 0;15,40° | 13;14 (!) | 0;56,48° | 0;31,20° |
| 30 | 0;2,23 | 0;15,44 | 13;12 | | |
| 60 | 0;2,25 | 0;15,58 | 13;13 (!) | | |
| 90 | 0;2,28 | 0;16,17 | 13;12 | 0;59,12 | 0;32,34 |
| 120 | 0;2,31[2] | 0;16,36 | 13;12 | | |
| 150 | 0;2,33 | 0;16,50 | 13;12 | | |
| 180 | 0;2,34 | 0;16,54 | 13;10 (!) | 1; 1,36 | 0;33,48 |

[1] This is the meaning of the Arabic title (*gedval elbuht*) common to all six tables in C and O. Tab. 62 to 66 has in the Latin version "Table of the motion of sun and moon per day, per hour, and per minute" (though only the hourly motion is tabulated).

[2] The text gives 0;2,30 from α = 105 to 120 and 0;2,31 from α = 121 to 125. The value 0;2,30 for 120° would, however, lead to a ratio 13;17 instead of the correct one for 0;2,31. Also the differences of the last digits (1, 2, 3, 3, 2, 1) require 0;2,31.

The mean value $r_{\odot} = 0;16,17°$ is the same which underlies the tables for solar eclipses (Tab. 78, below p. 127). The minimum value for the solar diameter, 0;31,20°, is identical with Ptolemy's value for the lunar diameter at maximum distance,[3] which in turn equals the constant solar diameter.[4] This coincidence seems accidental, however, particularly in view of the unusually high mean value and maximum.

**Column 4:** "Hourly motion of the moon". **Column 6:** "Half of the disc (i.e. radius) of the moon". The argument is the lunar anomaly α.[5] Tabulated is the lunar velocity

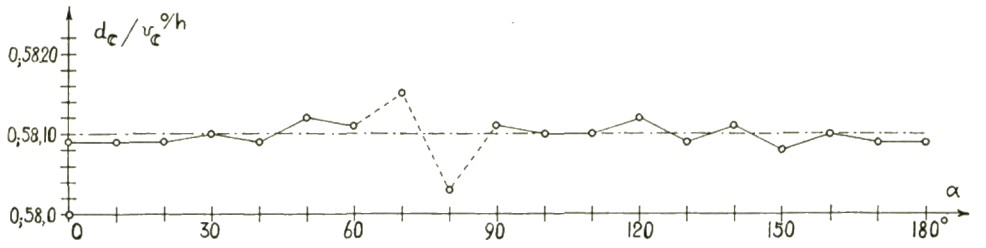

Fig. 31.

$v_{\mathbb{C}}^{\circ/h}$ and the apparent radius $r_{\mathbb{C}} = \frac{1}{2} d_{\mathbb{C}}$. According to ch. 30a the ratio $d_{\mathbb{C}}/v_{\mathbb{C}}^{\circ/h}$ should be a constant, presumably 0;58,10 if we follow the part of the text in ch. 30a that concerns the hourly lunar motion.[6] If one computes for every 10th degree of α the value of $d_{\mathbb{C}}/v_{\mathbb{C}}^{\circ/h}$ one finds indeed values $0;58,10 \pm 0;0,2$ (cf. fig. 31), with two exceptions which, however, are due to errors in the table as can be shown independently by forming differences.[7]

For the extrema one finds from the tables

| α | $v_{\mathbb{C}}^{\circ/d}$ | $d_{\mathbb{C}}$ |
|---|---|---|
| 0 | 12; 4,48°/d | 0;29,16° |
| 90 | 13;10,24 | 0;31,56 |
| 180 | 14;16, 0 | 0;34,34 |

These values seem unrelated to other values known to me.[8] They are again used in the eclipse tables Tab. 73 to 76 (cf. p. 118) and Tab. 78 (cf. p. 127).

**Column 7:** "First half of the disk of the shadow". **Column 8:** "Second half of the disk of the shadow" (lit. dragon). For the Hindu origin of this terminology cf. Bīrūnī, India II p. 75.

[3] Almagest V,14 (Heib. p. 421,5). Here, however, this value is 0;29,16.
[4] Heiberg p. 417,5 ff.
[5] We are dealing here, of course, with the pre-Ptolemaic model of the lunar motion.
[6] Cf. above p. 59.
[7] One would expect, for $\alpha = 2^{s}\ 10°$: $v = 0;31,56$ (instead of 0;31,54) and for $2^{s}\ 20°$: $v = 0;32,26$ (instead of 0;32,31). Also the intermediate values are in disorder.
[8] A mean lunar velocity of $13;10°/^{d}$ occurs in the Pança-Siddh. VIII,7 and in the Greek P. Mich. 149, col. II,20 (Mich. Pap. vol. III p. 66). A lunar mean motion of $13;10,24,52°/^{d}$ is given twice in the treatise *De dracone* of Petrus Alfonsus in the Latin version by Walcher of Malvern; cf. Millás-Vallicrosa [1] p. 25 and p. 29.

The diameter of the shadow during a lunar eclipse depends not only on the distance of the moon but also on that of the sun. Thus we find two functions, $s_1$ and $s_2$, tabulated, the first for the solar anomaly as argument, the second for the lunar anomaly.

Suter, based on al-Bīrūnī's description of the method in common to al-Khwārizmī and the Khaṇḍa-Khādyaka[9] discovered the structure of the tables and their use. He found that

$$s_1 = 5 \cdot v_{\odot}^{\circ/h} \qquad s_2 = \tfrac{8}{5} \cdot v_{☾}^{\circ/h} \tag{2a}$$

and that from these two components the radius $s$ of the shadow at the moon's place is given by

$$s = s_2 - s_1. \tag{2b}$$

This rule can now be fully confirmed from the Khaṇḍa-Khādyaka IV,2. From (2) one finds

$$s = \tfrac{1}{5}(8\, v_{☾} - 25\, v_{\odot})^{\circ/h} = \tfrac{1}{120}\,(8\, v_{☾} - 25\, v_{\odot})^{\circ/d}$$

and therefore for the diameter

$$2\, s = \tfrac{1}{60}\,(8\, v_{☾} - 25\, v_{\odot})^{\circ/d} \tag{3}$$

which is the rule of the Khaṇḍa-Khādyaka. It is also found in Hugo Sanctallensis' commentary.[10]

The smallest value of $s$ will be obtained by using in (3) the minimum of the lunar velocity and the maximum of the solar velocity; and conversely for the maximum of $s$. In this way one obtains from the above listed parameters for the diameter $2s$ of the shadow

$$\left.\begin{array}{rl} \text{min.:} & 1;10,58,24^{\circ} \\ \text{mean:} & 1;20,43,12 \\ \text{max.:} & 1;30,28 \end{array}\right\} \tag{4}$$

in good agreement with the values one can derive from the eclipse tables (Tab. 73 to 76, cf. p. 118).

[9] India II p. 79; cf. Suter p. 90.
[10] Se fol. 44r, 29ff.

## Tables 67 and 68

"Table for the equation of time". Cf. ch. 31, commentary 2 (p. 63).

**Column 1:** single degrees from 1 to 30, called *gradus regulares* in O, *gradus equales* in C.[1]

**Column 2:** for each zodiacal sign, beginning with Aries: "equation of time" in minutes and seconds. Fig. 32 gives a graph of the tabulated values as function of the longitude of the sun. The minimum 0 occurs at ♒ 22, the maximum 0;34,28h at ♏ 8

[1] Cf. Tab. 59 to 59b, p. 104 n. 2.

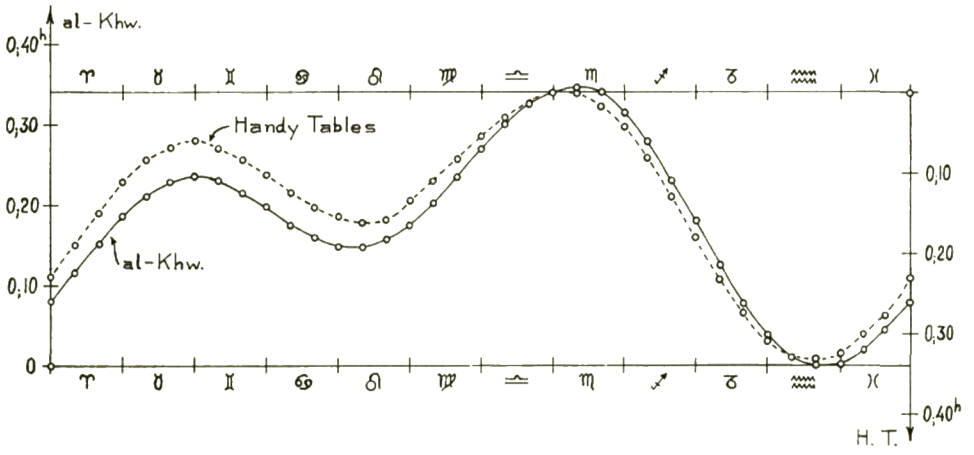


Fig. 32.

and 9. As far as I know no such detailed tabulation of the equation of time occurs in Hindu astronomy. The rules in Kh.-Kh. I,18 or in the Sūrya-Siddh. II, 46 concern only the velocity component. The complete theory of the equation of time is found in the Almagest III,8. The earliest extant tabulation, already normed for a minimum 0, is found in a fragment of P. Lond. 1278.[2] Such a table existed also in Ptolemy's "Handy Tables"[3] and is actually preserved in Theon's tables[4] (cf. fig. 32).

[2] Neugebauer [1] p. 101f., p. 109f.
[3] Opera II p. 162,23 to 163,6 ed. Heiberg.
[4] Halma I p. 148 to 155, column 3 of the tables for right ascensions.

## Tables 69 to 72

Cf. chapter 31 (above p. 59).

**Tab. 69:** "Table for the conjunctions of sun and moon in accumulated Arabic years for the meridian of Cordoba". Similar tables are contained in the "Toledan Tables" but the parameters are not the same.[1]

**Column 1:** "Weekday-numbers" (*notulae*), i.e. numbers with constant difference 5 mod. 7 as in Tab. 2 Sect. 1, col. 2 (cf. p. 85) except for one error in line 6: read 4 instead of 3; the same error also in Tab. 70.

**Column 2:** "Accumulated years", beginning with the *radix* A.H. 1, in 30-year steps to A.H. 511 (= A.D. 1117/18).[2] The years are current years; cf. the note in MS C to Tab. 2 (shows p. 84).

**Column 3:** "Days and hours" i.e. moments of the first mean conjunction in the year listed in column 2, expressed in days, hours, minutes and seconds elapsed since the noon from which Muḥarram 1 is counted. Thus the first entry $29^{d}\ 0;1,11^{h}$ means

[1] Cod. Vindob. 2385 fol. $30^{r}$ and $30^{v}$. Cf. Zinner [1] p. 761 Tafel 101 to 121.
[2] MS N continues this list to A.H. 571 (A.D. 1175/6); the corresponding entries were not published by Suter (cf. p. XIX,11).

that the mean conjunction takes place $29^d\ 0;1,11^h$ after Muḥ. 1 $0^h$ i.e. at Muḥ. 30 $0;1,11^h$ after noon (cf. fig. 33 where the two little circles at July 14 and August 12 represent the moments of conjunction).

The difference in this column is $0;13,45^h$ from line 1 to 5 and from line 15 to 18, but $0;13,44^h$ from line 6 to 14. This increment of $0;0,34,20^d$ resp. $0;0,34,22,30^d$ over an integer number of days during 30 years = 360 synodic months implies that one synodic month is assumed to contain between $29;31,50,5,43,20^d$ and $29;31,50,5,45^d$. According to al-Bīrūnī, India XXXV (trsl. Sachau I p. 350) the Hindus used a synodic month of $29\ \frac{189005}{356222}$ days, or, written sexagesimally, $29;31,50,5,43,22, \ldots{}^d$.

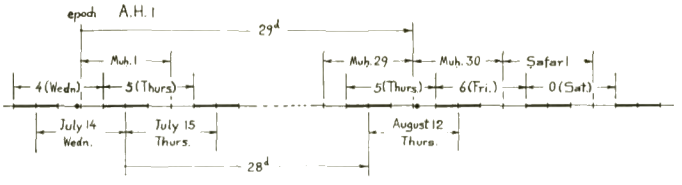


Fig. 33.

This close agreement with a parameter of Hindu astronomy speaks strongly in favor of the assumption that we are dealing here with a table from al-Khwārizmī's original work in spite of a later adaptation to the meridian of Cordoba.

Errors: line 2: read 0;14,56 instead of 0;14,46; line 17; read 3;41,2 instead of 3;40,2 (correct in C).

**Column 4:** "Mean (longitude) of sun and moon". Constant difference $1^s\ 7;57,37°$. Errors: line 2: read $6^s$ instead of $5^s$

line 5: read $24;1,37°$ instead of $14;1,37°$

line 8: read 17;54,28 instead of 17;54,38; correct in C

line 13: read 27;42,33 instead of 27;42,34; correct in C

the lines 14 to 18 should read:

| | | |
|---|---|---|
| 14. | 5;40,10 | not . . . ,14 |
| 15. | 13;37,47 | not . . . ,11 |
| 16. | 21;35,24 | not . . . ,48 |
| 17. $0^s$ | 29;33,1 | not $1^s$ 0;33,25 |
| 18. 2 | 7;30,38 | not 2 9;31,2 |

The error originated in line 13, then the last digit in line 14 was omitted and replaced from the next line; line 18 was finally filled with a correctly computed number.

**Column 5:** "Anomaly of the moon". Difference $9^s\ 24;4,55°$ from line 1 to 4 and 15 to 18, but $9^s\ 24;4,56°$ from line 5 to 14.

Errors: line 3: read $23;56,47°$ instead of 33; . . . (correct in C); line 8: read 24;21,26 instead of 24;21,27.

**Column 6:** "Argument of latitude". Difference: $8^s\ 1;25,22°$. Error: line 7: read 26;3,47 instead of 26;3,44. All subsequent lines are affected by this error.

**Table 70:** "Table for the opposition of sun and moon in accumulated Arabic years for the meridian of Cordoba".

**Columns 1 and 2:** same as in Tab. 69, including the error in line 6.

**Column 3:** "Days and hours" i.e. moments of the first mean opposition in the year listed in column 2, expressed in days, hours, minutes, and seconds elapsed since noon of Muḥarram 1. Thus the first entry $14^d$ 5;39,12$^h$ means that the mean opposition takes place on Muḥ. 15 5;39,12$^h$ after noon (cf. Tab. 69 col. 3).

In column 3 the difference is 0;13,45$^h$ in line 1 and from line 15 to 18, otherwise 0;13,44$^h$. Cf. Tab. 69. col. 3.

**Column 4:** "Mean (longitude) of sun and moon", actually the mean longitude of the sun at opposition. The difference is the same as in Tab. 69 col. 4.

Errors: line 14: read 21;7,1° instead of 21;7,2°; all subsequent lines are affected by this error. Line 17: read 14;59,53 instead of 15;59,53;[3] line 18: read 22;57,30 instead of 24;57,30.[3]

**Column 5:** "Anomaly of the moon". The difference is $9^s$ 24;4,55° from line 1 to 6, but $9^s$ 24;4,56 from line 7 to 14.

**Column 6:** "Argument of latitude". Difference: $8^s$ 1;25,22°.

Errors: line 2: read 3;36,51 instead of 4; . . . Lines 8 to 18 should read:

| | | | | | |
|---|---|---|---|---|---|
| 8. | 12; 9, 3 | not . . . ,2 | 14. | 20;41,15 | not . . . ,9 |
| 9. | 13;34,25 | not . . ;33,24 | 15. | 22; 6,37 | not . . . ,31 |
| 10. | 14;59,47 | not . . ;58,45 | 16. | 23;31,59 | not . . . ,53 |
| 11. | 16;25, 9 | not . . ;23, 6 | 17. | 24;57,21 | not . . . ,15 |
| 12. | 17;50,31 | not . . ;49,27 | 18. | 26;22,43 | not . . . ,37 |
| 13. | 19;15,53 | not . . ;14,48 | | | |

The error in the minutes in line 9 influences all lines until line 13.[4] The error in the seconds in line 8 influences line 9; from then on a difference of only 21 in the last place is used down to line 14 where 22 reappears.

### Mean Syzygies and Epoch Values

The values for the mean longitudes of sun and moon at epoch, A.H. 1 Muḥ. 1, are given in Tab. 4 and 6. From Tab. 5 and 8 one can obtain the mean motion during $29^d$ 0;1,11$^h$ which is the interval between epoch and the first mean conjunction in Muḥarram of A.H. 1 according to Tab. 69 col. 3. Adding the corresponding mean motion to the position at epoch one does not find the longitudes given in Tab. 69 col. 4, but smaller longitudes. Dividing the differences by the hourly motions one finds that agreement is obtained by motions during additional 4;12$^h$. Exactly the same result is found for the lunar anomaly, the lunar latitude and for the data at opposition given in Tab. 70. The explanation for this fact is obvious: the moments given in Tab.

[3] Ignoring the effect on the last place of the error in line 14.

[4] This assumes in line 11 the reading 16;24,9 as found in N and O.

69 and 70 are meant in Cordoba local time, whereas the epoch values in Tab. 4 and 6 are understood for a meridian 4;12$^h$ east of Cordoba. The corresponding difference in geographical longitude is 63° and is probably considered to represent the difference between Baghdad and Cordoba.[5]

As an example for the above mentioned relation between Tab. 69, 70 and Tab. 4 to 8 we derive the longitude for the first mean conjunction listed in Tab. 69:

| | | $\bar{\lambda}_\odot$ | | $\bar{\lambda}_☾$ |
|---|---|---|---|---|
| epoch: A.H. 1 Muḥ. 1 | Tab. 4: | 3$^s$ 23;25,48° | Tab. 6: | 3$^s$ 27;45,17° |
| motion during 29$^d$ | Tab. 5: | 28;34,57 | Tab. 7: | 0$^s$ 22; 6,52 |
| during 0;1,11$^h$ | | 0; 0, 3 | Tab. 8: | 0; 0,39 |
| | | 4$^s$ 22; 0,48 | | 4$^s$ 19;52,48 |
| motion during 4;12$^h$ | | 0;10,21 | | 2;18,21 |
| | | 4$^s$ 22;11, 9 | | 4$^s$ 22;11, 9 |

Thus $\bar{\lambda}_\odot = \bar{\lambda}_☾ = 4^s\ 22;11,9°$ exactly as given in Tab. 69 col. 4.

In the case of the lunar anomaly one obtains in the same way a value which is 0;0,1° lower than in Tab. 69 col. 5. For the argument of latitude ω one finds (cf. p. 21 fig. 3)

| | | | |
|---|---|---|---|
| epoch: A.H. 1 Muḥ. 1 | Tab. 19: | $-\lambda_☊$ = | 7$^s$ 23;47,38° |
| motion during 29$^d$ | Tab. 20: | | 1;32,14 |
| during 0;1,11$^h$ | | | 0; 0, 0 |
| | | | 7$^s$ 25;19,52 |
| motion during 4;12$^h$ | Tab. 20: | | 0; 0,34 |
| | | $-\lambda_☊$ = | 7$^s$ 25;20,26 |
| found before: | | $\bar{\lambda}_☾$ = | 4$^s$ 22;11, 9 |
| | | $\omega = \bar{\lambda}_☾ - \lambda_☊$ = | 0$^s$ 17;31,35 |

exactly as given in Tab. 69 col. 6.

In the same way one can check the values for the opposition of A.H. 1 Muḥ. 15 5;39,12$^h$ after noon and finds the following deviations:

+0;0,1° for $\bar{\lambda}$ of sun and moon, −0;0,2° for the lunar anomaly,
+0;0,2° for the argument of latitude.

**Tab. 71:** "Table for conjunction and opposition in single Arabic years".
**Column 1:** "Weekday-numbers" (*notulae*); cf. below p. 114.
**Column 2:** "Single (Arabic) years" from 1 to 30. These years are completed

[5] Al-Battānī gives for Baghdad the longitude of 80° (Nallino II p. 42 No. 168). In al-Khwārizmī's Geography, however, Baghdad is located at only 70° longitude (Nallino II, note ad No. 168, and p. 213). Our conjecture would then require for Cordoba a longitude of 17° or 7° respectively. Unfortunately Cordoba is only mentioned in the Spanish version of al-Battānī with a longitude of 9;9° (Nallino II p. 215); Cod. Vat. gr. 1291 gives 9 1/3° as longitude (Honigmann SK p. 194,19). To Prof. E. S. Kennedy I owe the following information: in the *Qanūn al-Mas'ūdī* of al-Bīrūnī and in the *zīj* of Ibn ash-Shāṭir Cordoba is given the longitude 8;40 ($\varphi$ = 35;0), while the *zīj-i Khāqānī* of Kashī gives 18;40 ($\varphi$ = 35;40). Cf. also p. 94 for a possible reference to the meridian of Baghdad.

years, in contrast to the 30-year tables (Tab. 69 and 70). The title given in MS N "Remaining (Arabic) years" would be more appropriate. Example: for a syzygy in A.H. 35 one has to enter col. 2 of Tab. 69 or 70 with the year 31, Tab. 71 col. 2 with the remaining 4 years.

**Column 3:** "Days to be subtracted"; cf. below p. 114.

**Column 4:** "Hours". Tabulated are the hours, minutes and seconds which are to be added for each completed year to the moment of the mean syzygies given in Tab. 69 and 70. This amount is given in the first line as $8;48,7^h$ but this is a scribal error for $8;48,27^h$.

The total for 30 years is given as $0;13,33^h$ which would be obtained if the yearly difference were $8;48,27,6^h$. From Tab. 69 and 70, col. 3, we know,[6] however, that the total for 30 years is $0;13,44^h$ or $0;13,45^h$ corresponding to a synodic month between $29;31,50,5,43,20^d$ and . . ,$5,45,0^d$. Consequently the yearly difference in col. 4 of Tab. 71 should be between $8;48,27,28^h$ and $8;48,27,36^h$.

Whatever difference one uses, the values given in Tab. 71 fluctuate too much to list the discrepancies. Computing, however, with the most plausible accurate value, 8;48,27,30, one obtains agreement with the text in the lines 3, 6, 8, and again in the lines 21 and 24. I see no reason for this peculiar fluctuation in the tabulated last place. All degrees and minutes are correct.

**Column 5:** "Mean (motion) of sun and moon". The constant difference is $11^s$ 19;15,55° per year. From Tab. 69 and 70, col. 4 one would obtain for one year $11^s$ 19;15,55,14°. The accumulated effect of the last digit is ignored in the computation of the present column.

Errors: line 3: read 27;47,45° instead of 26; . . .
line 5: read 6;19,35 instead of 5; . . .
line 6: read 25;35,30 instead of 25;15,30
line 7: read 14;51,25 instead of 14;51,35
line 24: read 12;22,0 instead of 12;24,0.

**Column 6:** "Anomaly of the moon". From Tab. 69 and 70 col. 5 one would obtain for the yearly difference a value near $10^s$ 9;48,9,51°. In the present column irregularly varying differences are used, usually slightly too small. Consequently the result for 30 years is 24;2,48° where one should expect on the basis of Tab. 69 and 70 the value 24;4,55°. Beside these irregular variations in the last place one finds several gross scribal errors in this column:

line 3: read 29;24,29° instead of 19; . . .
line 4: read 9;12,39 instead of . . . ,29
line 7: read 8;37, 7 instead of 8;27, . .
line 8: read 18;25,16 instead of 18;35, . .

[6] Cf. above p. 109.

line 16: read 6;50,31 instead of 5; . . .
line 28: read 4;28,25 instead of 4;38, . .
line 30: read 24; 4,48 instead of 24;2, . .

**Column 7:** "Argument of latitude". Tab. 69 and 70, col. 6, are based on a difference for 30 years of $8^s\ 1;25,22°$ which would require a yearly increment of $0^s\ 8;2,50,44°$. The present column, however, is consistently[7] computed with the difference $0^s\ 8;2,50°$. Consequently the total for 30 years is only $8^s\ 1;25,0°$.

Thus we see that all four columns, 4 to 7, which concern yearly motion, are computed with lower accuracy than the corresponding 30-year tables.

**Tab. 72:** "Table of (single) months of conjunction and opposition".

**Column 1:** "Weekday-numbers" (*notulae*; cf. p. 115). These numbers should have alternating differences 2 and 1 but the following errors occur:

line 4: read 5, not 3 line 11: read 1, not 7.
line 5: read 6, not 4 line 12: read 3, not 2.
line 9: read 5, not 4.

**Column 2:** "Arabic months"; to be entered with the current month for which the syzygy is under discussion.

**Column 3:** "Days to be subtracted"; cf. p. 115.

**Column 4:** "Hours". Beginning with $0;0,0^h$ and then using $12;44,3^h$ as constant difference modulo $24^h$. For 12 months[8] this would lead to $8;48,36^h$ while col. 4 of Tab. 71 is based on a yearly increment of $8;48,27^h$ or rather $8;48,27,30^h$. From the latter value would result a monthly difference of $12;44,2,17,30^h$.

Error: In line 10 read 18;36,27 instead of 18;37,27.

**Column 5:** "Mean (motion) of sun and moon". Beginning with zero, using a difference of about $29;6,19,22°$ per month, though the last digits of the single lines show irregular variations. From the yearly difference $11^s\ 19;15,55,14°$ in Tab. 71 col. 5[9] one would find $29;6,19,36,10°$ as monthly difference.

The following are probably scribal errors to be tentatively restored as follows:

line 3: read 28;12,39 not . . . ,29
line 6: read 25;31,36 not . . . ,56
line 8: read 23;44,14 not . . . ,54
line 9: read 22;50,14 not . . . ,54
line 11: read with N: 21;3,13 not . . . ,53.

**Column 6:** "Anomaly of the moon". From the yearly difference of $10^s\ 9;48,9,51°$ in Tab. 71 one would obtain a monthly difference of $25;49,0,49,15°$. Tab. 72 uses in the first six steps $25;49,1°$, thereafter $25;49,0°$.

[7] Scribal error in line 22: read 27;12,20 instead of 27;10,20.
[8] Because this table begins with zero, only 11 differences are accumulated in line 12.
[9] Cf. p. 112.

**Column 7:** "Argument of latitude". The text uses the constant difference $1^s$ 0;40,14° except for two errors at the beginning: line 2 read 0;40,14 instead of . . . ,15; line 3 read 1;20,28 instead of . . . ,29. Column 7 of Tab. 71 with a yearly difference of 8;2,50° or 8;2,50,44° would require here $1^s$ 0;40,14,10° or 0;40,14,13,40°.

### The Weekdays of the Calendaric Syzygies

The Tables 69 to 72 permit the computation of the moment of the mean syzygies and of the corresponding longitudes, anomalies, and arguments of latitudes as needed for the computation of eclipses. The interval between these astronomically accurate syzygies is a mean synodic month of 29;31,50,5 . . .$^d$ (cf. p. 109). The Islamic calendar, however, is based on a mean synodic month of exactly 29;31,50$^d$. The columns concerning weekdays in Tab. 69 to 72 are also based on this "calendaric mean synodic month" as will become clear from the subsequent description of the structure of these columns. The "calendaric conjunctions" and oppositions deviate only little from the corresponding astronomical syzygies, in the same sense as the calendaric 30-year cycle is a close approximation to the length of 30 lunar years.

Column 1 of Tab. 69 and 70 gives weekday-numbers $w$ (*notulae*) which begin with $w = 0$ for year 1 and increase for each 30-year step by 5 (mod. 7) because 30 Arabic years contain 10631 days and $10631 \equiv 5$ mod. 7 (cf. p. 85).

Column 3 introduces a correction to the *notulae* of column 1 due to the progress from year to year, not by either 354 or 355 days but by the constant amount of 354;22$^d$ which is the accurate length of the year that underlies the Islamic calendar.[10] This leads to the following procedure:

We know that the first conjunction of the year A.H. 1 falls on a Thursday ($w = 5$).[11] We start with the same weekday also for the calendaric conjunctions. In A.H. 1 no year is yet completed, so we give the year 0 the *notula* 5 and the correction (called *dies subtrahendi* "days to be subtracted") $c = 0$. Year 1 is an ordinary year and consequently its *notula* is $5 + 4 \equiv 2$. The corresponding calendaric conjunction, however, would fall at $5 + 4;22 \equiv 2;22$, thus within the day with $w = 2$ and the correction $c = 0$. Year 2 is a leap-year, therefore its *notula* is given by $w = 5 + 2 = 7$. The corresponding calendaric conjunction, however, will be at $2;22 + 4;22 = 6;44$ or still in weekday 6 and not in 7. Consequently we have here $c = -1$. In the next year we find $w = 7 + 4 = 4$ and a calendaric conjunction at $6;44 + 4;22 \equiv 4;6$ i.e. within the weekday 4 as is indicated by the *notula*. Thus $c = 0$. In this way one can obtain all the corrections listed in Tab. 71 col. 3.[12] For the sake of clarity I repeat once more this process

[10] Cf. ch. 1, p. 11 n. 1.

[11] Cf. p. 109 fig. 33.

[12] It might be remarked that only the initial values $w = 5;0$ and $5;1$ produce the corrections given in column 3. Thus it is virtually certain that the procedure started with exactly $w = 5$.

| completed years | *notula* | calendaric conjunction | *dies subtr.* |
|---|---|---|---|
| 0 | $w = 5$ | 5;0 thus in day 5 | $c = 0$ |
| 1 | 2 | 2;22 2 | 0 |
| 2* | 7 | 6;44 6 | −1 |
| 3 | 4 | 4; 6 4 | 0 |
| 4 | 1 | 1;28 1 | 0 |
| 5* | 6 | 5;50 5 | −1 |
| 6 | 3 | 3;12 3 | 0 |

in tabular form up to year 6. The entry for year 0 is not given in the table of the text. Leap-years I have marked with an asterisk.

A similar procedure explains the *notulae* and corrections for single months in Tab. 72 cols. 1 and 3 respectively. Column 1 gives the *notulae* beginning with $w = 0$ for month I. Month I is 30 days long, thus the next notula is $w = 0 + 30 \equiv 2$. For the scheme of the Islamic calendar one synodic month contributes $29;31,50^{d} - 1;31,50^{d}$. Thus we obtain for the consecutive conjunctions the following pattern:

| month | *notula* | calendaric conjunction | *dies subtr.* |
|---|---|---|---|
| I | $w = 0$ | 0 thus in day 0 | $c = 0$ |
| II | 2 | 1;31,50 1 | −1 |
| III | 3 | 3; 3,40 3 | 0 |
| IV | 5 | 4;35,30 4 | −1 |

etc., exactly as in Tab. 72 col. 1 for $w$ and in col. 3 for $c$.

The same numbers $w$ are assumed for 30-year steps, for single years, and for months in the case of oppositions (in Tab. 70, 71, and 72 respectively). Thus one assumed again as initial weekday $w = 5$ (Thursday) because the opposition 14 days after the first conjunction would fall on Thursday. Thus the calendaric oppositions can be obtained by the same numerical device as the conjunctions.

In order to find the weekday of any calendaric syzygy one has simply to add all the corresponding entries with their corrections. Example: weekday of the conjunction of A.H. 35 month II:

| | |
|---|---|
| Tab. 69: for 31 years | $w = 5$ |
| Tab. 71: compl. years 4 | $w = 4$ |
| | $c = 0$ |
| Tab. 72: month II | $w = 2$ |
| | $c = -1$ |
| | $w = 10 \equiv 3$ i.e. Tuesday. |

## Tables 73 to 76

Cf. chapter 33 (p. 66): The same tables are found in the Toledan Tables (T. fol. 35).

**Tab. 73 and 74, Columns 1 to 5:** "Lunar eclipses at far distance" i.e. for the moon at apogee. The heading does not apply to columns 6 to 8, for which see p. 117.

**Columns 1 and 2:** "Argument of latitude".[1] Tabulated in steps of $^1/_2$ degree for the argument of latitude $\omega$: Tab. 73 for positive latitudes from $\omega = 0$ to 10;30 and from 10;50 and from $\omega = 169;10$ and 169;30 to 180; Tab. 74 for negative latitudes from $\omega = 349$ to 359;30 and from $\omega = 180;30$ to 191. Thus the eclipse limits at the

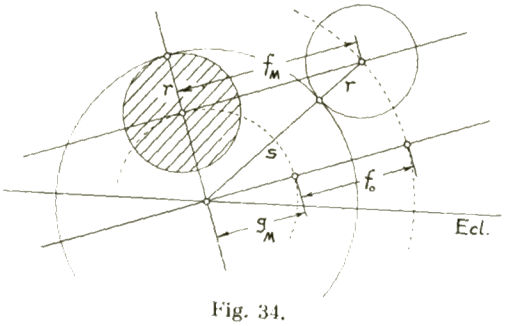


Fig. 34.

ascending node are $\omega = -11$ and $+10;50$, at the descending node $\omega = 180 - 10;50$ and $180 + 11$. Obviously $\pm 10;50$ should be the nodal distance in all cases.

**Column 3:** "Digits of the eclipse". The digits are linear digits, i.e. twelfths of the lunar diameter. The data refer to the middle of the eclipse. Maximum: 20;46 digits for $\omega = 0$ or 180 (Tab. 73).

Errors: Tab. 73 line 9: read 7;23 as given by C and Tab. 74
line 10: read 8;21 as given by C and Tab. 74
Tab. 74 line 18: read 3;35 as given by C and Tab. 73.

**Column 4:** "Minutes of immersion". Minutes of arc between the center of the moon at first contact and at inner contact or at mid-eclipse, measured with respect to the shadow (cf. fig. 34 and 35).

There are discrepancies between Tab. 73 and 74 in line 1, 6, 12 of Tab. 73. In Tab. 74 line 18 read 28;50 as given by C and Tab. 73 line 5.

**Column 5:** "Minutes of half totality". Minutes of arc between the center of the moon at inner contact and at mid-eclipse, measured with respect to the shadow (cf. fig. 34 and 35).

**Tab. 75 and 76, Columns 1 to 5:** "Lunar eclipses at near distance" i.e. for the moon at perigee. This heading does not apply to columns 6 to 8 for which see p. 118 and p. 128.

**Columns 1 and 2:** "Argument of latitude". Tabulated in steps of $^1/_2$ degree for the argument of latitude $\omega$: Tab. 75 for positive latitudes from $\omega = 0$ to 13 and 13;17 and from 166;43 and 167 to 180; Tab. 76 for negative latitudes from $\omega = 346;43$ and

[1] In Tab. 73: "motion of latitude".

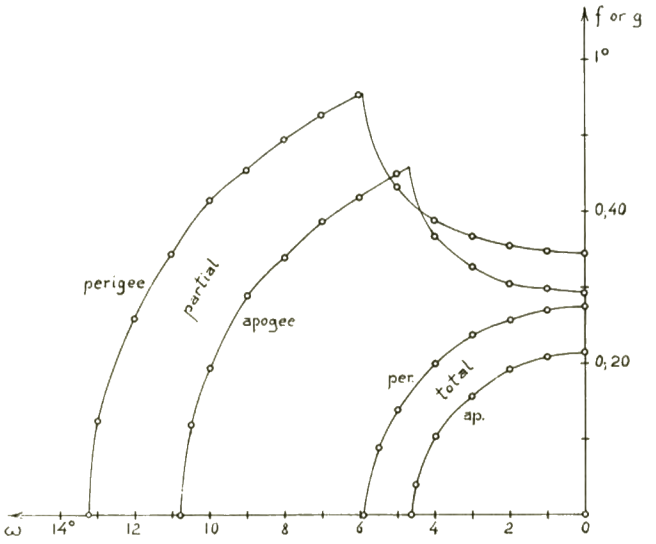


Fig. 35.

347 to 359;30 and from $\omega = 180;30$ to 193 and 193;17. Thus the eclipse limits at the ascending node are $\omega = -13;17$ and $+13;17$, at the descending node $180 - 13;17$ and $180 + 13;17$.

**Columns 3 to 5:** as in Tab. 73 and 74. In Tab. 75 col. 4 line 17 read 47;14 as in Tab. 76 line 11. In col. 5 lines 22, 24, 25 there exist discrepancies with Tab. 76. Maximum: 21;31 digits.

**Tab. 73 to 75, Columns 6 to 8:** "Table of proportion" i.e. coefficients of interpolation, varying between 0 and 1 (cf. fig. 36). Actually this table should not be associated spe-

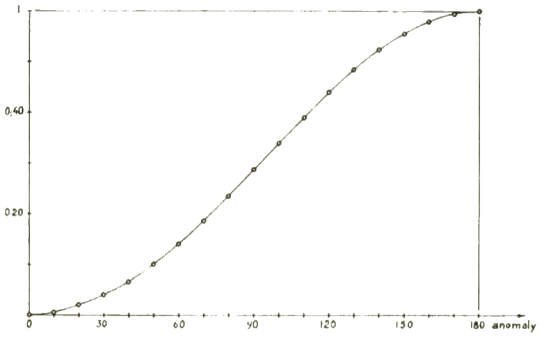


Fig. 36.

cifically with the lunar eclipses because it is equally needed for solar eclipses (Tab. 78).

**Columns 6 and 7:** "Parts of anomaly"[2] i.e. lunar anomaly in steps of 2 degrees from 2 to 180 and from 358 to 180.

[2] Lit. "parts of diversions" (*partitiones diversitatum*); this terminology is perhaps incorrect, suggested by the subsequent tables of parallaxes (Tab. 77 and 77a: *tabula diversitatis respectuum*). Cf. also ch. 35: *De diversitatibus defectuum solis* (above p. 73).

**Column 8:** "Minutes of the increments" i.e. fractions to be applied to the excess for perigee over apogee; cf. p. 68. A similar table is given in Almagest VI,8 (p. 522 Heiberg) with the only difference that the steps are 6°. The values of al-Khwārizmī's table are at the beginning sometimes smaller than in the table of the Almagest, toward the end occasionally larger. In the majority of cases the numbers are identical.

**Tab. 76, Columns 6 to 8:** Column 6: "digits of diameter" i.e. twelfths of the apparent diameter, or linear digits. Column 7: "twelfths of the sun", Column 8: "twelfths of the moon", i.e. twelfths of the area darkened[3] at the corresponding depth of immersion measured in linear digits in col. 6. These tables are identical with the tables in Almagest VI,8 (p. 522 Heiberg).

### Determination of the Parameters underlying Tab. 73 to 76, Columns 3 to 5

Let us assume that the horizontal line in fig. 34 represents the ecliptic in which the center of the shadow of radius $s$ is located. Let $r$ be the radius of the moon which during the eclipse moves on a straight line which is inclined at an angle $i$ toward the ecliptic. Then it is clear that the largest partial phase $f_M$ is given by

$$f_M^2 = (s+r)^2 - (s-r)^2 = 4\,sr \tag{1}$$

when the immersion at the mid-eclipse just produces totality. For deeper immersions the partial phase will decrease and reach a relative minimum

$$f_0 = 2\,r = d_{☾} \tag{2}$$

when the lunar node coincides with the center of the shadow. Then the totality reaches a maximum

$$g_M = s - r \tag{3}$$

and the argument of latitude $\omega$ is 0° or 180°. The values $f_0$ and $g_M$ can be found in the table

| | apogee Tab. 73 | perigee Tab. 75 |
|---|---|---|
| col. 4 | $f_0$ = 0;29,16° | $f_0$ = 0;34,34° |
| col. 5 | $g_M$ = 0;21,22 | $g_M$ = 0;27,27 |

and thus one finds for the radius $r$ of the moon and $s$ of the shadow

$$\left.\begin{array}{lll} r = 0;14,38° & s = 0;36,0° & \text{at apogee} \\ r = 0;17,17 & s = 0;44,44 & \text{at perigee.} \end{array}\right\} \tag{4}$$

The values for $r$ agree exactly with the values found from Tab. 61 to 66 (p. 106) whereas for $s$ we had previously (p. 107)

[3] A method for computing area-digits for given lunar latitude at mid-eclipse is explained by al-Bīrūnī in his book on chords (Suter [1] p. 46 No. 12). Also in Se fol. 57v–59v and clearer in Paris. gr. 2425 fol. 272v (both unpublished).

$$\left.\begin{aligned} s &= 0;35,29,12^\circ \quad \text{at apogee} \\ s &= 0;45,14 \quad\quad\ \text{at perigee.} \end{aligned}\right\} \tag{5}$$

For $f_M$ one would obtain from (1) and (4)

$$f_M \approx 0;45,52^\circ \quad \text{at apogee}$$
$$f_M \approx 0;55,30 \quad\ \text{at perigee.}$$

The nearest tabulated values are 0;44,52° and 0;55,20° respectively. Intermediate values for the partial and total phases can easily be constructed graphically and show good agreement with the tabulated values which are plotted in fig. 35.

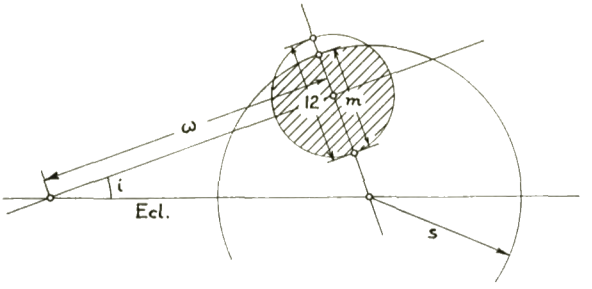


Fig. 37.

With $r$ and $s$ known we should now be able to determine the maximum value $m_M$ of the eclipse magnitudes, the greatest possible distance $\omega_0$ from the node, and the inclination $i$ of the lunar orbit to the ecliptic.

Indeed if $m$ denotes the eclipse magnitude in linear digits, $\omega$ the distance of the center of the moon from the nearest node, then we have (cf. fig. 37)

$$m = \frac{6}{r}(r + s - \omega \cdot \tan i) \tag{6}$$

and for the maximum

$$m_M = \frac{6}{r}(r + s) \tag{7}$$

whereas $m = 0$ for

$$\omega_0 = \frac{r + s}{\tan i}. \tag{8}$$

That we are entitled to operate here with plane instead of with spherical triangles is shown from the graph (fig. 38) of the values of $m$ which are given in Tab. 73 to 76 col. 3 and which vary practically linearly[4] with $\omega$, in agreement with (6).

In order to check the numerical values for $m_M$ we use in (7) the values (4). In this way we obtain

[4] If one actually computes $m$ as linear function between 0 at $\omega_0$ and $m_M$ at $\omega = 0$ one finds that the tabular values scatter irregularly around the computed values with derivations of rarely more than $\pm 0;4$. I could not detect any method which would lead exactly to the numbers in the extant tables.

| | | | |
|---|---|---|---|
| at apogee: | $m_M = 24;51$ | Tab. 73: | 20;46 |
| at perigee: | $m_M = 21;25$ | Tab. 75: | 21;31. |

The disagreement in the first case is, of course, far greater than can be explained by roundings in such simple computations as required by (7). The existence of a serious discrepancy between the two cases is once more revealed if one computes the ecliptic

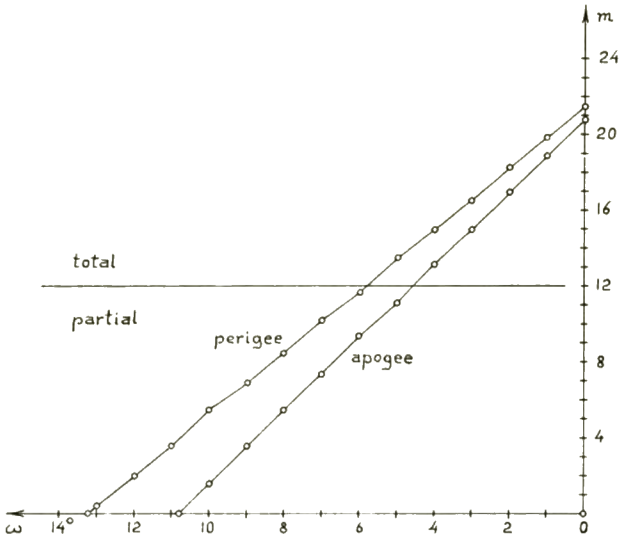

Fig. 38.

limit $\omega_0$ from (8). On the basis of Tab. 21 to 26, col. 6 (cf. p. 98) it seems reasonable to assume

$$i = 4;30°.$$

Then one obtains

| | | | |
|---|---|---|---|
| at apogee: | $\omega_0 = 12;51°$ | Tab. 73, 74: | 10;50° |
| at perigee: | $\omega_0 = 13;9°$ | Tab. 75, 76: | 13;17°. |

Conversely, if one uses the values of $\omega_0$ taken from the tables, to compute $i$ one finds for the apogee $i = 5;19°$ which is certainly wrong, while the parameters for the perigee give $i = 4;27°$ which is close enough to the expected value 4;30°. The limits for solar eclipses, given in Tab. 78, lead for apogee and perigee alike to $i = 4;27°$.

Thus there can be no doubt that the tables for apogee and for perigee belong to different sets of parameters. The fact that for the apogee $\omega_0 = 10;50°$ is almost the same value as in the Almagest VI, 8 ($\omega_0 = 10;48°$), combined with an inclination nearer to 5° than to 4;30° suggests that these tables were adjusted on the basis of the Almagest or of related tables, whereas the tables for the perigee reflect a different – presumably Hindu – tradition.

## Tables 77 and 77a

"Table of parallaxes of the moon in hours". This title is incorrect insofar as only the longitudinal component of the parallax is measured in hours, whereas the units of the latitudinal component are degrees. Furthermore we are dealing here not with the lunar parallax alone but with the "adjusted parallax",[1] i.e. the difference between solar and lunar parallax. Cf. also ch. 34 (p. 69).

[Addition in N by a later hand to Tab. 77 a:] "The greatest latitudinal parallax of the moon is 1;27° when the sun (?) [read: moon] is at nearest distance of the epicycle". I do not know of tables which show this value but the value itself suggests a table of the type of Almagest V,18 (maximum 1;44).

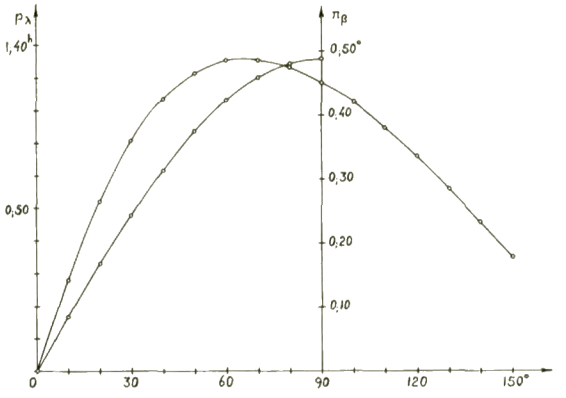

Fig. 39.

Prof. E. S. Kennedy discovered a duplicate of these tables, including the rules for their use similar to our chapter 34 (p. 69), in the *zīj* of ibn ash-Shāṭir, written probably in Damascus c. A.D. 1350.[2]

**Column 1:** "Column of numbers".

**Column 2:** cf. after column 3 (p. 123).

**Column 3** in Tab. 77 and Tab. 77a left section: "parallax in latitude".[3] This column (cf. fig. 39) is computed by means of

$$\pi_\beta = 0;48{,}45 \sin \bar{a} \qquad (1\,a)$$

where $\bar{a}$ is the zenith distance of the highest point of the ecliptic (V in fig. 40) and the coefficient

$$0;48{,}45^\circ = \pi_0 \qquad (1\,b)$$

[1] For this term cf. Kennedy [6] p. 35.

[2] Kennedy [6] p. 48 and Survey p. 125 No. 11 and p. 163, J.

[3] The heading "Hours of (parallax in) latitude" should have been omitted as well as the corresponding heading of the preceding column (cf. note 8), quite apart from the fact that the parallax in latitude is measured in degrees, not in hours.

represents the horizontal parallax, i.e. parallax at the horizon. According to the Khaṇḍa-Khādyaka V, 1, 2[4] the horizontal parallax corresponds to a time of 4 nadis i.e. 0;4[d]. Using for the mean velocities of sun and moon the values found in Tab. 61 to 66[5] we find indeed for the increase of elongation during 0;4[d]

$$(13;10{,}24 - 0;59{,}12)^{o/d} \cdot 0;4^{d} = 12;11{,}12^{o/d} \cdot 0;4^{d}$$
$$= 0;48{,}44{,}48^{\circ} \approx 0;48{,}45^{\circ} = \pi_0$$

i.e. the value (1b) for the adjusted horizontal parallax.

The same value of $\pi_0$ is then used in V, 3, 4 to find the latitudinal component from

$$\pi_\beta = \frac{13}{40}\,\mathrm{Sin}_{150}\,\bar{a} = \frac{13 \cdot 150}{40}\sin\bar{a} = 0;48{,}45\sin\bar{a}$$

or exactly the formula

$$\pi_\beta = \pi_0 \sin\bar{a} \tag{1}$$

which served also for the computation of our tables.[6]

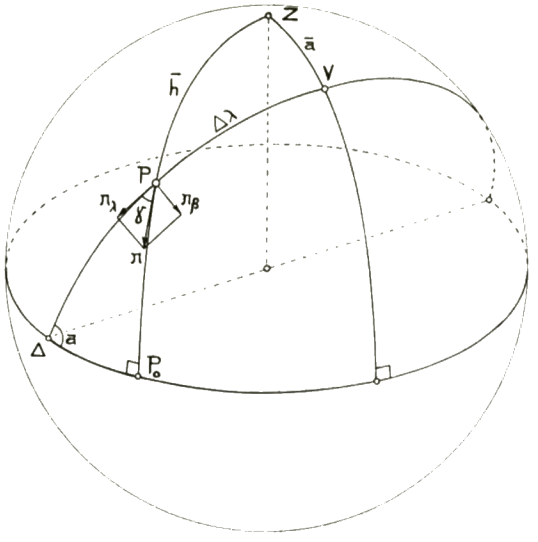


Fig. 40.

We now have to derive (1) which implies that the latitudinal parallax is constant along the ecliptic for a fixed position of the latter.

Let $\pi$ denote the total parallax which is a vector in the circle of altitude, pointing away from the zenith. With $\pi_0$ we mean as before the horizontal parallax, i.e. the value of $\pi$ for a zenith distance $\bar{h}$ of 90°; finally we call $\gamma$ the angle between ecliptic

[4] Similarly, e.g., in Sūrya-Siddh. V,10. Cf. below p. 123.

[5] Cf. p. 105.

[6] The Sūrya-Siddh. uses in V,11 the approximation $\pi_\beta = 0;49 \cdot \sin\bar{a}$, Ḥabash in his zīj (about A.D. 850) uses $0;48 \cdot \sin\bar{a}$ (Kennedy [6] p. 51).

and circle of altitude (cf. fig. 40 p. 122). Between the extrema for $\bar{h} = 0$ and $\pi = \pi_0$ for $\bar{h} = 90$ we assume a *sinusoidal variation*[7] of $\pi$:

$$\pi = \pi_0 \sin \bar{h}. \tag{2}$$

For the latitudinal component we have

$$\pi_\beta = \pi \sin \gamma \tag{3}$$

and for the spherical triangle PVZ, using (2):

$$\sin \gamma = \frac{\sin \bar{a}}{\sin \bar{h}} = \frac{\pi_0}{\pi} \sin \bar{a}$$

and thus with (3)

$$\pi_\beta = \pi_0 \sin \bar{a} \tag{1}$$

q.e.d. Thus the constancy of $\pi_\beta$ along the ecliptic is the equivalent of the approximation (2).

**Column 2:** "Hours of lunar parallax in longitude"[8] giving hours, minutes, and seconds of $p_\lambda$ (cf. fig. 39 p. 121).

Before discussing the problem of the computation of these tables we shall derive formulae which are the counterpart of (1) and (2).

Considering again the parallelogram formed by the components of parallax as plane and using for any angle $\Theta$ the notation

$$\overline{\Theta} = 90^\circ - \Theta$$

we find from (1) and (2)

$$\pi_\lambda^2 = \pi^2 - \pi_\beta^2 = \pi_0^2 (\sin^2 \bar{h} - \sin^2 \bar{a}) = \pi_0^2 (\sin^2 a - \sin^2 h).$$

Furthermore, from fig. 40 p. 122, in the triangle $\Delta P_0 P$ where the arc from $\Delta$ to P is $\overline{\Delta\lambda}$ because the arc from $\Delta$ to V is $90^\circ$

$$\sin a = \frac{\sin h}{\sin \overline{\Delta\lambda}}.$$

Thus

$$\pi_\lambda^2 = \pi_0^2 \sin^2 a \cdot (1 - \sin^2 \overline{\Delta\lambda}) = \pi_0^2 \sin^2 a \cdot \sin^2 \Delta\lambda$$

or

$$\pi_\lambda = \pi_0 \sin a \cdot \sin \Delta\lambda. \tag{4}$$

This is indeed the relation on which the method of computing $p_\lambda$ is based in the Khaṇḍa-Khādyaka V, 1, 2. Let us assume that the moon is exactly in the ecliptic, i.e.

[7] Actually one would find instead of (2) for the moon at mean distance and for a spherical earth

$$\sin \pi = \tan \pi_0 \sin (\bar{h} + \pi).$$

As we shall see (p. 124) Kepler's table of parallaxes is still based on (2) and (1).

[8] The heading "hours of (parallax in) longitude" in Tab. 77, next to the last column, is a duplication of the column title (cf. also note 3). Also the heading "longitude and latitude" given by N and O in Tab. 77a right section should have been omitted, following C.

$\beta = 0$. Then the rule given for the longitudinal parallax under the assumption of a horizontal parallax of

$$p_0 = 4 \text{ nadis} = 0;4^{\text{d}}$$

is

$$p_\lambda = \frac{\text{Sin } \Delta\lambda}{\left(\frac{R}{2}\right)^2 \frac{1}{\text{Sin } a}} \text{ nadis} = 4 \frac{\text{Sin } \Delta\lambda \cdot \text{Sin } a}{R^2} = 4 \sin \Delta\lambda \cdot \sin a$$

or

$$p_\lambda = p_0 \sin a \cdot \sin \Delta\lambda \tag{4a}$$

which is the exact equivalent of (4), only counted in time instead of in degrees of arcs[9]. For $\beta \neq 0$ the modification of sin $a$ to sin $a'$ takes place with $a' = a + \beta$ exactly as in our chapter 34 (above p. 70).

**Remark.** Kepler's table of parallaxes[10] is based on our three formulae

$$\pi = \pi_0 \sin \bar{h} \tag{2}$$

$$\pi_\beta = \pi_0 \sin \bar{a} \tag{1}$$

$$\pi_\lambda = \pi_0 \sin a \cdot \sin \Delta\lambda. \tag{4}$$

Since the elegance of Kepler's device is obscured by his way of presentation and since his procedure was partly misinterpreted by the editor Caspar[11], I shall give here a simple description of Kepler's rules.

His table is a table of double entry: the left column gives single degrees of a variable $\Theta$ between 0° and 90°; the top row represents a parameter $c$, measured in seconds of arc, from 1″ to 66″. Tabulated in each column is $c$ sin $\Theta$. This explains Kepler's rules:

To find the total parallax $\pi$: take $c = \pi_0$, the horizontal parallax, and $\Theta = \bar{h}$, the zenith distance of the moon. Then the table gives $\pi$ because of (2).

To find the latitudinal component $\pi_\beta$: take $c = \pi_0$ and $\Theta = \bar{a}$, the zenith distance of the "nonagesimal" V. Then the table gives $\pi_\beta$ because of (1).

To find the longitudinal component $\pi_\lambda$: take $c = \pi_0$ and $\Theta = a$, the altitude of the nonagesimal. Then the table gives max $\pi_\lambda = \pi_0 \sin a = \pi_{0\lambda}$. Enter the table once more but now with $c = \pi_{0\lambda}$ and with $\Theta = \Delta\lambda$, the distance of the moon from the nonagesimal. Then the table gives

$$\pi_{0\lambda} \cdot \sin \Delta\lambda = \pi_0 \sin a \cdot \sin \Delta\lambda = \pi_\lambda$$

because of (4).

Thus Kepler's theory of parallax is identical with the theory of the Khaṇḍa-Khādyaka.

[9] Measurement of longitudinal parallax in time is also found in the Sūrya-Siddhānta V,8, obviously because it affects the moment of the middle of an eclipse.

[10] Kepler, Werke 2, incorrectly placed with fol. 275 (p. 240/241) instead of with p. 275.

[11] Werke 2 p. 454 ad 275.29; correct in Werke 3 p. 463 ad 124.36.

We now return to column 2 in al-Khwārizmī's Tab. 77 and 77a. Kennedy succeeded[12] in discovering the process of computation of this table. He found that for a value $\Delta\alpha$ of the argument, as defined in chapter 34 as the right ascensional difference between the nonagesimal and the moon (cf. (2) p. 71), the longitudinal component of the parallax $p_\lambda$, measured in time, was found from

$$p_\lambda = 1;36 \sin \Theta \tag{5}$$

where $\Theta$ is a solution of the equation[13]

$$\Theta - 24 \sin \Theta = \Delta\alpha. \tag{6}$$

Kennedy also found that this equation was solved by an iteration process, described explicitly by Ḥabash in his *zīj* of about 850 A.D.[14] Since

$$1^d = 24^h = 6{,}0°$$

we can write for the horizontal parallax of $0;4^d$ assumed in Hindu astronomy[15]

$$\pi_0 = 6{,}0° \cdot 0;4 = 24°$$
$$p_0 = 24^h \cdot 0;4 = 1;36^h$$

and thus from (5) and (6)

$$p_\lambda = p_0 \sin \Theta \tag{7a}$$

$$\Theta - \pi_0 \sin \Theta = \Delta\alpha. \tag{7b}$$

This rule for finding $p_\lambda$ we have now to compare with the relation derived above

$$p_\lambda = p_0 \sin a \cdot \sin \Delta\lambda. \tag{4}$$

In other words we should show that

$$\sin \Theta = \sin a \cdot \sin \Delta\lambda \tag{8}$$

if $\Theta$ is a solution of

$$\Theta - \pi_0 \sin \Theta = \Delta\alpha. \tag{7b}$$

I see no way to prove the equivalence of $\sin a \cdot \sin \Delta\lambda$ with $\sin \Theta$ under the condition (7b). In fact I suspect that the text is misleading us by interpreting $\Delta\alpha$ as the difference in right ascension of the nonagesimal (V) and the moon (P) because the corresponding longitudinal difference can only vary between 0° and 90° and therefore $\Delta\alpha$ can not exceed about 96°. The argument of column 2, however, ranges between 0° and 150° instead of, at most, between 0° and 100°. In fact I see no reason for any argument in the computation of parallaxes to go essentially beyond 90°.[16]

[12] Kennedy [6] p. 50.

[13] This equation is known as the "Kepler equation" from its appearance in Kepler's theory of elliptic motion of a planet (Werke 3, p. 301).

[14] Kennedy [6] p. 51 and Survey p. 154, J. Cf. also Kennedy-Transue [1].

[15] Cf. above p. 122.

[16] If $\Delta\alpha$ be taken to represent the right ascensional arc which corresponds to the arc of the ecliptic between its culminating point and the moon, angles up to 150° can of course occur at sufficiently high northern latitudes. But the text of ch. 34 clearly defines the highest point of the ecliptic and not the culminating point.

The iteration process for the computation of the longitudinal parallax as described by Ḥabash[17] which is the equivalent of (7a) and (7b) proceeds as follows: compute a

$$\left.\begin{array}{l}\text{"first parallax": } \pi_1 = \pi_0 \sin \Delta\alpha\,;\\ \text{form: } \Delta\alpha + \pi_1 = \Delta\alpha + \pi_0 \sin \Delta\alpha \text{ and with this argument the}\\ \text{"second parallax": } \pi_2 = \pi_0 \sin (\Delta\alpha + \pi_1);\\ \text{form: } \Delta\alpha + \pi_2 = \Delta\alpha + \pi_0 \sin (\Delta\alpha + \pi_1) \text{ and with this argument the}\\ \text{"third parallax": } \pi_3 = \pi_0 \sin (\Delta\alpha + \pi_2);\\ \text{form: } \Delta\alpha + \pi_3 = \Delta\alpha + \pi_0 \sin (\Delta\alpha + \pi_2) \text{ and with this argument the}\\ \text{"fourth parallax": } \pi_4 = \pi_0 \sin (\Delta\alpha + \pi_3);\\ \text{form: } \Delta\alpha + \pi_4 = \Delta\alpha + \pi_0 \sin (\Delta\alpha + \pi_3) \text{ and with this argument the}\\ \text{"fifth parallax": } \pi_5 = \pi_0 \sin (\Delta\alpha + \pi_4).\end{array}\right\} \quad (9)$$

This is now considered to be a sufficiently close approximation, such that we can say that the longitudinal component, measured in time, is given by

$$p_\lambda \approx p_0 \sin (\Delta\alpha + \pi_4). \quad (10)$$

Each successive "parallax" in this procedure is of the form

$$\pi_n = \pi_0 \sin \Theta$$

which suggests that it is either the total parallax $\pi$, or the latitudinal component $\pi_\beta$, corresponding to a zenith distance $\Theta$ (cf. (2) and (1) p. 123). This points to a geometric procedure that modifies in successive steps $\pi$ or $\pi_\beta$ to $\pi_\lambda$. I do not see, however, how this could be done[18], nor do I understand how $\Theta$ could ever come near to 150°.

[17] Kennedy [6] p. 51.

[18] Similar approximation processes are not unknown from Hindu astronomy. Cf., e.g. the successive corrections for variation in latitude in the computation of the duration of an eclipse in Sūrya-Siddh. IV,12 to 15.

## Table 78

Left section: "Solar eclipse at far distance" i.e. when the moon is at the apogee of the epicycle.

Right section: "Solar eclipse at near distance" i.e. when the moon is at the perigee of its epicycle. Cf. chapter 35, later addition (p. 74).

Columns 1 and 2: "Argument of latitude". Limits:

$$\text{for apogee: } \begin{cases} 353 = 360 - 7 \leq \omega \leq 6;37° \\ 173;23 = 180 - 6;37 \leq \omega \leq 187° \end{cases}$$

$$\text{for perigee: } \begin{cases} 352;49 = 360 - 7;11 \leq \omega < 7;11° \\ 172;49 = 180 - 7;11 \leq \omega \leq 187;11°.^1 \end{cases}$$

[1] I follow MS N against C which gives the round numbers 352;30 and 187;30. Cf. Suter's apparatus p. 193. The limits given by the Toledan Tables Cod. Vindob. 2385 fol. 34r are: 353;33 and 6;37, 173;23 and 187;37, 352;30 and 7;11, 172;49 and 187;11.

**Column 3:** "Digits of the eclipse". Maxima:

at apogee: 10;48 at perigee: 12;44.

For the derivation of these values cf. the commentary to column 4.

The tabulated functions should be exactly linear, with a difference of almost 0;49 per line in the case of the apogee, of about 0;53,11 in the case of the perigee. The text, however, in both cases uses irregularly differences between 0;50 and 0;54

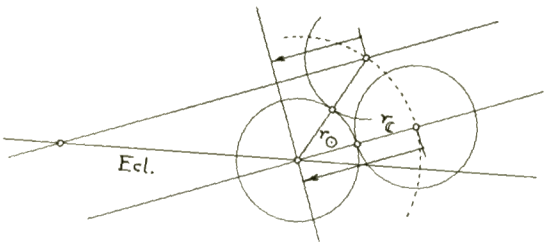


Fig. 41.

with an average of about 0;52 which corresponds to a maximum of 12 digits and limits of $\pm 7°$.

**Column 4:** "minutes of immersion". The extremal values given in this column are

for apogee: 0;30,55° for perigee: 0;33,34°.

We know from Tab. 61 to 66 that the radius of the sun had been assumed to be 0;16,17° (cf. p. 106) while the same tables as well as Tab. 73 to 76 (cf. (4) p. 118) give for the moon

at apogee: $r_\mathbb{C} = 0;14,38°$ at perigee: $r_\mathbb{C} = 0;17,17°$.

Thus we obtain

at apogee: $r_\odot + r_\mathbb{C} = 0;30,55°$ at perigee: $r_\odot + r_\mathbb{C} = 0;33,34°$

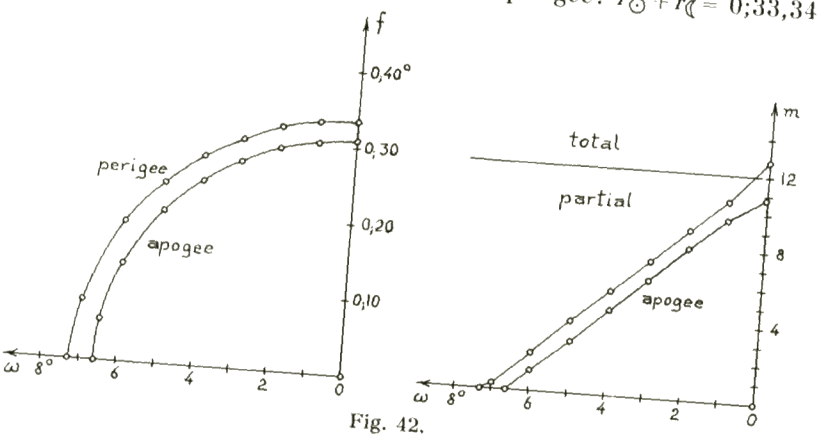


Fig. 42.

thus exactly the maximum values given in our column 4. This shows that column 4 tabulates the distance from the center of the moon at first contact to the middle of the eclipse (cf. fig. 41 and 42). The closeness of the values for solar and lunar diameters makes it unnecessary to distinguish between partial and total phase and leads us to identify the time of immersion with half duration.[2]

With the values for the apparent size of sun and moon securely established we can now compute the maximum eclipse magnitude $m_M$, measured in digits, i.e. twelfths of the solar diameter. According to this definition we have

$$m_M = 12 \frac{r_☾}{r_☉}$$

and substituting the above given values we find

$$\text{at apogee: } m_M = \frac{12 \cdot 0;14,38}{0;16,17} = \frac{2,55,36}{16,17} = 10;47,2,\ldots \qquad \text{col. 3: } 10;48$$

$$\text{at perigee: } m_M = \frac{12 \cdot 0;17,17}{0;16,17} = \frac{3,27,24}{16,17} = 12;44,17,\ldots \qquad \text{col. 3: } 12;44.$$

The ecliptic limits given in column 1 and 2

$$\text{for apogee: } \omega_0 = 6;37 \qquad \text{for perigee: } \omega_0 = 7;11$$

combined with the values for $r_☉ + r_☾$ in column 4, lead to a determination of the value which was assumed for the inclination $i$ of the lunar orbit toward the ecliptic, because

$$\tan i = \frac{r_☉ + r_☾}{\omega_0}.$$

In this way one finds both for apogee and perigee $i = 4;27°$ which is exactly the same value which we had found from the tables for lunar eclipses (above p. 120).

For the tables of interpolation for cases between apogee and perigee cf. the tables which are now associated as columns 6 to 8 with Tab. 73 to 75 (above p. 117).

[2] *On the basis of the present parameters and the corresponding velocities found in Tab. 61 to 66 (cf. p. 106) one finds for the greatest possible half duration of an annular eclipse about* $0;3,30^h$ *and similarly for totality about* $0;1,50^h$.

## Tables 79 to 90

"Equation of the twelve houses for the sign of Aries", ". . . Taurus", etc. to ". . . Pisces" respectively. Cf. chapter 36 (p. 78). MS C, which gives the same heading in its Arabic form, continues after Aries with the words "for the latitude" and Suter supplied "of Cordoba"[1]. Exactly the same tables (except for a few scribal variants)

[1] Suter p. 242 s.v. *Fi*; cf. also Tab. 91 (p. 129).

are preserved in the Toledan Tables cod. Vindob. 2385 fol. 8r to 10v under the title *Equatio domorum ad signum arietis* (etc.) *super latitudinem ciuitatis toleti*[2].

**Column 1:** single degrees from 1 to 30 called in Tab. 79 "*gradus aequales*[3] of the ascendant (in) Aries"; the subsequent tables (from Taurus to Pisces) say "rising" (*ortus*) instead of "ascendant" (*horoscopus*).

**Columns 2 to 6:** "The second (house[4]) Taurus" etc. to "The sixth (house) Virgo" in Tab. 79. The next table begins with Taurus and Gemini in column 2 and so forth. Tabulated: degrees and minutes within zodiacal signs.

Assuming that one enters column 1 with the longitude λ (H) of the ascendant one finds in the corresponding line of column 2 the beginning of the second house, in column 3 of the third, etc., to the sixth house in column 6. The beginning of the seventh house and its followers need not be tabulated since they are diametrically opposite to the corresponding first six houses. The length of each house is two seasonal hours, as explained in chapter 32 (cf. p. 77). The beginning of the fourth house is the culminating point of the ecliptic.

The geographical latitude for which these tables were computed is not mentioned. Suter assumed Cordoba since this place is repeatedly mentioned[5], thus φ = 38;30° which is presumably al-Majrīṭī's value for the latitude of Cordoba[6]. Since we know, however, that the same tables are also part of the Toledan Tables one has probably to assume φ = 39;54[7].

[2] Cf. Zinner [1] p. 749; also Millás-Vallicrosa, Est. Azar. p. 64.
[3] Cf. Tab. 59 to 59b p. 104 note 2.
[4] So one has to supply in spite of the fact that *casa* is feminine and the ordinal calls for a neuter.
[5] E.g. in ch. 31 (p. 61) or in Tab. 69 (p. 108).
[6] Suter p. 97 note 3 and Tab. 91 below. The Spanish version of al-Battānī gives φ = 38;20 (Nallino II p. 215); Cod. Vat. gr. 1291 has φ = 38;12 (Honigmann SK p. 194). The correct latitude would be 37;53°.
[7] Cod. Vindob. 2385 fol. 6r; Zinner [1] p. 749. The correct value is 39;52.

## Tables 91 to 114

"The casting of rays by the stars"[1]; at the top of the first sub-table: "for the latitude 38;30", presumably of Cordoba[2]. Cf. chapter 37 (p. 78).

There are six sub-tables (two facing pages in Suter) in each of the twelve main tables, one for each zodiacal sign, beginning with Aries.

Example of headings of a sub-table (first of Tab. 91): "Five degrees of Aries. Sextile 49;3° – Quartile 73;34° – Trine 96;6°".

Column 1: "equal degrees"[3] giving 5°, 10°, . . . , 30°.

Columns 2 to 6: "Aries, Libra, horoscopus" etc. for the remaining pairs of signs. For the name "*horoscopus*" cf. p. 78 n. 6 and the commentary to ch. 37 p. 80.

Marginal note to Tab. 91, 92[4]: "According to Eumathius (i.e. Ammonius) the

[1] Cf. ch. 37 p. 79 n. 11.
[2] Cf. above n. 5.
[3] *gradus aequales.*
[4] Suter p. 207 and p. 236 note 5.

radiation from the zodiac falls upon the equator and is emitted according to the latitude of this degree'. This apparently has to do with the doctrine mentioned by Suter p. 99.[5]

We use here the following abbreviations for the numbers given in the headings of the sub-tables for the aspects:

$s$ . . . sextile, $q$ . . . quartile, $t$ . . . trine.

Then one finds that as a rule the following relations hold – except for roundings and scribal errors:

$$s = \tfrac{1}{2}\, t \qquad q = \tfrac{3}{4}\, t = \tfrac{3}{2}\, s \tag{1}$$

or

$$s : q : t = 60 : 90 : 120 \tag{2}$$

as to be expected from the geometric significance of the aspects. This leads to the following corrections in the tables:[6]

| | | | | | | |
|---|---|---|---|---|---|---|
| Tab. 91, Aries | 10°: | read $s$ = | 49;11° | instead of | 41;11 | |
| | | read $t$ = | 98;22 | instead of | 98;32 | – correct in C |
| Tab. 93, Taurus | 10°: | read $s$ = | 51;34 | instead of | 51;24 | – correct in C |
| Tab. 97, Cancer | 5°: | read $t$ = | 122;22 | instead of | 122;12 | – correct in O |
| Tab. 106, Scorpio | 20°: | read $q$ = | 93;27 | instead of | 94;27 | – correct in C |
| Scorpio | 25°: | read $s$ = | 60;41 | instead of | 60;40 | – correct in O |

Best agreement with the given numbers is obtained if one assumes that the values of $t$ were computed first and then $s$ and $q$ by means of (1) and rounded to minutes.

I shall not discuss here in any detail the investigation of the numbers $t$ as function of the longitude $\lambda$ at which they are listed. They vary in a sinus-shaped but asymmetric fashion around the value of 120° as shown by the following characteristic numbers:

| | | | |
|---|---|---|---|
| $\lambda$ = ♌ | 5° | $t$ = | 96;39 = 120 − 23;21 |
| ♋ | 0 | | 119;16 |
| ♍ | 15 | | 139;18 = 120 + 19;18 |
| ♐ | 0 | | 119;22. |

There are many small irregularities among the intermediate numbers which seem to have been found by stretchwise linear interpolation, possibly with values at the midpoints of each sign actually computed. I would guess that these numbers were obtained as differences between oblique ascensions corresponding to a longitudinal arc of 120°, probably for the latitude of 38;30° mentioned at the beginning[7]. The possible

[5] Latitude in connection with the "projection of rays" is also mentioned by al-Khāzinī (about A.D. 1120) with reference to al-Bīrūnī; cf. Kennedy, Survey, p. 161 section O.

[6] There are more errors, but they are consistent with (1); e.g. for Gemini 5 (Tab. 95) one should have = 110;19 but $s$ and $q$ agree with the incorrect value $t$ = 109;19.

[7] Cf. p. 129.

results do not seem to me to be worth the effort of reconstructing the corresponding tables.

A similar situation prevails with the sub-tables. A check of the differences, which are always near 5° in each column, shows so many scribal errors and inconsistencies, common to all manuscripts, that a restoration of the original table would require a disproportional amount of computation.

## Table 115

"[Table of conversion for the years of Nativity]"[1].

Left section, column 1: "decades" from 10 to 100 Egyptian years;
column 2: "conversions of decades of years of Nativity";
column 3: "single years" i.e. Egyptian years from 1 to 10;
column 4: "conversion of single years of Nativity".

Right section: same headings as in left section.

This table gives the excess of sidereal years over Egyptian years. The first entry, e.g., in column 2 of the left section, means

$$10 \text{ Egyptian years} + 2^{d}\,14;1^{h} = 10 \text{ sidereal years}.$$

Using the entry $25^{d}\,20;15^{h}$ for 100 years one finds for one year the excess

$$\frac{25^{d}\,20;15^{h}}{100} = \frac{10,20;15^{h}}{1,40} = 6;12,9^{h} = 0;15,30,22,30^{d}$$

and thus

$$1 \text{ sidereal year} = 365;15,30,22,30^{d}. \qquad (1)$$

Exactly this value is also used by Brahmagupta in his Brahmasiddhānta[2] (written about A.D. 625), a work which is probably the "Sindhind" of the Arabic tradition[3]. Indeed al-Hāshimī (about A.D. 950) ascribes the value (1) to the Sindhind[4] while al-Bīrūnī quotes it repeatedly as a parameter in Hindu astronomy[5]. It appears again in Bhaskara's Siddhānta Śiromaṇi[6] (about A.D. 1150).

The right hand section of Tab. 115 is the exact equivalent of the left hand section since all entries are obtainable from the left section by multiplication with 15 (mod. 360°). Thus the units are now time degrees instead of hours. This expression for the difference between $365^{d}$ and a sidereal or a tropical year is known in Islamic astronomy as the "excess of revolution"[7].

[1] This title seems to have been supplied by Suter on the basis of column 2. This and the next table are found only in MS O and have no counterpart in the introductory text. Cf. Suter p. 102.

[2] J. Burgess [1] p. 722 or Nallino, Battānī I p. 206.

[3] Kennedy, Survey p. 129 No. 28 and Bīrūnī, Transits, p. 141 ad 27:1–4. Cf. also Davidian, Journal of the American Oriental Society 80 (1960) p. 332.

[4] Kennedy, Survey p. 151 Sect. O.

[5] India XXXV or LII (written about A.D. 1030); trsl. Sachau I p. 350, II p. 39.

[6] J. Burgess [1] p. 722 or Nallino, Battānī I p. 206.

[7] Nallino, Battānī I p. 211, or Kennedy, Survey p. 147.

Also the anonymus commentary of Hugo Sanctallensis refers implicitly to the relation (1): first[8] by ascribing to the Sindhind a number of 157791645000 days as common period of all planets, which, as we know from Hindu astronomy, is one kalpa of 4320000000 years; secondly[9] by assuming the *conversio mundanorum annorum*, i.e. the excess over 365[d], as 93;2,15°. Both relations are exactly equivalent to (1).

[8] Se fol. 17[r], 11 ff. The same value in Bīrūnī, India I p. 368 ff.
[9] Se fol. 61[v], 15 ff.

## Table 116[1]

**Column 1.** "Signs" from Aries to Pisces; common for all sub-tables.

**First sub-table:** "Houses and kings and separations and servants" which means, in the terminology of classical astrology, "houses", "exaltations", "oppositions to the houses", "depressions" respectively. The same terms appear again as headings of the four columns respectively. Listed are the the seven planets in the classical arrangement for each of the above mentioned concepts, but with the addition of the lunar nodes.

**Second sub-table**: left: "Triangles during daytime; their judges[2] in daytime"; *right*: *similar for nightime. Each triangle is given three rulers, as in al-Battānī (Nallino* II p. 299). This procedure is critisized by al-Bīrūnī, Astrol. § 445[3].

**Third sub-table:** "Decans" (*facies*) giving the ruler, following the classical order[4].

**Fourth sub-table:** "Terms according to the Medes". The table follows the order commonly known as the "Egyptian", e.g. in the Tetrabiblos[5], but the reference to the "Medes" (= Persians?) is peculiar.

Errors: Aquarius, column 3: read Jupiter 7, not 3 as is incorrectly given by Suter against the text. Column 4: read Mars 5, instead of 9.

[1] Contained only in O, apparently without title. Cf. Tab. 115 (p. 131 n. 1).
[2] The "rulers" in classical terminology.
[3] Trsl. Wright p. 259.
[4] Bouché-Leclercq, AG p. 228.
[5] I,20 Robbins = I,21 Boll-Boer. Also al-Bīrūnī, Astrol. § 453, trsl. Wright p. 265.

### Explicit

"Here ends the *zīj* – i.e. work[1] – of al-Khwārizmī – (i.e.) name of the author[1] – translated from the Arabic by Althelard of Bath".

This explicit is only found in MS O.

[1] Gloss, written above the line.

APPENDIX

# The MS. Corpus Christi College 283 (Fol. 114$^{r}$ to 145$^{r}$)

## Introduction

Suter, in a supplementary note[1] written in 1914 during the printing of his work, reported that C. H. Haskins had informed him of the existence in Oxford of another copy of al-Khwārizmī's tables. Haskins himself, in his Studies in the History of Mediaeval Science, gave (1927) some details about the new manuscript[2], quoting in particular a long passage from fol. 142$^{v}$ beginning with *dixit Petrus Anfulsus* and suggesting a mix-up between Adelard's translation and some work of Petrus Alphonsi.

Through the courtesy of the Bodleian Library I had the opportunity of investigating photocopies of these tables[3]. They cover Tables 1 to 56 of Suter's text, followed by Chapters 7 to 17 of the introduction. This material corresponds to the tables for mean and true longitudes of sun, moon and planets. Then follows the preface of Petrus Alphonsi and four short calendaric chapters.

Missing in the new text – henceforth cited as CC – are all tables concerning spherical and plane trigonometry, the tables of lunar phases and eclipses, and the astrological tables. That the latter are not genuine has been remarked by Suter; that the eclipse tables come from the Almanac of Azarqiel has been shown by Millás-Vallicrosa[4]. It seemed therefore tempting to think that CC represented the real nucleus of the tables of al-Khwārizmī.

Unfortunately this hypothesis cannot be upheld. A closer investigation of CC shows that the bulk of the tables is based on a rather unskillful adaptation to non-Islamic conditions: all the Arabic years and months are replaced by julian years and months and the numerical data are changed accordingly. The change from the Arabic lunar months to the julian calendar makes it impossible to use the methods of al-Khwārizmī for any of the lunar phenomena which depend on syzygies. Equally senseless is the use

[1] p. 236.

[2] In particular, p. 23 and p. 117f. Haskins' references to fol. 113 must now be changed to fol. 114, because of a recent renumbering.

[3] According to A. van de Vyver, Osiris 1 (1935) p. 688 n. 148, the part of the MS which concerns us here (fol. 114$^{r}$ to 145$^{r}$) was written by a hand of the end of the 12th century. The same date is given by Thorndike HM II p. 69 n. 3 (1923).

[4] Estudios sobre Azarquiel p. 144 note 3.

of the steps of 30 julian years instead of lunar years. It is quite obvious that under these conditions CC could not imitate its archetype beyond the determination of mean and true longitudes of the planets and hence we see that CC is not nearest to the Islamic original but on the contrary farthest removed from it.

Nevertheless CC is a manuscript of great interest. One of its owners disliked – rightly – the transformation to the julian norm of tables constructed for the Arabic calendar, and, writing between the lines of his manuscript, reverted many tables to Arabic years and months. These interlinear numbers[5] – which I shall cite as version D – are taken, however, not from the tables of al-Khwārizmī but, with minor exceptions, from the Toledan Tables. Thus CC provides us with some tables, which, needless to say, have never been published in extenso in modern times[6].

Finally the new manuscript raises many new questions, in particular by its repeated references in the text and on the margin to Petrus Alphonsi, to Azarqiel and to others. With almost no texts published the history of mathematical astronomy in the late Middle Ages will nevertheless remain largely unknown.

[5] While the original uses Roman numerals, the later owner preferred Hindu numerals.

[6] Solar tables and a table of the lunar anomaly were given by Herz, Bahnbest. II p. 49–52 (from a MS in Vienna) and a general summary from 52 unpublished manuscripts is found in a study of Zinner [1].

## Notes to the Transcription

I have made no attempt to adopt a consistent mode of transcription. In a few critical cases I have indicated abbreviation by enclosure in ( ). In general I have, of course, followed the spelling of the manuscript but I did not try to distinguish everywhere between *u* and *v*, *i* and *j*, capitals and minuscles, etc.

In the original text (CC) all numbers are Roman numerals unless I explicitly stated otherwise. The numbers in the interlinear version (D) are Hindu numerals. For the marginal notes and similar additions I have always indicated, in the notes or in the commentary, whether Roman or Hindu numerals had been used.

In the transcription of sexagesimal numbers I have ordinarily used the notation *a*;*b*,*c* to separate degrees (*a*) from minutes (*b*) and seconds (*c*). In the headings of tables I represent the units by the following abbreviations

| | | |
|---|---|---|
| *s.* | *signa* | (signs) |
| *g.* | *gradus* | (degrees) |
| *p.* | *puncta* | (minutes) |
| *m.* | *minutie* | (seconds). |

The numbering of tables, columns, and lines is, of course, my addition.

## Folio 114r [1]

### 1. Left Table, corresponding to Suter Table 2

**A. Transcription** **Plate 1**

| | Tabula ad cognoscendum in qua feria incipiat quilibet mensis arabum. | | | | |
|---|---|---|---|---|---|
| | Annales | Ferie | Singulares anni | Ferie | |
| 1. | 30 | 5[1] | 1 | 4[2] | 1. |
| | 60 | 3 | 2 | 1 | |
| | 90 | 1 | 3 | 6 | |
| | 120 | 6 | 4 | 3 | |
| 5. | 150 | 4 | 5 | 7 | 5. |
| | 180 | 2 | 6 | 5 | |
| | 210 | 7 | 7 | 2 | |
| | 240 | 5 | 8 | 7 | |
| | 270 | 3 | 9 | 4 | |
| 10. | 300 | 1 | 10 | 1 | 10. |
| | 330 | 6 | 11 | 6 | |
| | 360 | 4 | 12 | 3 | |
| | Menses arabum. | | 13 | 7 | |
| | | | 14 | 5 | |
| | | | 15 | 2 | 15. |
| 1. | Almoharam | 1[4] | 16 | 7[3] | |
| | zaphar | 3 | 17 | 4 | |
| | Rabe pr. | 4 | 18 | 1 | |
| | Rabe sec. | 6 | 19 | 6 | |
| 5. | Iumet pr. | 7 | 20 | 3 | 20. |
| | Jumet sec. | 2 | 21 | 7 | |
| | Raieb | 3 | 22 | 5 | |
| | Sabben | 5 | 23 | 2 | |
| | Ramazan | 6 | 24 | 6 | |
| 10. | Sauhel | 1 | 25 | 4 | 25. |
| | Zilchecl | 2 | 26 | 1 | |
| | Zilhege | 4 | 27 | 6 | |
| | | | 28 | 3 | |
| | | | 29 | 7 | |
| | | | 30 | 5 | 30. |

[1] Here and in the subsequent lines, down to 270, written in the form $\overset{a}{5}$. *feria*; in the last three lines *1. feria* etc.

[2] Written in the whole column in the form *feria* $\overset{a}{4}$ etc.

[3] Sic, instead of *6*; correct in Suter's tables.

[4] Written in the whole column in the form $\overset{a}{1}$ etc.

[1] Previously called fol. 113 and as such quoted by Coxe, Catal. II p. 121, by Haskins SHMS (p. 24; p. 117 n. 11, 12; p. 118 n. 14), and by Thorndike HM II p. 69 n. 3, etc.

**B. Translation and Comments**

"Table for knowing in what weekday any Arabic month begins".
**Section 1,** Column 1: "(Accumulated) years";[2] column 2: "Weekdays".
**Section 2,** Column 1: "Single years"; column 2: "Weekdays".
**Section 3:** "Arabic months".

For the structure and use of these tables cf. above p. 84, as well as the duplicate fol. 142[r] (p. 206).

[2] This use of *annales* for *anni collecti* is not known to me from other MSS.

## 2. Right Upper Table, corresponding to Suter Table 2a

**A. Transcription** **Plate 2**

| | Nomina mensium persarum. | Tabula ad cognoscendum in qua feria incipiat quilibet mensis persarum.[1] numerus annorum persarum. | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1. | Affrodinmah | 3[2] | 4 | 5 | 6 | 7 | 1 | 2 |
| | Ardenhastmah | 5 | 6 | 7 | 1 | 2 | 3 | 4 |
| | Ardarmah | 7 | 1 | 2 | 3 | 4 | 5 | 6 |
| | Nirmah[3] | 2 | 3 | 4 | 5 | 6 | 7 | 1 |
| 5. | Mardarmah | 4 | 5 | 6 | 7 | 1 | 2 | 3 |
| | Zaberermah | 6 | 7 | 1 | 2 | 3 | 4 | 5 |
| | Maharmah | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | Abenmah | 3 | 4 | 5 | 6 | 7 | 1 | 2 |
| | Adarmah | 3 | 4 | 5 | 6 | 7 | 1 | 2 |
| 10. | Zimah | 5 | 6 | 7 | 1 | 2 | 3 | 4 |
| | Baharmah | 7 | 1 | 2 | 3 | 4 | 5 | 6 |
| | Bazspadar | 2 | 3 | 4 | 5 | 6 | 7 | 1 |
| | mah | | | | | | | |

[1] Here the text (b) p. 138 is written by a later hand.

[2] All numbers of the first six lines of entries in the tables are written in the form *feria.* $\overset{a}{3}$ etc., the remaining lines, however, write only *feria. 1.* etc.

[3] Sic, instead of Tiirmah.

**B. Translation and Comments**

"Table for knowing in what weekday any Persian month begins". Column 1: "Names of the Persian months"; column 2: "Number of the Persian years".

For structure and use of this table cf. above p. 86; for additions by later hands to the heading cf. below p. 138.

### 3. Right Lower Table, corresponding to Suter Tab. 1 Sect. 2

**A. Transcription** **Plate 3**

| | Tabula ad cognoscendum quantum temporis secundum omnes subscriptos terminos restat usque ad principium huius operis.[1] | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | nomina terminorum[6] | milia anno rum. | Cente naria | Dece naria. | Uni tatis | Men ses. | Dies. | ferie prin cipii an norum. |
| 1. | Diluuium. | 4 | 2 | 2 | 0 | 0[2] | 6 | 6 [5] |
| 2. | donosor Nabucha | 1 | 8 | 7 | 8 | 8 | 13 | 4 |
| 3. | Alexandr. | 1 | 4 | 2 | 7 | 0 | 1 | 2 |
| 4. | hera. | 1 | 1 | 5 | 3 | 9 | 1 | 1 |
| 5. | Incarnat. | 1 | 1 | 1 | 5 | 9 | 7 | 7 |
| 6. | egip. | 0 | 8 | 4 | 7 | 3 | 8 | 6 |
| 7. | Arabum. | 0 | 5 | 0 | 9 | *or* 4 | 22 | 5 |
| 8. | Persar. | 0 | 4 | 9 | 9 | 5[4] | 13 | 3 |

[1] Here the text (c) p. 138 is written by a later hand.
[2] Sic, instead of 6.
[3] i.e. *quatluor*.
[4] Reading very uncertain.
[5] Cf. the marginal notes (d) p. 138.
[6] Written on diagonals.

**B. Translation and Comments**

"Table for knowing how much time is left for each of the epochs listed below to the beginning of this work".

Column 1: "Names of the years"; column 2: "Millennia"; column 3: "centuries"; column 4: "decades"; column 5: "units" (i.e. single years); column 6: "months"; column 7: "days"; column 8: "Weekdays of the beginning of the years".

This table is similar to the table of eras in Suter, Tab. 1, Sect. 2, with the exception that fewer eras are mentioned and that all intervals are given with respect to the epoch "of this work" (A.D. 1116). There is furthermore added a column which gives the weekday of the first day of each era. For details see the commentary below p. 143 ff.

### 4. Marginal Notes by Later Hands

**A. Transcription**[3]

**(a)** Upper margin:

*tot sunt dies ab incarnatione domini usque ad principium operis huius Petri Amphulphi* .407534. *quia . . . . est et . . . . ad . . .*

*Tot sunt dies persarum usque ad principium huius operis* 182299.

[3] All numbers in these marginal notes are written with Hindu numerals, unless stated otherwise.

**(b)** Below title of right upper table:

*Secundum Petrum*
*Differencia annorum Christi et persarum id est a nativitate Christi usque ad principium*
*Annorum persarum tot sunt dies .225235. Qui sunt in annos re*
*ducti Anni tot .616. et praeterea dies tot .241. et sic incepit primus*
*annus persarum .$\overset{o}{10}$*[4] *kalendis Septembris Vigilia sancti Bartholomi super*
*d. letteram. lettera dominicali .B*
*ciclo manus mee*(?) *solari .$\overset{o}{10}$.*[4]

**(c)** Below title of right lower table:

*Michi nihil aliud istud usque inclusive . . . . Nos non sumus de hac opi*
*nione Petri sed incipimus annos domini ab annuntiacione domini . . . et dicimus*
*quod anno sequenti conceptus est et natus dominus natus . . . . secundum nos.*

**(d)** Margin beside right lower table:[5]

beside lines 1 and 2: *tholomaus an*[*nos*
1111.[6]
beside line 5: . . . . *mens annos* [
*secundum*(?) *Petrum annos* [
beside line 7: . . . . *Arabum an*[*nos*
beside line 8: 54(?)

**(e)** Margin below the tables:

*Differencia annorum Nabugudoni et Christi anni* 762(?) *et Januarii*
*ultimi dies scilicet finales Januarii dies* 21
*deinde menses 10.*[7] *deinde decembris dies* 24.

### B. Translation and Comments

The *Petrus Amphulsus* mentioned in (a), or simply *Petrus* in (b) (c) (d), is in all probability the *Petrus Anfulsus* of fol. 142$^{r}$ and 143$^{v}$ (below p. 175 and 216), also known as the author of the *Disciplina clericalis* and of the *Dialogi cum Iudeo.*[8]

**(a)** "The number of days from the incarnation of the Lord to the beginning of the work of Petrus Alphonsi is 407534 . . . . The number of days of the Persians to the beginning of this work is 182299".

The first number is repeated in the marginal note (b) to fol. 142$^{r}$ (cf. p. 210).

[4] Written with Roman numerals.
[5] Endings lost by trimming of page.
[6] If read as Hindu numeral; otherwise Roman 4.
[7] Written with Roman numeral.
[8] Cf. Thorndike HM II p. 68ff. and the monograph on Petrus Alphonsi by Millás-Vallicrosa [1].

**(b)** "According to Petrus the difference between the Christian and Persian years, i.e. from the birth of Christ to the beginning of the Persian years, is 225235 days. These, reduced to years, are 616 years and an additional 241 days; and thus the first year of the Persians began on the 10th calends of September, on the day before (the day) of St. Bartholomew, with the letter *d*; dominical letter *B*, on the 10th solar cycle . . . . .".

The difference of 225235 days between Christian and Persian era is consistent with the two numbers given in (a). It is also correct that this difference corresponds to 616 julian years plus 241 days. The 10th calends of September correspond to August 23, the day before the feast of St. Bartholomew, August 24.[9] Thus the epoch of the "Persian years" is assumed to be 617 Aug. 23.[10] Subtracting 225235 days we obtain as epoch of the Christian era year 0 Dec. 25.[11] Adding to this date the 407534 days given in (a) we find for the epoch of the work of Petrus Alphonsi A.D. 1116 Oct. 1.

The "dominical letter" of A.D. 617 is B, the "lettera calendarum" of August 23 is *D*, the solar cycle is 10,[12] exactly as stated in our text.

**(c)** is almost unintelligible to me. In only understand that "we are not of the opinion of Petrus, but we begin the years of the Lord from the annunciation of the Lord (i.e. from March 25[13]) and we say that in the following year the Lord was conceived and born, . . . . . according to us". A similar statement is found on the lower margin of fol. 123$^{v}$ (below p. 181).

**(d)** refers in the line for the era Nabonassar to Ptolemy, in the line for the years of the incarnation to Petrus (Alphonsi), and in the line for the Arabic era to the years of the Arabs, all for reasons unknown.

**(e)** "The difference of the years of Nabonassar and of Christ (amounts to) 762[14] years and the last days of January, namely 21 final days of January, then 10 months, then 24 days of December".

Accepting the above reading we obtain as epoch for the era Nabonassar – 762 Jan. 10, instead of – 746 Febr. 26, i.e. an error of 16 years and 47 days, or of 5890 days. A similar, though slightly smaller error, results from the table itself (cf. Pl. 3 p. 137), according to which the era Nabonassar begins 5410 days too early, i.e. – 761 May 6 (cf. p. 143). This latter error is consistent with errors in the epoch of the era Yazdigerd and the Hijra.[15] Thus it is clear that at least parts of the chronological system in our text is badly distorted.

[9] Cf., e.g., Grotefend, Zeitr. I p. 15.

[10] Julian day 1946652, *feria* 3 (Tuesday); cf. below p. 143.

[11] Julian day 1721417; cf. for this norm Ginzel, Hdb. III p. 167.

[12] Ginzel, Hdb. III p. 129, p. 126, and p. 125 respectively.

[13] Ginzel Hdb. III p. 161 ff., or Grotefend, Zeitr. I p. 7 ff.

[14] This reading is uncertain because the middle digit is amost invisible under an inkblot. Nevertheless it seems certain to me that the traces exclude the correct figure 746.

[15] This consistency is based on the use of Egyptian years for the interval from (γ) to (π) on p. 144. If one uses julian years the error increases to 5880 days and the corresponding date, – 762 Jan. 20, becomes almost identical with the date obtained from (e).

## Folio 114v

### 1. Table, corresponding to Suter Table 3a

**A. Transcription** **Plate 4**

Tabula ad cognoscendum in qua feria incipiat quilibet mensis romani anni secundum illos qui uolunt bissextum esse in mense decembri:

| Numerus annorum Romanorum singularium | Tesserin prim. | Tesserin sec. | kenun prim. | kenun sec. | Sabat | Iadgar | Nicen | hiiar | Agiran | Thamus | Ab | helul | Bissextus[1] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Octob. | Nov. | Dec. | Ian. | Febr. | Mar. | Apr. | Mai. | Iuni. | Iuli. | Aug. | Sept. | |
| | 31 | 30 quarta | 31 quarta | 31 | 28 | 31 | 30 | 31 | 30 | 31 | 31 | 30 | |
| | fer. | fer. | fer. | fer. | fer. | fer. | fer. | fer. | fer. | fer. | fer. | fer. | |
| | | | | | | | | | | | | | |

same as in Suter, Tab. 3a[2]

[1] In Suter, Tab. 3a, this column is correctly placed between December and January.
[2] All numbers and *bissexti* as in Suter Tab. 3a, the only exception being year 18 June and year 16 July where CC has 6 instead of the correct 7 of Suter.

**B. Translation**

"Table for knowing on what weekday any month of the Roman year begins according to those who wish to put the leap-day in the month of December".

Column 1: "Number of the single Roman years".

Main table: heading line 1: Syriac month names from "First Tesserin" to "Helul". Line 2: Roman month names from October to September. Line 3: number of days in these months; in November "one fourth 30" (probably an error), in December "31 one fourth". Line 4: "Weekday".

Last column: "Leap-days".

For the contents of this table cf. above p. 89; for the system of intercalation see below p. 224.

### 2. Marginal Notes by Later Hands

**A. Transcription**

**(a)** Upper margin:

*hec tabula subscripta grecorum est. forte vocat eos romanos a romania*

**(b)** Lower margin, beginning to the left of last line (year 28):

[*I*]*n hoc anno ultimo*
[*i*]*ncipit opus petri.*

[. .] *iste erat ciclus anno primo de tabulis planetarum sequentibus. Iste etiam ultimus ciclus fuit millesimo* $^{o}$*200 anno ab incarnatione domini et secundum nos in [h]oc anno deberet interseri dies bisextilis. secundum grecos vero in anno precedenti. Nota ergo quod principium huius ultime linee fuit annus ab incarnatione .1.*$^{o}$*200. finis vero .*$^{o}$*1201.*[1] *Nota quod ciclus lunaris grecorum non excedit 8.*

**B. Translation and Commentary**

**(a)** "The table below is for the Greek (years). Perhaps he calls them Roman for Romania". This remark is justified insofar as we are dealing here with the Syrian version of the Seleucid era (cf. above p. 89). This era is called "Era of the Greeks" by Syrian and Arabic authors.[2]

**(b)** "In the last year (the year 28 of the table) begins the work of Petrus". For Petrus Alphonsi cf. above p. 138. Since the epoch of this work is known to be 1116 Oct. 1 (above p. 138), and since year 1 of the Seleucid era begins = 311 Oct. 1 (above p. 89), we find for the epoch of Petrus Alphonsi the year S.E. 1428. And because $1428 \equiv 28$ mod. 28[3] it is correct to say that the epoch of Petrus corresponds to the last, the 28th, line of the table.

What follows next I do not really understand and my translation is therefore only tentative: "This was the [. .] cycle in the first year of the following planetary tables; it was also the last cycle in the 1200th year from the incarnation of the Lord[4], and according to us the leap-day should be inserted in this year, according to the Greeks in the preceding year. Note that the beginning of this last line (of the table, namely October) was the year 1200 from the incarnation, the end (September), however, (the year) 1201. Note that the Greek lunar cycle does not exceed 8 (years)".

This means that the julian years of the Seleucid era, beginning with October 1, straddle the years of the incarnation which begin with December 25.[5]

The use of an octaeteris up to the 15th century by Byzantine chronologers has been demonstrated by Schissel.[6] This cycle seems to be mentioned again in the notes to fol. 145$^{r}$.[7]

[1] Written with Hindu numerals; all other numbers are written with Roman numerals.
[2] *Tarīkh al-yūnāniyyīn*; cf. Taqizadeh [2] p. 125.
[3] Cf. also Tab. 1, Sect. 1 (above p. 82).
[4] The year A.D. 1200 = S.E. 1512 corresponds again to the 28th year of the table because $1512 \equiv 28$ mod. 28.
[5] Cf. below p. 143.
[6] Cf. Schissel [1].
[7] Below p. 227.

## Folio 115[r]

### 1. Table corresponding to Suter Table 2b

**A. Transcription** **Plate 5**

Tabula ad cognoscendum in qua feria incipiat quilibet mensis annorum egiptiacorum qui arabice dicuntur Kepthie

| Numerus annorum egiptiacorum singularium | 30 Tot fer. | 30 hobbe fer. | 30 haton fer. | 30 Kihac fer. | 30 Toba fer. | 30 amsir fer. | 30 barha mech fer. | 30 barmu da fer. | 30 bessir fer. | 30 Thona fer. | 30 Abib fer. | et quarta 35 Ma- cere fer. | Bissexti |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | [2] |

same as in Suter, Tab. 26 b[1]

[1] Deviations from Suter Tab. 2b: year 5 month 7: CC has 3 (instead of 2), year 6 month 7: CC has 4 (instead of 3), year 6 month 9: CC has 6 (instead of 7); Suter's values are the correct ones.
[2] All leap years are marked one line lower than in Suter's table, the latter being correct.

**B. Translation**

"Table for knowing on what weekday any month of the Egyptian years begins, which in Arabic are called Coptic".

Column 1: "Number of single Egyptian years".

Main table: Line 1: "30" for the first 11 months, "35 and one fourth" for the last month. Line 2: Names of the Coptic months from "*Tot*" to "*Macere*" (= Mesore). Line 3: "*Feria*".

Last column: "Leap-days".

For the contents of this table cf. above p. 86 f.

### 2. Marginal Notes by Later Hand

**A. Transcription**

*Differentia annorum Christi et Egiptiorum. Dies tot.* .98069. *Qui sunt anni* .268.[1] *postea dies* .182.[1] *Et sic incipit*
*ciclus egiptius in crastino nativitatis sancti Johannis feria* 6.[2] *letera dominicali C.*

[1] Traces of writing above the numerals.
[2] Roman numeral; all other numerals are Hindu numerals.

**B. Translation and Comments**

"The difference between Christian and Egyptian years is 98069 days which are 268 years and 182 days. And thus the Egyptian cycle begins on the day following the nativity of St. John, weekday 6 (Friday), dominical letter *C*".

It is correct that 98069 days are 268 julian years and 182 days. The nativity of John the Baptist is celebrated on June 24.[3] If we subtract from A.D. 269 June 25[4] the 98069 days, we find as before (p. 139) for the beginning of the Christian era 0 Dec. 25.

It is also correct that *C* is the dominical letter of the year A.D. 269[5] and that 269 June 25 has the weekday number 6.

[3] Ginzel, Hdb. III p. 191.
[4] Julian day 1819486.
[5] Ginzel, Hdb. III p. 129.

**The Chronological System of Petrus Alphonsi**

The chronological table on fol. 114r, lower right half,[1] presents the following eras and epoch dates to which we add the modern equivalents[2] and the corresponding weekday numbers *w*:

(β) "Diluvium": −3101 Febr. 17 = jul. day 588465 *w* = 5, text: 6
(γ) "Nabonassar": −746 Febr. 26 = j.d. 1448638 *w* = 4
($\varepsilon_2$) "Alexander" (Seleucid era[3]): −311 Oct. 1 = j.d. 1607739 *w* = 2
(ζ) "Era" (Spanish era): −37 Jan. 1 = j.d. 1707544 *w* = 1
(η′) "Incarnation": 0 Dec. 25 = j.d. 1721417 *w* = 2, text: 7
(θ) "Egypt" (Diocletian): 284 Aug. 29 = j.d. 1825050 *w* = 6
(ι) "Arab" (A.H.): 622 July 15 = j.d. 1948439 *w* = 5
(κ) "Persian" (era Yazd.): 632 June 16 = j.d. 1952063 *w* = 3.

To this list we add the epoch date "of the present work" as derived from the marginal notes[4]:

(π) "The present work" of Petrus Alphonsi: 1116 Oct. 1 = j.d. 2128951 *w* = 1.

The table on fol. 114r (cf. Pl. 3 p. 137) gives the following intervals between the different eras and "the present work" (π):

| | | | |
|---|---|---|---|
| (π) − (β): | 4220 years | 0 months | 6 days |
| (π) − (γ): | 1878 | 8 | 13 |
| (π) − ($\varepsilon_2$): | 1427 | 0 | 1 |
| (π) − (ζ): | 1153 | 9 | 1 |
| (π) − (η′): | 1115 | 9 | 7 |
| (π) − (Θ): | 847 | 3 | 8 |

[1] Cf. above p. 137.
[2] Cf. also our discussion of Suter, Table 1 Section 2 (p. 82ff.).
[3] For the various names of this era cf., e.g., Taqizadeh [1] p. 124f.
[4] Cf. p. 139.

| | | | |
|---|---|---|---|
| (π) – (ι): | 509 | 4 | 22 |
| (π) – (κ): | 499 | [5] | 13(?) |

The entry for (η′) confirms in a general fashion the date of (π). Because the year 0 is a leap-year there are 7 days between 0 Dec. 25 and 1 Jan. 1; then 9 months from Jan. 1 to Oct. 1[5], and 1115 julian years from A.D. 1 to 1116. But there are also flagrant contradictions; e.g. for the era Hijra one would obtain the year 1115 – 509 = 606 instead of 622. We must, accordingly, discuss the entries one by one.

(β). If we assume Egyptian years, as in Suter Tab. 1, we find for 4220 years 6 days the equivalent of 1540306$^{d}$ instead of 1540486$^{d}$. The difference amounts to exactly 180$^{d}$ = 6 months and is easily explicable as a scribal error.

(γ). Operating with Egyptian years we find that (π) – (γ) is given 5410$^{d}$ = 14 years 10 months longer than it should be. It seems very implausible to ascribe this deviation to a copyist's error and it is indeed possible to show that the error is of a systematic character and not an accidental scribal mistake.

The marginal note (b) of this folio[6] gives as interval from (η′) to (κ) the number of 225235 days whereas actually this interval is 230646 days, or 5411 days longer. Since 5411 ≡ 0 mod. 7 the weekday remains the same as in the correct list.

If we ignore a difference of one day[7] the error in the distance from (η′) to (κ) is the consequence of placing (γ) too early (dotted line in fig. 43) but using the right distance from (γ) to (κ).

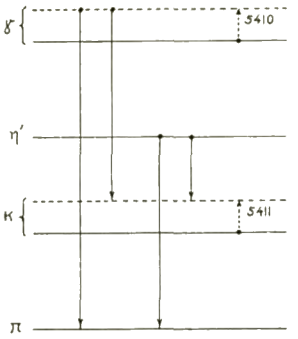


Fig. 43.

($\varepsilon_2$). The given difference (π) – ($\varepsilon_2$) is 521212 $^{3}/_{4}$ days if we operate with julian years, as is to be expected from Suter's table.[8] This is correct if we ignore the $^{3}/_{4}$ of one day.

(ζ). The difference (π) – (ζ) is correct within 1 day if we reckon January to October as 9 months in the same fashion as with (η′).

[5] But not 9 schematic months of 30 days each as in Suter's Tab. 1.

[6] Above p. 139.

[7] This one day may have to do with the one day involved in the norm of (π) which is mentioned in the marginal note (b) of fol. 142$^{r}$ b 8 (below p. 211). The difference 5411 is supported by the correctness of the weekday.

[8] Cf. above p. 83.

(η′). Cf. above p. 143.

(Θ). Computing with julian years and schematic months of 30 days we find for the given interval: (π) – (Θ) = 309464 $^3/_4$ days or for (Θ) the date 269 June 25 which is exactly the date given in the marginal note of fol. 115$^r$.[9]

(ι). If one computes the difference (ι) – (Θ) one finds 337 julian years and 321 $^1/_4$ days = 123410 $^1/_2$ days. The correct difference between (Θ) and (ι) would be 123409 days. Thus, except for one day, (ι) is consistent with (Θ) and we have to assume A.D. 607 May 11 or 12 ($w$ = 5 or 6 respectively) as epoch for the era of the Arabs.

(κ). As epoch of the "Persian" era we have already found A.D. 617 Aug. 23.[10] Combining this with (ι), we would find: (π) – (κ) = 499 years 3 months 7 $^1/_2$ days. The text gives 499 years 5(?) months 13(?) days, thus a nearly correct interval.

Our results accumulated so far can be summarized in the following list of epochs:

(β) "Diluvium": – 3101 Febr. 17 $w$ = 6 (error for $w$ = 5)
(γ) "Nabonassar": – 761 May 5 or 6[11]; May 5: $w$ = 4 (instead of – 746 Febr. 26, $w$ = 4)
($\varepsilon_2$) "Alexander" (Sel. era): – 311 Oct. 1 $w$ = 2
(ζ) "Era" (Spanish): – 37 June 1 $w$ = 1
(η′) "Incarnation": 0 Dec. 25 $w$ = 7 (error for $w$ = 2)
(Θ′) "Egyptian": 269 June 25 $w$ = 6 (instead of 284 Aug. 29, $w$ = 6)
(ι′) "Arabs": 607 May 11 $w$ = 5 (instead of 622 July 15, $w$ = 5)
(κ′) "Persian": 617 Aug. 23 $w$ = 3 (instead of 632 June 16, $w$ = 3)
(π) "Present work of Petrus Alphonsi": 1116 Oct. 1.

This shows that the era Nabonassar, the Diocletian era, and the eras of the Hijra and of Yazdigerd were placed about 15 years earlier than would have been correct, though not with identical deviations. We shall see later on[12] that other parts of our manuscript ascribe to Petrus Alphonsi the correct date for the era Hijra. I have no plausible explanation to offer for these discrepancies.

[9] Above p. 143.
[10] Above p. 139.
[11] For another possibility (– 762 Jan. 20 or Jan. 10) cf. above p. 139 and note 15 there.
[12] Cf. p. 211.

## Folios 115$^{v}$, 116$^{r}$

### 1. Tables for the Mean Motion of the Sun

#### A. The Original Text (CC)

Transcription of the original text, headed "Table for finding the mean motion of the center of the sun", later amplified by a partial interlinear version[1] and extensive marginal notes[2]:

**Plate 6**

Tabula ad cognoscendum motum medium centri solaris

| | Anni adiuncti | signa | gradus | puncta | minutie |
|---|---|---|---|---|---|
| | funda- | | | | |
| 1. | mentum | 6 | 15 | 53 | 50 |
| | 30 | 6 | 15 | 47 | 47 |
| | 60 | 6 | 15 | 34 | 58 |
| | 90 | 6 | 15 | 22 | 9 |
| 5. | 120 | 6 | 15 | 9 | 20 |
| | 150 | 6 | 14 | 56 | 31 |
| | 180 | 6 | 14 | 43 | 42 |
| | 210 | 6 | 14 | 30 | 53 |
| | 240 | 6 | 14 | 18 | 4 |
| 10. | 270 | 6 | 14 | 5 | 15 |
| | 300 | 6 | 13 | 52 | 26 |
| | 330 | 6 | 13 | 39 | 37 |
| | 360 | 6 | 13 | 26 | 48 |
| | 390 | 6 | 13 | 13 | 59 |
| 15. | 420 | 6 | 13 | 1 | 10 |
| | 450 | 6 | 12 | 48 | 21 |
| | 480 | 6 | 12 | 35 | 33 |
| | 510 | 6 | 12 | 22 | 44 |
| | 540 | 6 | 12 | 9 | 55 |

| | Anni singulares | signa | gradus | puncta | minutie |
|---|---|---|---|---|---|
| | 1 | 11 | 29 | 45 | 46 |
| | 2 | 11 | 29 | 31 | 33 |
| | 3 | 11 | 29 | 17 | 19 |
| | 4 | 0 | 0 | 2 | 13 |
| | 5 | 11 | 29 | 48 | 0 |
| | 6 | 11 | 29 | 33 | 47 |
| | 7 | 11 | 29 | 19 | 33 |
| | 8 | 0 | 0 | 4 | 28 |
| | 9 | 11 | 29 | 50 | 15 |
| | 10 | 11 | 29 | 36 | 1 |
| | 11 | 11 | 29 | 21 | 47 |
| | 12 | 0 | 0 | 6 | 42 |
| | 13 | 11 | 29 | 52 | 29 |
| | 14 | 11 | 29 | 38 | 15 |
| | 15 | 11 | 29 | 24 | 1 |
| | 16 | 0 | 0 | 8 | 5[6] |
| | 17 | 11 | 29 | 54 | 43 |
| | 18 | 11 | 29 | 40 | 29 |
| | 19 | 11 | 29 | 26 | 15 |
| | 20 | 0 | 0 | 11 | 10 |
| | 21 | 11 | 29 | 56 | 56 |
| | 22 | 11 | 29 | 42 | 42 |
| | 23 | 11 | 29 | 28 | 29 |
| | 24 | 0 | 0 | 13 | 24 |
| 25. | 25 | 11 | 29 | 59 | 12[1] |
| | 26 | 11 | 29 | 44 | 58 |
| | 27 | 11 | 29 | 30 | 44 |
| | 28 | 0 | 0 | 15 | 38 |
| | 29[2] | 0 | 0 | 1 | 25 |
| 30. | 30 | 11 | 29 | 47 | 11[2] |

motus centri corporis solaris in singulis mensibus medius

| | menses Romani | signa | gradus | puncta | minutie | |
|---|---|---|---|---|---|---|
| 1. | Octob. | 1 | 0 | 33 | 19 | 1. |
| | Novb. | 2 | 0 | 7 | 29 | |
| | Decemb. | 3 | 0 | 40 | 48 | |
| | Ianua | 4 | 1 | 14 | 7 | |
| 5. | Febr. | 4 | 28 | 50 | 1 | 5. |
| | Mart. | 5 | 29 | 23 | 20 | |
| | Aprilis | 6 | 28 | 57 | 30 | |
| | Maius | 7 | 29 | 30 | 49 | |
| | Iunius | 8 | 29 | 4 | 59 | |
| 10. | Iulius | 9 | 29 | 38 | 18 | 10. |
| | Augustus | 11 | 0 | 11 | 37 | |
| | Sept. | 11 | 29 | 45 | 47 | |

motus centri solaris corporis quando Sol intrat in primum punctum uniuscuiusque signi

| | nomina signorum[4] | s. | g. | p. | m. | |
|---|---|---|---|---|---|---|
| 1. | Aries | 11 | 27 | 48 | 12 | 1. |
| | Taurus | | | | | |
| | Gemini | | | | | |
| | Cancer | 3 | 4 | 28 | 20[3] | |
| 5. | Leo | | | | | 5. |
| | virgo | | | | | |
| | Libra | 6 | 2 | 9 | 37 | |
| | Scorpius | | | | | |
| | Sagittar. | | | | | |
| 10. | Capricor. | 8 | 29 | 33 | 42 | 10. |
| | Aquarius | | | | | |
| | Pisces | | | | | |

Notes to the transcription of fol. 115$^{v}$ (cf. Pl. I):

[1] Sic, instead of 11.
[2] The last two lines are crossed out.
[3] Sic, instead of 0;28,20 as correctly given in Suter, Tab. 4.
[4] Written on diagonals.

[1] Cf. below p. 149.
[2] Cf. below p. 151.

**Plate 7**

## Tabula ad cognoscendum medium motum centri solaris

| | Numerus dierum | Motus in singulis diebus | | | In singulis horis | | | | In singulis punctis horarum | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | signa | gradus | puncta minutie | Numerus horarum | signa | gradus | puncta minutie | Numerus punctorum horarum | signa | gradus | puncta minutie | |
| 1. | 1 | 0 | 0;59, 8 | | 1 | 0 | 0; 2,28 | | 2 | 0 | 0; 0, 5 | | 1. |
| | 2 | 0 | 1;58,16 | | 2 | 0 | 0; 4,56 | | 4 | 0 | 0; 0,10 | | |
| | 3 | 0 | 2;57,25 aug | | | | | | | | | | |
| | 4 | 0 | 3;56,33 | | | | | | | | | | |
| 5 | 5 | 0 | 4;55,42 | | | same as in Suter, Tab. 5[3] | | | | | | | 5. |
| | 6 | 0 | 5;54,50a[1] | | | | | | | | | | |
| | 7 | 0 | 6;53,58 | | | | | | | | | | |
| | 8 | 0 | 7;53, 6 | | | | | | | | | | |
| | 9 | 0 | 8;52,15a | | | | | | | | | | |
| 10. | 10 | 0 | 9;51,23 | | | | | | | | | | |
| | 11 | 0 | 10;50,31 | | | | | | | | | | |
| | 12 | 0 | 11;49,40a | | | | | | | | | | |
| | 13 | 0 | 12;48,48 | | | | | | | | | | |
| | 14 | 0 | 13;47,56 | | | | | | | | | | |
| 15. | 15 | 0 | 14;47, 5a | | | | | | | | | | |
| | 16 | 0 | 15;46,13 | | | | | | | | | | |
| | 17 | 0 | 16;45,21 | | | | | | | | | | |
| | 18 | 0 | 17;44,30a | | | | | | | | | | |
| | 19 | 0 | 18;43,35 | | | | | | | | | | |
| 20. | 20 | 0 | 19;42,46 | | | | | | | | | | |
| | 21 | 0 | 20;41,55a | | | | | | | | | | |
| | 22 | 0 | 21;41, 3 | | | | | | | | | | |
| | 23 | 0 | 22;40,11 | | | | | | | | | | |
| | 24 | 0 | 23;39,20a | | | | | | | | | | |
| 25. | 25 | 0 | 24;38,28 | | | | | | | | | | |
| | 26 | 0 | 25;37,36 | | | | | | | | | | |
| | 27 | 0 | 26;36,45 | | | | | | | | | | |
| | 28 | 0 | 27;35,53 | | | | | | | | | | |
| | 29 | 0 | 28;34, 1[2] | | | | | | | | | | |
| 30. | 30 | 0 | 29;34,10a | | | | | | | | | | |
| | 31 | 1 | 0;33,19a | | | | | | | | | | |

Notes to the transcription of fol. 116[r]:

[1] Sic; the note *a* (= *augmentum*) should have been written in line 5.
[2] Sic, instead of 28;35,1.
[3] Except for the last column where Suter's text gives incorrectly in line 2: 0;0,8 (instead of 0;0,10) and in line 16: 0;1,25 (instead of 0;1,24).

Translation of the column headings:

fol. 115$^{v}$: Section 1, column 1: "Accumulated years".
Section 2, column 1: "Single years".
Section 3: "Mean motion of the center of the body of the sun in single months".
Section 4: "Motion of the center of the body of the sun, when the sun enters the first point of each single sign".
fol. 116$^{r}$: Section 1: "Motion in single days".
Section 2: "In single hours".
Section 3: "In single minutes of hours".

In the table for the daily motion a letter *a* is added when 0;59,9 is taken as difference instead of 0;59,8. In line 3 we find *aug* instead of *a*, apparently meaning *augmentum*.

The present tables correspond to Suter's Tab. 4 and 5 of solar mean motion, but in Suter's text the years are Arabic years, here, however, julian years. Only section 4 of fol. 115$^{v}$ is an excerpt from Suter's text, giving the mean solar longitudes which correspond to the true longitudes 0°, 90°, 180°, and 270°.

From the mean motion during 30 julian years (fol. 115$^{v}$ sect. 1 or 2) one finds a daily motion of about 0;59,8,11,4° but from the total for 31 days (fol. 116$^{r}$ sect. 1) 0;59,8,21°. The latter value is close to a value used by al-Battānī.[3]

For the value given as *fundamentum* in fol. 115$^{v}$ sect. 1, cf. the discussion of the marginal note (d) p. 153.

[3] Cf. Kennedy, Survey p. 156.

**B. Version D: Fol. 115$^{v}$ and 116$^{r}$, Interlinear Text**

The following changes are made above the corresponding lines of the original version. The numbers are written with Hindu numerals.

**Plate 8**

fol. 115$^{v}$

Section 1

| | | | |
|---|---|---|---|
| 1. | | 3 | 23;41,11 |
| | | 5 | 1;41,32 |
| | | 6 | 9;41,53 |
| | | 7 | 17;42,13[1] |
| 5. | | 8 | 25;42,35 |
| 10. | | | |
| 15. | | 9 | 15 46, 5[2] |
| | | 10 | 23;46,26 |
| | | 0 | 1;46,47 |
| | | 1 | 9;47, 8 |
| | | 2 | 17;47,19[3] |

[4]

Section 3

| | | | |
|---|---|---|---|
| 1. | Martius | | 18 [5] |
| | Aprilis | | 28 |
| | M. | | 46 |
| | I. | | |
| 5. | I. | | |
| | Augustus | | |
| | S | | |
| | O | | |
| | N | | |
| 10. | D | | |
| | Jan. | | |
| | Februarius | | |

fol. 116$^{r}$

Section 1

| | | | |
|---|---|---|---|
| 1. | | 0 | 0;59, 8 |
| | | | 1;58,16 |
| | | | 2;57,24 |
| | | | 32 |
| 5. | | | 40 |
| | | | 49 |
| | | | 57 |
| | | | 5 |
| | | | 13 |
| 10. | | | 21 |
| | | | 30 |
| | | | 38 |
| | | | 46 |
| | | | 54 |
| 15. | | | 2 |
| | | | 11 |
| | | | 18 |
| | | | 27 |
| | | | 35 |
| 20. | | | 43 |
| | | | 52 |
| | | | 0 |
| | | | 8 |
| | | | 16 |
| 25. | | | 24 |
| | | | 32 |
| | | | 41 |
| | | | 49 |
| | | | 28;34,57 |
| 30. | | | 34, 5 |
| | | | 33,13 |

Notes to the transcription (cf. also Pl. I):

[1] Sic, instead of 14.
[2] Correctly continued from line 5 with the difference of $1^{s}$ 8;0,21° per line. Cf. below p. 154.
[3] Sic, instead of 29.
[4] Continued on fol. 116$^{r}$, marginal note (f); cf. below p. 152.
[5] The 18 is once more written on the margin.

The interlinear table of fol. 115$^{v}$ sect. 1 is based on intervals of 30 Arabic years whereas the original text (CC) operates with 30 julian years. A table for single Arabic years is not written interlinearly but in a column by itself on the right margin of fol. 116$^{r}$ (transcribed below in Pl. 9).

In the table for single days, fol. 116$^{r}$ sect. 1, changes are indicated for the last one or two digits by interlinear Hindu numerals. These changes agree exactly with the Toledan Tables and in part with Suter's Tab. 5 sect. 1, except for the lines 4, 5, 10, and 17 where the last digit is one unit higher than in S and for lines 21, 22, and 27 where the values in S are one unit lower; there is no line 31 in S or in T.

## C. Version D: Fol. 116$^{r}$, Marginal Tables

**Plate 9**

below section 2:

| | | |
|---|---|---|
| Almuharam | 0 | 29;34, 5 |
| Zaphar | 1 | 28; 9, 2 |
| Rabe primus | 2 | 27;43, 8 |
| Rabe secundus | 3 | 26;18, 5 |
| Jumedi | 4 | 25;52,12 |
| Jumedi secundus | 5 | 24;27, 9 |
| Raueh | 6 | 24; 1,15 |
| Saben | 7 | 22;36,13 |
| Ramazan | 8 | 22;10,18 |
| Sauhel | 9 | 22;45,16[1] |
| *Zilchecl* | 10 | 20;19,22 |
| Zilhege | 11 | 18;54,19 |

Right margin:[2]

| | Anni Arabum expansi | |
|---|---|---|
| 1. | 11 | 18;54,19 |
| | ·11 | 8;47,4[7] |
| | 10 | 27;42, 7 |
| | 10 | 16;36,2[6] |
| 5. | ·10 | 6;29,54 |
| | 9 | 25;24,14 |
| | ·9 | 15;17,4[1] |
| | 9 | 4;12, 1 |
| | 8 | 23; 6,2[1] |
| 10. | ·8 | 12;59,49 |
| | 8 | 1;54, 8 |
| | 7 | 20;48,2[8] |
| | ·7 | 10;41,56 |
| | 6 | 29;36,16 |
| 15. | 6 | 18;30,35 |
| | ·6 | 8;24, 3 |
| | 5 | 27;14,33[3] |
| | ·5 | 17;11,51 |
| | 5 | 6; 6,10 |
| 20. | 4 | 25; 0,29 |
| | ·4 | 14;53,58 |
| | 4 | 3;48,17 |
| | 3 | 22;42,36 |
| | ·3 | 12;36, 6 |
| 25. | 3 | 1;30,26 |
| | ·2 | 21;23,5[4] |
| | 2 | 10;18,1[3] |
| | 1 | 29;12,33 |
| | ·1 | 19; 6, 1 |
| 30. | 1 | 8; 0,2[1] |

Notes to the transcription:

[1] Sic, instead of 20;45,16.

[2] Trimming of the page has cut away or half destroyed several of the last digits. The restorations are made from the Toledan Tables, Cod. Vindob. 2385 fol. 11$^{r}$.

[3] Sic, instead of 27;18,23.

The table for the solar mean motion during the 12 Arabic months is based on a daily motion of 0;59,8,11,21, . . °/d.

The table for the yearly mean motion indicates leap-years by a dot.[4] The differences show irregular variations from the expected values in the last digits. The total for 30 years, $1^{s}$ 8;0,21°, which is also used for the interlinear version of fol. $115^{v}$ sect. 1,[5] assumes a daily motion of 0;59,8,11,28, . .°.

## 2. Marginal Notes

### A. Transcription[6]

*Fol. $115^{v}$ (cf. photo Pl. I)*

**(a)** Upper margin:

1. *Nota cum incipiat primus annus omnium annorum Arabum die Jovis secundum has tabulas toletanas illa dies*
2. *incipit die m(er)curii precedente hora meridiei. et sic omnes dies sequentes et secundum hoc. operare*(sic) *est. et Nota quod est*
3. *prima radix super meridiem tolete super primam diem Anni Arabum quae dies incipit die m(er)curii. signa .3. gradus vero* 23. *puncta* 41 *secunda* 11. . . .[6a]
4. *Illa vero ordine dies principalis quae principium est terminatur secundum tabulas die Jovis licet dies Jovis sit prima dies annorum arabum.*

**(b)** Left margin, near beginning of table, in part cut away by trimming of the page:

1. [*Secundum*] *Petrum*
2. [*Anno* 1]$\overset{o\,o\,o}{116}$ *ab incar*
3. ]*natione prima*
4. ]*die octobris*
5. [*es*]*t fundamentum*
6. [11]$\overset{o\,o}{46}$
7. [117]$\overset{o}{6}$
8. [1]206

**(c)** Below section 1:

1. *Melius est ut haec tabula consideretur secundum 28. annos*
2. *ne faciat bissextus ordinem numeri*(?) *vacillare.*
3. *Anno domini* 46 *super Aren prima die octobris haec*
4. *Radix. sic intellige* $\overset{o\,o\,o\,o}{1146}$ *.6. signa gradus .15.*

[4] The same thing occurs in section 2 of Pl. 28 from fol. $142^{r}$.
[5] Cf. Pl. 8.
[6] Numbers printed in *italics* are written with Roman numerals, all other numbers with Hindu numerals.
[6a] Three doubtful letters.

5. *47. puncta. et totidem minuta. Si incipiamus a rad*
6. *[ic]e precedentis secundum 28. annos. Anno igitur domini*
7. *1.1.16. ut supra.*

[11]16. *Radix* 6 *16; 0,36*
1144. 6 *16;16,14*
1172. 6 *16;31,52*
1200 6 *16;47,30*
1228 6 *17; 3, 8*
1256 6 17;18,46

**(d)** On lower margin:

1. *Credo quod male correxi Radicem supremam scilicet fundamentum in quo prius fuerunt signa sex. gradus 15 puncta 53. minutie .50.*
2. *credidi quod error esset in fundamento et hoc per secundum. Nunc melius credo quod erraretur(?) in secundo.*
3. *Radix super principium annorum arabum super meridiem toleti* 3 23 41 11. *transactis vero annis* 30 5.1.41 32
4. *et hoc ab initio arietis stellarum transactis vero annis* 480. 0 1 46 47
5. *Anni expansi primus* 11.18 54 19 *Secundus vero* 11 8 47 47 30 1 8 0 21[7]

*Fol. 116r*

**(e)** Upper margin:

*Veris motibus motus octave spere est addendus*

**(f)** Lower margin; actually continuing interlinear version (D) of fol. 115v sect. 1 (above p. 149 Pl. 8):

*Radix super toletum*
*Anni*
570 3 25;47,51
600 5 3;48,12
630 6 11;48,33

**B. Translation and Commentary**

**(a)** "Note: since the first year of all Arabic years begins on a Thursday according to these Toledan Tables, this day begins on the preceding Wednesday at noontime, and thus are all subsequent days to be handled. Note also that the first *radix* (is made for) noon at Toledo and the first day of the Arabic year, which day begins on Wednesday:

[7] Lower half of the last numbers cut away by trimming of page.

3 signs, 23 degrees, 41 minutes, 11 seconds. . . . According to this arrangement the main day, which is the beginning, ends according to the tables on a Thursday, even if Thursday is the first day of the Arabic years".[8]

For the meaning of the first sentence cf. fig. 1 p. 10. The epoch value for the mean sun, $3^s$ 23;41,11, is the value given in the interlinear version on fol. $115^v$ sect. 1 (Pl.8). Thus our interlinear version D is taken from the Toledan Tables. This is confirmed by comparison with extant Toledan Tables.[9]

**(b)** This marginal note identifies the *radix* and the next three lines of the original tables as the years A.D. 1116, 1146, 1176, 1206 of the epoch date October 1, used by Petrus Alphonsi. That the copyist of the main text denoted these years as 0, 30, 60, 90 respectively shows that he did not understand that this meant the years of the Hijra while his table was operating with julian years beginning A.D. 1116 Oct. 1.

**(c)** "It would be better to have this table arranged according to 28 years, such that the intercalation does not make the sequence of(?) . . . oscillate. A.D. (11)46 from Arin on the first day of October is this *radix*; thus understand (for the year) 1146: 6 signs, 15 degrees, 47 minutes and as many seconds.[10] If we begin with the *radix* of the preceding (30 years) – thereafter (proceeding) with 28 years – thus with A.D. 1116 as above:

| | |
|---|---|
| 1116 *radix* | $6^s$ 16; 0,36° |
| 1144 | 6 16;16,14 |
| 1172 | 6 16;31,52 |
| 1200 | 6 16;47,30 |
| 1228 | 6 17; 3, 8 |
| 1256 | 6 17;18,46 ." |

**(d)** "I think I corrected wrongly the uppermost *radix*, namely the *fundamentum* which was previously given as six signs, 15 degrees, 53 minutes, 50 seconds. I thought that the error was in the *fundamentum*, and this because of the second (number). Now I think that the error lies in the second (number). The *radix* for the beginning if Thursday is the first day of the Arabic years".[8]
of the Arabic years (and) for noon at Toledo (is) $3^s$ 23;41,11°, and after 30 years $5^s$ 1;41,32 and this (reckoned) from the beginning of the stars of Aries. After the passing of 480[11] years: $0^s$ 1;46,47°. Single years: first $11^s$ 18;54,19°, second: $11^s$ 8;47,47°, 30th: $1^s$ 8;0,21°."

The first two lines of section 1 of the original text (CC, i.e., the text of Petrus Alphonsi) are (cf. Pl. 6)

| | |
|---|---|
| *fundamentum* | $6^s$ 15;53,50 |
| 30 | 6 15;47,47. |

[8] Similar marginal notes are found on fol. $122^r$ (below p. 176) and on fol. $123^r$ (p. 181).
[9] Zinner [1] p. 750, M $11^r$, giving the first two entries.
[10] This is indeed the value found in the table for the year "30" (Pl. 6 p. 146).
[11] A.H. 480 = A.D. 1087/1088. This date precedes by one 28-year cycle the epoch date of 1116.

As remarked before, these years are julian years but incorrectly associated with the numbers 0, 30, 60 etc. as if representing Arabic years of the Hijra. Furthermore the difference between the two given lines is not the value −0;12,49° which is applied in all subsequent lines. Since the year "30" is actually A.D. 1146 one should have as beginning:

| | | |
|---|---|---|
| 1116 | $6^s$ 16; 0,36 | as given in (c) |
| 1146 | 6 15;47,47. | |

It is furthermore impractical to proceed in steps of 30 julian years because this interval does not represent an integer number of days. Therefore a 28-year step is more advantageous, since it also leaves the weekdays unchanged. Thus in (c) a new table is suggested

| | | |
|---|---|---|
| *radix*: A.D. 1116 (Oct. 1) | $6^s$ 16; 0,36 | |
| 1144 | 6 16;16,14 | |

etc. – based on a mean motion of 0;59,8,20,46, . . .$^{o/d}$.

This is certainly the meaning of the note (c). In note (d) at the end is repeated and amplified what we know from note (a): For the Toledan Tables the following holds:

| | |
|---|---|
| *radix* (A.H. 0) | $3^s$ 23;41,11° |
| 30 | 5 1;41,32 |
| 480 | 0 1;46,47 |

as indeed given in version D of fol. 115$^v$ sect. 1 (Pl. 8 p. 149) and in T (fol. 11$^r$). Similarly the motions for single Arabic years

| | |
|---|---|
| year 1 | $11^s$ 18;54,19 |
| 2 | 11 8;47,47 |
| 30 | 1 8; 0,21 |

agree with the table on fol. 116$^r$, right margin (Pl. 9 p. 150).

(e) "The motion of the 8th sphere is to be added to the true motions". About the precessional correction cf. below fol. 142$^r$, marginal note (b).[12]

(f) This short table is the continuation of the table for accumulated Arabic years of the interlinear version (D) of fol. 115$^v$ sect. 1 (above p. 149 Pl. 8). The same values are found in the Toledan Tables (T fol. 11$^r$).[13] The final year, A.H. 630, corresponds to A.D. 1232/1233.

[12] Cf. p. 211 and p. 213.

[13] Except for the last digit in the last line where our text has incorrectly 38 instead of 33.

## Folios 116$^v$ to 117$^v$

### Tables for the Mean Motion of the Moon

#### A. The Original Text (CC)

Transcription of fol. 116$^v$ and 117$^r$, headed "Table for finding the mean motion of the body of the moon and of its anomaly":

fol. 116v lune **Plate 10**

Tabula ad cognoscendum motum medium corporis[1] lune et sue heze

| | Motus medius centri lune in annis adiunctis | | | Motus heze lune in annis adiunctis | | | Anni Singulares | Motus medius centri lune in singularibus annis | | | Motus heze lune in singularibus annis | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Anni adiuncti | Signa | Gradus puncta minutie | signa | gradus puncta minutie | | | signa | gradus puncta minutie | | signa | gradus puncta minutie | | |
| 1. | Fundamentum | 3 | 11;12,13 | 4 | 20;58,11[3] | | 1 | 4 | 9;22,55 | | 2 | 28;43,18 | | 1. |
| | 30 | 3 | 24;53,48 | 1[4] | 19; 4,19[5] | | 2 | 8 | 18;45,50 | | 5 | 27;26,36 | | |
| | 60 | 4 | 8;35,23 | 8 | 12;10,47 | | 3 | 0 | 28; 8,40 | | 8 | 26; 9,54 | | |
| | 90 | 4 | 22;16,58 | 4 | 5;17, 5 | | 4 | 5 | 20;42,15 | | 0 | 7;57, 6 | | |
| 5. | 120 | 5 | 5;58,33 | 11 | 28;23,23 | | 5 | 10 | 0; 5,10 | | 3 | 6;40,24 | | 5. |
| | 150 | 5 | 19;40, 8 | 7 | 21;29,41 | | 6 | 2 | 9;28, 5 | | 6 | 5;23,42 | | |
| | 180 | 6 | 3;21,43 | 3 | 14;35,59 | | 7 | 6 | 18;51, 0 | | 9 | 4; 7, 0 | | |
| | 210 | 6 | 17; 3,18 | 11 | 7;42,17 | | 8 | 11 | 11;42,30 | | 0 | 15;54,12 | | |
| | 240 | 7 | 0;44,53 | 7 | 0;48,35 | | 9 | 3 | 20;47,25 | | 3 | 14;35,30 | | |
| 10. | 270 | 7 | 14;26,28 | 2 | 23;54,53 | | 10 | 8 | 5;10,20 | | 6 | 13;20,48 | | 10. |
| | 300 | 7 | 28; 8, 3 | 10 | 17; 1,11 | | 11 | 0 | 9;33,15 | | 9 | 12; 4, 6 | | |
| | 330 | 8 | 11;49,38 | 6 | 10; 7,29 | | 12 | 5 | 2; 6,45 | | 0 | 23;51,18 | | |
| | 360 | 8 | 25;31,13 | 2 | 3;13,47 | | 13 | 9 | 11;29,40 | | 3 | 22;30,36 | | |
| | 390 | 9 | 9;12,48 | 9 | 26;20, 5 | | 14 | 1 | 20;52,35 | | 6 | 21;17,54 | | |
| 15. | 420 | 9 | 23; 4,23[2] | 5 | 19;26,23 | | 15 | 6 | 0;15,30 | | 9 | 20; 1,12 | | 15. |
| | 450 | 10 | 6;45,58 | 1 | 12;34,41[6] | | 16 | 10 | 22;49, 0 | | 1 | 1;48,24 | | |
| | 480 | 10 | 20;27,33 | 9 | 5;40,59 | | 17 | 3 | 2;11,55 | | 4 | 0;31,42 | | |
| | 510 | 11 | 4; 9, 8 | 4 | 28;47,17 | | 18 | 7 | 11;34,50 | | 6 | 19;15, 0 | | |
| | | | | | | | 19 | 11 | 20;57,45 | | 9 | 27;58,18 | | |
| | | | | | | | 20 | 4 | 13;31,15 | | 1 | 9;45,30 | | 20. |
| | | | | | | | 21 | 8 | 22;54,10 | | 4 | 8;28,48 | | |
| | | | | | | | 22 | 1 | 2;17, 5 | | 7 | 7;12, 6 | | |
| | | | | | | | 23 | 5 | 11;40, 0 | | 10 | 5;55,24 | | |
| | | | | | | | 24 | 10 | 4;13,30 | | 1 | 17;42,36 | | |
| | | | | | | | 25 | 2 | 13;36,25 | | 4 | 16;25,54 | | 25. |
| | | | | | | | 26 | 6 | 22;59,20 | | 7 | 15; 9,12 | | |
| | | | | | | | 27 | 11 | 2;22,15 | | 10 | 13;52,30 | | |
| | | | | | | | 28 | 3 | 24;55,45 | | 1 | 25;39,42 | | |
| | | | | | | | 29 | 8 | 4;18,11 | | 4 | 24;23, 0 | | |
| | | | | | | | 30 | 0 | 13;41,35 | | 7 | 23; 6,18 | | 30. |

Notes to the transcription of fol. 116v:

[1] The word *corporis* is omitted on fol. 117r.
[2] Sic, instead of 22;54,23. This error influences the subsequent lines.
[3] Sic, instead of 25;58,11, assuming that the subsequent lines are correct.
[4] Sic, instead of 0. Isolated error.
[5] Sic, instead of 19;4,29. Isolated error.
[6] Sic, instead of 12;32,41. This error influences the subsequent lines.

fol. 117r lune **Plate 11**

Tabula ad cognoscendum motum medium lune et sue heze

| | Menses Romani | Motus medius centri lune per singulos menses | | Motus heze lune in singulis mensibus | | | Numerus Dierum | Motus centri lune in Singulis Diebus | | Motus heze lune in Singulis Diebus | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | s. | g. p. m. | s. | g. p. m. | | | s. | g. p. m. | s. | g. p. m. | |
| 1. | Octob. | 1 | 18;28, 5 | 1 | 15; 0,53 | | 1 | 0 | 13;10,35 | 0 | 13; 3,54 | |
| | N | 2 | 23;45,35 | 2 | 16;57,52 | | 2 | 0 | 26;21,10 | 0 | 26; 7,48 | |
| | | 4 | 12;13,40 | 4 | 1;58,45 | | 3 | | | | | |
| | | 6 | 0;41,45 | 5 | 16;59,38 | | 4 | similar to Suter | | | | |
| 5. | | 6 | 9;38, 5 | 5 | 22;47,49 | | 5 | Tab. 7[1] | | | | |
| | | 6 | 25; 5,10 | 7 | 7;49,42 | | 6 | | | | | |
| | (blank) | 9 | 3;23,40 | 8 | 9;46,41 | | | | | | | |
| | | 10 | 21;51,45 | 9 | 23;47,34 | | | | | | | |
| | | 11 | 27; 9,15 | 10 | 26;44,33 | | | | | | | |
| 10. | | 1 | 15;27,20 | 0 | 12;45,26 | | | | | | | |
| | | 3 | 4; 4,25 | 1 | 26;46,19 | | | | | | | |
| | | 4 | 8;22,55 | 2 | 28;43,18 | | | | | | | |

Notes to the transcription of fol. 117r:

[1] The last digits of the mean longitudes deviate from Suter's table from line 5 on, insofar as the present table operates with the fixed difference 13;10,35. A last line for 31 days ($1^s$ 18;28,5°) is only given in CC. The anomalies agree exactly with Suter Tab. 7, sect. 2 col. 3, except for the last digit 54 in the first line (53 in Suter) and line 31 which is not given by al-Khwārizmī, because no Arabic month has 31 days: $1^s$ 15;0,53°.

Fol. 117v is practically identical with Suter Tab. 8. The only diviations in numbers are: section 1, column 2, line 4: 2;40,46 (a scribal error for 2;11,46: *xl* instead of *xi*); section 2, column 2, line 13: 0;14,16 (Suter, incorrectly: 0;14,12); section 2, column 3, line 21: 0;22,52 (Suter, incorrectly: 0;23,52); line 22: 0;23,57 (Suter, incorrectly: 0;24,57).

In section 1, column 3, lines 2, 6, 10, 14, 18, 22 the word *minus* is added, indicating the use of 0;32,39 as difference instead of 0;32,40 in all other cases.

The headings are slightly different from Suter's. On the top of the page is written *L* (for *lune*). The main title is the same as in fol. 116v. The column headings are:

Section 1: *Numerus horarum*; *motus centri lune in singulis horis medius*; *motus heze lune in singulis horis medius.*

Section 2: *Numerus punctorum horarum*; *motus centri lune in singulis punctis horarum medius*; *motus heze lune in singulis punctis horarum medius* (proceeding in steps of 2 minutes from 2 to 60).

All tables of the version CC on fol. 116v to 117v are based on the round value 13;10,35 for the daily motion in longitude[1] (*motus medius centri lune*) while as mean motion in anomaly (*heze lune*) 13;3,53,58,3, . .$^{o/d}$ is used as compared with al-Khwārizmī's 13;3,53,58,49, . .$^{o/d}$.

[1] Al-Khwārizmī's value is 13;10,34,53, ... (cf. p. 92).

**B. Interlinear Version (D)**

In returning to the Arabic calendar the interlinear version of fol. 116$^{v}$, 117$^{r}$ is close to Suter's Tables 6 and 7, whereas fol. 117$^{v}$ is practically identical with Suter Tab. 8 and needed no revision.

fol. 116$^{v}$ **Plate 12**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | | 4 | 0;58,18 | 4[2] | 18; 8,39 | | | 11 | 14;25,49 | 10 | 5; 0,14 |
| | | 5 | 8;48,33[1] | 2 | 11;58,37 | | cab | 11 | 12; 2,13 | 8 | 23; 4,22 |
| | | 6 | 16;38,48 | 0 | 5;48,35 | | | 10 | 26;27,59 | 6 | 28; 4,36 |
| | | 7 | 24;29, 3 | 9 | 29;38,34 | | | 10 | 10;53,46 | 5 | 3; 4,51 |
| 5. | | | | 7 | 23;28,37 | | · | 10 | 8;30, 9 | 3 | 21; 8,59 |
| | | | | 4· | 17;18,30 | $\dot{5}$[3] | | 9 | 22;55,56 | 1 | 26; 9,13 |
| | | 11 | 17;59,14 | | | | · | 9 | 20;32,20 | 0 | 14;13,21 |
| | | 0 | 25;49,39 | | | | | 9 | 4;58, 6 | 10 | 19;13,35 |
| | | | | | | | | 8 | 19;23,54 | 8 | 24;13,49 |
| 10. | | | | | | | · | 8 | 17; 0,17 | 7 | 12;17,54 |
| | | 5 | 27;10,25 | | | | | 8 | 1;26, 5 | 5 | 17;18,11 |
| | | | | 3 | 10;18,20 | | | 7 | 15;51,53 | 3 | 22;18,25 |
| | | 7 | 5; 0,37 | 1 | 4; 8,29[11] | | · | 7 | 13;28,15 | 2 | 10;22,33 |
| | | | | 10 | 27;58,17 | | | 6 | 27;58, 2[7] | 0 | 15;22,47 |
| 15. | | 9 | 24;41, 0 | 8 | 21;48,15 | | | 6 | 12;19,50 | 10 | 20;23, 2 |
| | | 10 | 28;31,11 | 6 | 15;38,14 | | · | 6 | 9;56,13 | 9 | 8;27,10 |
| | | 0 | 6;21,23 | 4 | 9;28,12 | | | 5 | 24;22, 1 | 7 | 13;27,24 |
| | | 1 | 14;11,34 | 2 | 3;18,10 | | · | 5 | 21;58,24 | 6 | 1;31,32 |
| | Tabula medii cursus lune in annis Arabum coll. | | | Tabula partis lune in Annis Arabum coll. | | | | 5 | 6;24,11 | 4 | 6;31,46 |
| | | | | | | | | 4 | 20;49,58 | 2 | 11;37, 0 |
| | | | | | | | · | 4 | 18;26,21 | 0 | 29;36, 8 |
| | Radix | 4 | 0;58,18 | 4[4] | 18; 8,39 | | | 4 | 2;52, 7 | 11 | 4;36,23 |
| | 30 | 5 | 8;48,33 | 1 | 11;58,37 | | | 3 | 17;17,55 | 9 | 9;36,37 |
| Longe infra | 480 | 0 | 6;21,23 | 4 | 9;28,12 | | · | 3 | 4;54,17[8] | 7 | 27;40,45 |
| infra | 570 | 3 | 29;53, 3 | 9 | 20;58, 6[6] | | | 2 | 29;20, 5 | 6 | 2;40,59 |
| | 600 | 5 | 7;43,18 | 8[5] | 14;47,59 | | · | 2 | 26;56,29 | 4 | 20;46, 8 |
| | 630 | 6 | 15;33,33 | 6 | 8;37,57 | | | 2 | 11;22,15 | 2 | 25;45,22 |
| | | | | | | | | 1 | 25;48, 2 | 1 | 0;45,36 |
| | | | | | | | · | 1 | 23;24,26 | 11 | 18;49,44 |
| Corrections: | | | | | | | | 1 | 7;50,15[9] | 9 | 23;49,58 |
| left margin: | 540 | 2 | 22; 1,46 | | | | | | | | |
| | 570 | 3 | 29;51,48 | | | | | | | | |
| | 600 | 5 | 7;42, 0 | | | | | | | | |
| upper margin: | | | 540 | 11 | 27; 8, 8 ... | | | | | | |
| | | | 570 | 9 | 20;58, 6 | | | | | | |
| | | | 600 | 7 | 14;48, 4 ..... | | | | | | |

Notes to the transcription of fol. 116$^{v}$:

[1] Corrected from 38(?). On the margin: 33. The Toledan Tables (Zinner [1] p. 750 M 12$^{v}$) give here 8;48,29.

[2] Sic, instead of 3. Also the subsequent four lines give values one sign too high. The 4 in line 6 is correct, but changed to 5 in a marginal note.

[3] Referring to the 4 in the same line; cf. note 2.
[4] Sic, instead of 3. The next line, however, is correct. Cf. note 2. Both lines 1 and 2 are correctly given in the Toledan Tables (Zinner [1] p. 750 M 12$^{v}$).
[5] Sic, instead of 7. Consequently the next line should have 5 instead of 6. Cf. the correction on the upper margin.
[6] The 6 is corrected, above the line, to 1; cf. the correction on the upper margin.
[7] Sic, instead of 27;54,2.
[8] Sic, instead of 14;54,17.
[9] The numbers 50 and 15 are corrections, written over some other numbers. On the margin, once more: 50,15.
[10] 58 repeated on the margin.
[11] Sic, instead of 4;8,19.

**Plate 13**

fol. 117$^{r}$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1. | elmuharam ..... | 1 | 5;17,27 | 1 | 1;56,58 | 0 | 13;10,35 | 0 | 13; 3,54 | 1. |
| | [Z]aphar ..... | 1 | 27;24,18 | 1 | 20;50, 2 | | | | | |
| | Rabe ..... | 3 | 2;41,45 | 2 | 22;47, 0 | | | | | |
| | [R]abe secund..... | 3 | 24;48,36 | 3 | 11;40, 4 | | | | | |
| 5. | [J]umedi ..... | 5 | 0; 6, 3 | 4 | 13;37, 2 | | | | | 5. |
| | [J]umedi ..... | 5 | 22;12,55 | 5 | 2;30, 6 | | | | | |
| | [Ra]uel ..... | 6 | 27;30,21 | 6 | 4;27, 4 | | | | | |
| | [Sa]hben ..... | 7 | 19;37,13 | 6 | 23;20, 8 | | | | | |
| | [R]amazan ..... | 8 | 24;54,39 | 7 | 25;17, 7 | | | | | |
| 10. | [Sa]uchel..... | 9 | 17; 1,31 | 8 | 14;10,11 | | | | | 10. |
| | [Zi]lchecl..... | 10 | 22;18,57 | 9 | 16; 7, 9 | | | | | |
| | [Zil]hege ..... | 11 | 14;25,49 | 10 | 5; 0,14 | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | |

The agreement with the Toledan Tables (T) is not good, in spite of the fact that the radices for mean motion and anomaly are identical. The mean longitude for A.H. 600 is in T 5$^{s}$ 7;42,10 as compared with 7;43,18 or 7;42,0 in our interlinear version. For the anomaly the agreement is much better. There are no deviations for the first 510 years. But for A.H. 600 T gives 7$^{s}$ 14;48,5 as compared with 14;47,59 in the interlinear version and 14;48,4 in the marginal table.

## Folio 118

### Tables for the Mean Motion of Saturn

**A. The Original Text (CC)**

fol. 118$^{r}$ Saturni **Plate 14**

Tabula ad cognoscendum medium motum centri corporis Saturni

| | Anni adjuncti | Motus saturni medius in annis adiunctis s. | g. p. m. |
|---|---|---|---|
| | Funda mentum | 1 | 4;15,57[2] |
| | 30 | 1 | 8;38,57 |
| | 60 | 1 | 9; 1,57 |
| | 90 | 1 | 13;24,57 |
| 5. | 120 | 1 | 17;47,57 |
| | 150 | 1 | 22;10,58 |
| | 180 | 1 | 26;33,58 |
| | 210 | 2 | 0;56,58 |
| | 240 | 2 | 5;19,58 |
| 10. | 270 | 2 | 9;42,59 |
| | 300 | 2 | 14; 5,59 |
| | 330 | 2 | 18;28,59 |
| | 360 | 2 | 22;51,59 |
| | 390 | 2 | 27;14,59 |
| 15. | 420 | 4[1] | 1;38, 0 |
| | 450 | 4 | 6; 1, 0 |
| | 480 | 4 | 10;24, 0 |
| | 510 | 4 | 14;47, 0 |

| | Anni Singulares | Motus Saturni medius in annis singularibus s. | g. p. m. |
|---|---|---|---|
| | 1 | 0 | 12;12,18 |
| | 2 | 0 | 24;24,36 |
| | 3 | 1 | 6;36,54 |
| | 4 | 1 | 18;51,12 |
| 5. | 5 | 2 | 1; 3,30 |
| | 6 | 2 | 13;15,48 |
| | 7 | 2 | 25;28, 6 |
| | 8 | 3 | 7;42,24 |
| | 9 | 3 | 19;54,42 |
| 10. | 10 | 4 | 2; 7, 0 |
| | 11 | 4 | 14;19,18 |
| | 12 | 4 | 26;33,36 |
| | 13 | 5 | 8;45,54 |
| | 14 | 5 | 20;58,12 |
| 15. | 15 | 6 | 3;10,30 |
| | 16 | 6 | 15;24,48 |
| | 17 | 6 | 28;37, 6[3] |
| | 18 | 7 | 9;49,24 |
| | 19 | 7 | 22; 1,42 |
| | 20 | 8 | 4;16, 0 |
| | 21 | 8 | 16;28,18[13] |
| | 22 | 8 | 27;40,36[4] |
| | 23 | 9 | 10;52,54 |
| | 24 | 9 | 23; 7,12 |
| 25. | 25 | 10 | 5;19,30 |
| | 26 | 10 | 17;31,48 |
| | 27 | 10 | 27;44, 6[5] |
| | 28 | 11 | 9;48,24 |
| | 29 | 11 | 20;10,42 |
| 30. | 30 | 0 | 4;23, 0 |

| | Menses Romani | Motus Saturni medius in qualibet mense s. | g. p. m. |
|---|---|---|---|
| | Octob. | 0 | 1; 2,11 |
| | Novb. | 0 | 2; 2,22 |
| | Decb. | 0 | 3; 4,35 |
| | Ianu. | 0 | 4; 6,46 |
| 5. | Febr. | 0 | 5; 2,53 |
| | Mart. | 0 | 6; 5, 6 |
| | April. | 0 | 6; 5,10[6] |
| | Maius | 0 | 7; 7,30 |
| | Iunius | 0 | 8; 7,40 |
| 10. | Iulius | 0 | 10; 9,52 |
| | Aug. | 0 | 10;12, 3 |
| | Sept. | 0 | 11;12,15[7] |

| | Dies et hore | heza Saturni in diebus et horis s. | g. p. m. |
|---|---|---|---|
| | Dies | 0 | 0;57, 8 |
| | 5 | 0 | 4;45,39 |
| | 10 | 0 | 9;31,18 |
| | 15 | 0 | 14;16,56 |
| | 20 | 0 | 19; 2,35 |
| 5. | 25 | 0 | 23;48,14 |
| | 30 | 0 | 28;33,54 |
| | 40[8] | 0[9] | 27; 7,48 |
| | hora 1 | 0 | 0; 2,23 |
| | 3 | 0 | 0; 7,19[10] |
| | [11] | 0 | 0;14,17 |
| | 12 | 0 | 0;29,34[12] |

Notes to the transcription of fol. 118r:

[1] Sic, instead of 3. Similarly in the subsequent lines.
[2] Assuming lines 3 ff. to be correct one should have in line 1 0;15,57, in line 2 4;38,57.
[3] Sic, instead of 27;37,6. Isolated error.
[4] Sic, instead of 28;40,36. Isolated error.
[5] The last four entries should be: 29;44, 6
11;58,24
24;10,42
6;23, 0.
The incorrect final value 4;23,0 has been used as difference in section 1.
[6] Sic, instead of 7;5,10. All subsequent longitudes should be one degree higher, except for line 10.
[7] The final number should be 12;12,18 as is shown in section 2, line 1.
[8] Sic, instead of 60.
[9] Sic, instead of 1.
[10] Sic, instead of 0;7,9.
[11] Blank; restore 6.
[12] Sic, instead of 0;28,34.
[13] Originally written 16;33,18, then corrected to 16;28,18.

Section 1 of fol. 118r gives the mean motion of Saturn in accumulated groups of 30 julian years, section 2 in single julian years, section 3 in single months. Section 4 tabulates the mean anomaly for days and hours.

Not only do these tables contain many scribal errors, but the table for single years ends in an incorrect number which then is used as difference for the 30-year section. This mean motion for 30 years, 6,4;23,0° would correspond to a daily motion of only 0;1,59,43 . .° instead of 0;2,0,22, . .°. The epoch value and the final longitudes in section 1 are inconsistent with each other.

All numbers in the tables on fol. 118v are identical with Suter's Table 10. The main heading is the same as on fol. 118r. The subheadings in section 1 are: *numerus dierum*; *motus centri corporis Saturni in singulis diebus medius*. Similarly for the *numerus horarum* in section 2 and for the *numerus punctorum horarum* in section 3.[1]

[1] The word *Saturni* is omitted in the last column.

**B. Interlinear Version (D)**

foll. 118[r] **Plate 15**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | |
| 1. | | 3 | 25;51,15 | | 0 | 11;50,36 | | 1; 0,13 | Alm | 1. |
| | | 3 | 21;31,46 | | 0 | 23;43,12 | | 1;58,26 | Zaph | |
| | | 3 | 17;12,17 | | 1 | 15;33,47[2] | | 2;58,39 | Rabe 1 | |
| | | | | | 1 | 17;24,21 | | 3;56,52 | Rabe 2 | |
| 5. | | | | | 1 | 29;16,57 | | 4;57, 5 | Jumedi | 5. |
| | | | | | 2 | 11; 7,32 | | 5;55,18 | Jumedi 2[us] | |
| | | | | | 2 | 23; 0, 7 | | 6;55,31 | Raeel | |
| | | | | | 3 | 4;50,42 | | 7;53,44 | Sahabe | |
| | | | | | 3 | 16;41,17 | | 8;53,57 | Ramazan | |
| 10. | | | | | 3 | 28;33,52 | | 9;52,10 | Sauhel | 10. |
| | | | | | 4 | 10;24,27 | | 10;52,23 | Zilched | |
| | | | | | 4 | 22;16, 2 | | 11;50,36 | Zilhege | |
| | | | | | 5 | 4; 8,36 | | | | |
| | | | | | 5 | 15;59,12 | | | | |
| 15. | | | | | 5 | 17;49,47[3] | | | | |
| | | | | | 6 | 9;42,22 | | | | |
| | | 1 | 26;39,30[1] | | 6 | 21;32,57 | | | | |
| | | 1 | 12;20, 1 | | 7 | 3;25,32 | | | | |
| | 540 | 1 | 8; 0,32 | | 7 | 15;16, 7 | | | | |
| 20. | 570 | 1 | 3;41, 3 | | 7 | 27; 6,42 | | | | |
| | 600 | 0 | 29;21,34 | | 8 | 8;59,17 | | | | |
| | | | | | 8 | 20;49,52 | | | | |
| | | | | | 9 | 2;40,26 | | | | |
| | | | | | 9 | 14;33, 1 | | | | |
| | | | | 25. | 9 | 26;23,36 | | | | |
| | | | | | 10 | 8;16,11 | | | | |
| | | | | | 10 | 20; 6,46 | | | | |
| | | | | | 11 | 1;57,21 | | | | |
| | | | | | 11 | 13;49,56 | | | | |
| | | | | 30. | 11 | 25;40,31 | | | | |

Notes to the transcription of fol. 118[r] (D):

[1] Sic, instead of 16;39,30.
[2] Sic, instead of 5;33,47.
[3] Sic, instead of 27;49,47.

The entries for A.H. 0 and 30 in section 1, lines 1 and 2, agree with the Toledan Tables.[2] The mean motion in longitude, resulting from the entry for 30 Arabic years is 0;2,0,26,35, . .$^{o/d}$ whereas the motion for 30 days (fol. 118[v] sect. 1) leads to 0;2,0,28$^{o/d}$.

Fol. 118[r] has on the upper margin the remark: *super toletum ad annos arabum per algorismum* "For Toledo and Arabic years according to al-Khwārizmī". Cf. also the marginal note on fol. 122[r] (below p. 176).

[2] Zinner [1] p. 751 M 16[r].

Fol. 118$^{v}$, section 1 (daily motion) gives corrections of the last digits of the original table which agrees with Suter's Table 10. The changes made begin in line 6 and continue in all lines from line 8 on. They amount to $+0;0,1^{\circ}$ or $+0;0,2^{\circ}$ up to line 29, while the last line is changed from 1;0,11 to 1;0,14.

Section 2 of fol. 118$^{v}$ (hourly motion) is left unchanged. In section 3 one more sexagesimal place is added on the margin with the heading *Secunda*$^{(?)}$ *quas*$^{(?)}$ *aliter*(?) *tertia*(?) "second(s), otherwise (called) third(s)" (?). In the first three lines is added 10, 20, 30 respectively. Then three lines are left free, to be followed by 10, 20, 30 in the next three lines. This scheme is followed through the whole column. It is, of course, a useless pedantry to add a fourth sexagesimal place to the (rounded) figures for the mean motion of Saturn during minutes.

# Folio 119

## Tables for the Mean Motion of Jupiter

### A. The Original Text (CC)

fol. 119r lovis **Plate 16**

Tabula ad cognoscendum motum medium corporis lovis

| | Anni adiuncti | Motus medius corporis lovis in annis adiunctis s. | g. p. m. | | Anni singulares | Motus medius corporis lovis in singulis Annis s. | g. p. m. | | Menses Romani | Motus medius corporis lovis in singulis mensibus s. | g. p. m. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | fundamentum | | | | | | | | | | | |
| 1. | 30 | 7 | 0;31,46 | | 1 | 1 | 0;19,54 | | Octob. | 0 | 2;34,34 | 1. |
| | 60 | 1 | 10;44,39 | | 2 | 2 | 0;39,48 | | Novemb. | 0 | 5; 4, 9 | |
| | 90 | 7 | 20;56,32[1] | | 3 | 3 | 0;59,42 | | Decemb. | 0 | 7;38,43 | |
| | 120 | 2 | 1;10,25 | | 4 | 4 | 1; 5,35[5] | | Januar. | 0 | 10;12,17[7] | |
| 5. | 150 | 8 | 11;23,18 | | 5 | 5 | 1;25,29 | | Februar. | 0 | 12;31,53 | 5. |
| | 180 | 2 | 21;36,11 | | 6 | 6 | 1;45,23 | | Martius | 0 | 15; 6,27 | |
| | 210 | 9 | 1;49, 4 | | 7 | 7 | 2; 5,17 | | Aprilis | 0 | 18;26, 2 | |
| | 240 | 3 | 12; 1,57 | | 8 | 8 | 2;30,10 | | Maius | 0 | 20;10,36 | |
| | 270 | 9 | 22;14,50 | | 9 | 9 | 2;50, 4 | | Junius | 0 | 22;40, 1 | |
| 10. | 300 | 4 | 2;17,43[2] | | 10 | 10 | 3; 9,58 | | Julius | 0 | 25;15,15 | 10. |
| | 330 | 10 | 12;40,37[3] | | 11 | 11 | 3;29,52 | | August. | 0 | 27 19[8] | |
| | 360 | 4 | 22;53,29 | | 12 | 0 | 3;54,45 | | Septemb. | 1 | 0;19,54 | |
| | 390 | 11 | 3; 6,22 | | 13 | 1 | 4;14,39 | | | | | |
| | 420 | 5 | 13;19,15 | | 14 | 2 | 4;34,33 | | | | | |
| 15. | 450 | 11 | 23;32, 8 | | 15 | 3 | 4;54,27 | | | | | |
| | 480 | 5[12] | 3;45, 1 | | 16 | 4 | 5;19,20 | | | | | |
| | 510 | 11 | 13;57,54 | | 17 | 5 | 5;39,14 | | | | | |
| | | 5 | 24;10,47 | | 18 | 6 | 5;59, 8 | | | | | |
| | | | | | 19 | 7 | 6;19, 2 | | dies et hore | heza lovis in diebus et horis s. | g. p. m. | |
| | | | | 20. | 19[4] | 8 | 6;43,55 | | dies | 0 | 0;54, 9 | 1. |
| | | | | | 20 | 9 | 7; 3,49 | | 5 | 0 | 4;30,45 | |
| | | | | | 21 | 10 | 7;23,43 | | 10 | 0 | 9; 1,30 | |
| | | | | | 22 | 11 | 7;43,37 | | 15 | 0 | 13;32,15 | |
| | | | | | 23 | 0 | 8; 8,30 | | 20 | 0 | 18; 3, 0 | 5. |
| | | | | 25. | 24 | 1 | 8;28,24 | | 25 | 0 | 22;33,45 | |
| | | | | | 25 | 2 | 8;48,18 | | 30 | 0 | 17; 4,30[11] | |
| | | | | | 26[4] | 3 | 9; 8,12 | | 60[9] | 0[10] | 24; 9, 0 | |
| | | | | | 28 | 4 | 9;28, 5[6] | | hora 1 | 0 | 0; 2,16 | |
| | | | | | 29 | 5 | 9;52,49 | | 3 | 0 | 0; 6,47 | 10. |
| | | | | 30. | 30 | 6 | 10;12,53 | | 6 | 0 | 0;13,32 | |
| | | | | | | | | | 12 | 0 | 0;27, 4 | |

Notes to the transcription of fol. 119r:

[1] Sic, instead of 20;57,32. Isolated error.
[2] Sic, instead of 2;27,43. Isolated error.
[3] Sic, instead of 12;40,36. Isolated error.
[4] The numbers from 19 to 26 should be one higher.
[5] Sic, instead of 1;24,35. This error influences all subsequent lines, including the final result which should be 10;31,53. Nevertheless 10;12,53 is used as difference for the 30-year periods (section 1).

[6] Sic, instead of 9;33,5. Isolated error.
[7] Consistent use of the differences as obtainable from the table of daily motion (fol. 119$^{v}$ sect. 1 = Suter Tab. 12) would lead to the following values in line 4 ff.:

| | | |
|---|---|---|
| I 10;13,17 | IV 17;37, 2 | VII 25;15,35 |
| II 12;32,53 | V 20;11,36 | VIII 27;50, 9 |
| III 15; 7,27 | VI 22;41, 1 | IX 0;19,44. |

[8] The second place is left blank.
[9] Written *xlx*, i.e. 40 corrected to 60.
[10] Sic, instead of 1.
[11] Sic, instead of 27;4,30. Isolated error.
[12] Sic, instead of 6. This error influences the next two lines.

The tables of fol. 119$^{r}$ of the original version (CC) contain sufficiently many errors to show how they were computed and to suggest that these tables were the work of a person who did not make independent checks of his computations.

Section 1 gives the mean motion of Jupiter in 30 julian years with a constant difference of 6$^{s}$ 10;12,53° which would correspond to a daily motion of only 0;4,59,2,38°/$^{d}$ instead of 0;4,59,9,8. The error originated in a simple adding error in section 2 (motion in single julian years) and this vitiated the final value for 30 years, then being used as difference in section 1. Also the table of the monthly mean motion (section 3) is full of errors. Again the final number is the difference for the next section. Obviously the computer did not quite succeed in arranging the differences for the single months properly, such that the desired yearly motion resulted.

The little table for the anomaly (section 4) is the same as in Suter Tab. 11.

The tables of fol. 119$^{v}$ are identical with Suter's Tab. 12, except for the addition of a 31st line in the first section (daily motion)[1] and one correction in section 3.[2]

The main title in fol. 119$^{v}$ is *Iovis* above the line, then *Tabula ad cognoscendum medium motum centri corporis stelle Jovialis.* The subtitles are *motus Iovis in singulis diebus* (and *horis*, and *punctis horarum*) *medius.*

[1] The value is 0$^{s}$ 2;34,34°.
[2] Line 11 (22 minutes) gives 0;0,4 as in Suter, but a 5 is written above the 4.

**B. Interlinear Version (D)**

fol. 119r **Plate 17**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |
| | | | | | | | | | | | |
| 1. | | 11 | 1;39,37 | | 0 | 29;24,51 | | 0 | 2;29,34 | Almuharam | 1. |
| | | 4 | 14;54,56[1] | | 1 | 28;54,42 | | | 4;54, 9 | | |
| | | 9 | 28;20,15 | | 2 | 28;19,32 | | | 7;24,44[4] | | |
| | | | | | 3 | 27;44,33 | | | 9;48,18 | | |
| 5. | | | | | 4 | 27;14,13 | | | 12;18,52[5] | | 5. |
| | | | | | 5 | 26;39, 4 | | | 14;42,27 | | |
| | | | | | 6 | 26; 8,55 | | | 17;12, 1 | | |
| | | | | | 7 | 25;33,45 | | | 19;36,26[6] | | |
| | | | | | 8 | 24;58,36 | | | 22; 6, 8[7] | | |
| 10. | | | | | 9 | 24;18,26[2] | | | 24;31,43[8] | | 10. |
| | | | | | 10 | 23;52,16[3] | | | 27; 0,17 | | |
| | | | | | 11 | 23;18, 8 | | 0 | 29;24,51 | Zilhege. | |
| | | | | | 0 | 22;47,58 | | | | | |
| | | | | | 1 | 22;12,49 | | | | | |
| 15. | | | | | 2 | 21;37,40 | | | | | |
| | | | | | 3 | 21; 7,30 | | | | | |
| | | 2 | 5; 4,49 | | 4 | 20;37,21 | | | | | |
| | | 5 | 18;25, 8 | | 5 | 20; 2,11 | | | | | |
| | 540 | 1 | 1;45,28 | | 6 | 19;27, 2 | | | | | |
| 20. | 570 | 6 | 15; 5,48 | | 7 | 18;51,53 | | | | | |
| | 600 | 11 | 28;26, 7 | | 8 | 18;21,43 | | | | | |
| | | | | | 9 | 17;46,34 | | | | | |
| | | | | | 10 | 17;11,25 | | | | | |
| | | | | | 11 | 16;41,15 | | | | | |
| | | | | 25. | 0 | 16; 6, 5 | | | | | |
| | | | | | 1 | 15;35,56 | | | | | |
| | | | | | 2 | 15; 0,47 | | | | | |
| | | | | | 3 | 14;25,38 | | | | | |
| | | | | | 4 | 13;55,28 | | | | | |
| | | | | 30. | 5 | 13;20,18 | | | | | |

Notes to the transcription of fol. 119r (D):

[1] Sic, instead of 14;59,56 as correctly given in the Toledan Tables. Isolated error.
[2] Sic, instead of 24;28,26. Isolated error.
[3] Sic, instead of 23;53,16. Isolated error.
[4] Sic, instead of 7;23,44. Isolated error.
[5] Sic, instead of 12;17,52. Isolated error.
[6] Sic, instead of 19;36,36. Isolated error.
[7] Sic, instead of 22;6,11. This error influences the subsequent lines.
[8] Sic, instead of 24;30,43. Isolated error.

The interlinear table of 30-year motion (section 1) is based on a value of $5^s$ 13;20,19,30° for each 30-year cycle, whereas section 2 ends only with the value $5^s$ 13;20,18° for 30 years. The larger of these two values corresponds to a daily motion of 0;4,59,7,37, . .° which is still slightly smaller than the mean value used in section

1 of fol. 119$^{v}$, namely 0;4,59,10$^{o/d}$. The epoch values for A.H. 0 and 30 agree with the Toledan Tables.[3]

The computation of the table for single years (section 2) is carried out with little concern for the proper position of the intercalary years.

A note on fol. 119$^{r}$ upper margin says *super toletum ad annos Arabum* and below section 2 of fol. 119$^{v}$ is written *idem super toletum*, as is indeed confirmed by extant Toledan Tables (T).

On fol. 119$^{v}$ the following modifications are indicated: in section 1 line 1: 4,59 is written above the same number of the original text but in line 30 version D gives 2;29,34 over the original 2;29,35. Section 2 is kept unchanged. Section 3 has one more column, headed by *Secunde*, on the margin. But only selected numbers are given which can be obtained as follows. In line 1 is written 22 (meaning 0;0,0,22° as motion during 0;2$^{h}$). If we add to this number 22 for each line and write down only the residues mod. 60 whenever 60 is transgressed, we obtain all the numbers given in the first 17 lines, i.e. 6, 12, 18, 2, 8, 14. Continuing this procedure one would obtain 20, 4, 10, 16 and back to the 22 of line 1. Actually, however, only the 4 is found where expected (line 22). Instead of the three remaining numbers one finds one line higher than required the notes

*defectus* [2] (corresponding number: 58)
*defectus* 12 ( » » 48)
*defectus* 6 ( » » 54)

which obviously must mean something like $-2$, $-12$, $-6$ (namely $58 = 60 - 2$ etc.), but I do not understand the reason for this procedure; nor do I know what the number 26 means which is written in line 30 (where the residue is zero).

[3] Zinner [1] p. 751 M 18$^{v}$.

## Folio 120

### Tables for the Mean Motion of Mars

**A. The Original Text (CC)**

fol. 120r martis **Plate 18**

Tabula ad cognoscendum medium motum centri corporis stelle martis

| Anni Adiuncti | Motus martis in annis Adiunctis | | Anni Singulares | Motus martis in annis Singularibus | | menses Romani | Motus martis in Singulis mensibus | |
|---|---|---|---|---|---|---|---|---|
| | s. | g. p. m. | | s. | g. p. m. | | s. | g. p. m. |
| fundamentum | 4 | 2;40,11 | 1 | 6 | 11;15,57 | Octob. | 0 | 16;14,40 |
| 30 | 3 | 14;18,43 | 2 | 0 | 22;31,54 | Novemb. | 1 | 1;57,54 |
| 60 | 2 | 25;56,15[2] | 3 | 7 | 3;47,51 | Decemb. | 1 | 18;12,34 |
| 90 | 2 | 7;35,47 | 4 | 1 | 15;35,14 | Ianua. | 2 | 4;27,14 |
| 120 | 1 | 19;14,19 | 5 | 7 | 26;51,11 | Febru. | 2 | 19; 7,35 |
| 150 | 1 | 0;52,51 | 6 | 2 | 8; 7, 8 | Mart. | 3 | 5;22,15 |
| 180 | 0 | 12;31,23 | 7 | 8 | 19;23, 5 | April. | 3 | 21; 4,29 |
| 210 | 11 | 24; 9,55 | 8 | 3 | 1;10,28 | Maius | 4 | 7;20, 9 |
| 240 | 11 | 5;48,28[3] | 9 | 9 | 12;26,25 | Iunius | 4 | 23; 3,23 |
| 270 | 10 | 17;27, 0 | 10 | 3 | 23;42,22 | Iulius | 5 | 9;18, 3 |
| 290[1] | 9 | 29; 5,32 | 11 | 10 | 4;58,19 | August. | 5 | 25;32,43 |
| 330 | 9 | 10;44, 4 | 12 | 4 | 16;45,42 | Sept. | 6 | 11;15,57 |
| 360 | 8 | 22;22,36 | 13 | 10 | 28; 1,39 | | | |
| 390 | 8 | 3;51, 8[4] | 14 | 5 | 9;17,36 | | | |
| 420 | 7 | 15;29,40 | 15 | 11 | 20;33,33 | | | |
| 450 | 6 | 27; 8,12 | 16 | 6 | 2;20,56 | | | |
| 480 | 6 | 8;46,44 | 17 | 0 | 13;36,53 | | | |
| 510 | 5 | 20;25,16 | 18 | 6 | 24;52,50 | | | |
| | | | 19 | 1 | 6; 8,47 | | | |
| | | | 20 | 6 | 17;56,10 | | | |
| | | | 21 | 1 | 29;12, 7 | | | |
| | | | 22 | 8 | 10;28, 4 | | | |
| | | | 23 | 2 | 21;44, 1 | | | |
| | | | 24 | 9 | 3;31,24 | | | |
| | | | 25 | 3 | 14;47,21 | | | |
| | | | 26 | 9 | 26; 3,18 | | | |
| | | | 27 | 4 | 7;19,15 | | | |
| | | | 28 | 10 | 19; 6,38 | | | |
| | | | 29 | 5 | 0;22,35 | | | |
| | | | 30 | 11 | 11;38,32 | | | |

| Dies et hore | heza martis in diebus et horis | |
|---|---|---|
| | s. | g. p. m. |
| Dies | 0 | 0;27,42 |
| 5 | 0 | 2;18,29 |
| 10 | 0 | 4;36,57 |
| 15 | 0 | 6;55,25 |
| 20 | 0 | 8;13,54[5] |
| 25 | 0 | 11;32,22 |
| 30 | 0 | 13;50,51 |
| 60 | 0 | 27;41,42 |
| hora 1 | 0 | 0; 1,10 |
| 3 | 0 | 0; 3,28 |
| 6 | 0 | 0; 6,55 |
| 12 | 0 | 0;13,51 |

Notes to the transcription of fol. 120r:

[1] Sic, instead of 300.
[2] Sic, instead of 25;57,15. Isolated error.
[3] Sic, instead of 5;48,27. All subsequent lines are influenced by this error.
[4] Sic, instead of 4;1,8. All subsequent lines are influenced by this error.
[5] Sic, instead of 9;13,54. Isolated error. Correct in Suter, Tab. 13.

The daily motion used in the computation of the tables on fol. 120$^{r}$ is 0;31,26,22,16, . .$^{o/d}$ whereas the table on fol. 120$^{v}$ section 1, gives 0;31,26,28$^{o/d}$.

The tables on fol. 120$^{v}$ agree with Suter Tab. 14, except for a few minor variants:

Section 1: line 8: 4;11,22 where Suter has correctly 4;11,32.
The last digits in lines 16, 18, 20, 22, and 24 are one unit lower than in Suter's table.
line 27: 14; 8,55 while Suter has 14;8,54.
line 28: 14;40,21 where Suter has incorrectly 14;40,51.
line 31: 15;14,41 not in Suter.

Section 3: line 8: 0; 0,21 corrected from 0;0,20.
line 9: 0; 0,23, as in Suter N and O; Suter C: 0;0,24.

**B. Interlinear Version (D)**

fol. 120$^{r}$ **Plate 19**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| 1. | | 7 | 1;24,59 | | | 6 | 5;30,32[2] | | | 15;43,16 | Almuharam | 1. |
| | | 0 | 22;28, 0 | | | 0 | 11;31,35[6] | | 1 | 0;55, 6 | | |
| | | 6 | 13;31, 1 | | | 6 | 17; 3, 9 | | 1 | 16;38,22 | | |
| | | | | | | 0 | 22;33,44 | | 2 | 1;50,12 | | |
| 5. | | | | | | 6 | 28;35,45 | | 2 | 17;38,28[4] | | 5. |
| | | | | | | 1 | 4; 6,19 | | 3 | 2;45,17 | | |
| | | | | | | 7 | 10; 8,19 | | 3 | 18;28,33 | | |
| | | | | | | 1 | 15;38,54 | | 4 | 3;40,23 | | |
| | | | | | | 7 | 21; 9,29 | | 4 | 19;23,39 | | |
| 10. | | | | | | 1 | 27;11,30 | | 5 | 4;25,29[5] | | 10. |
| | | | | | | 8 | 2;42, 4 | | 5 | 20;18,45 | | |
| | | | | | | 2 | 8;12,39 | | 6 | 5;30,34 | Zilhege. | |
| | | | | | | 8 | 14;14,39 | | | | | |
| | | | | | | 2 | 19;45,14 | | | | | |
| 15. | | | | | | 8 | 25;15,48 | | | | | |
| | | | | | | 3 | 1;17,49 | | | | | |
| | | 2 | 8;13,15 | | | 9 | 6;18,23[3] | | | | | |
| | | 7 | 11;22,18 | | | 3 | 12;50,24 | | | | | |
| | 540 | 1 | 20;19,17 | | | 9 | 18;20,58 | | | | | |
| 20. | 570 | 7 | 11;22,18 | | | 3 | 23;51,33 | | | | | |
| | 600 | 1 | 2;25,17 | | | 9 | 29;53,33 | | | | | |
| | | | | | | 4 | 5;24, 8 | | | | | |
| | | | | | | 10 | 10;54,42 | | | | | |
| | | | | | | 4 | 16;56,43 | | | | | |
| | | | | 25. | | 10 | 22;27,17 | | | | | |
| | | | | | | 4 | 28;29,18 | | | | | |
| | | | | | | 15[1] | 3;59,52 | | | | | |
| | | | | | | 5 | 9;30,26 | | | | | |
| | | | | | | 11 | 15;32,27 | | | | | |
| | | | | 30. | | 5 | 21; 3, 1 | | | | | |

Notes to the transcription of fol. 120$^{r}$ (D):

[1] Sic, instead of 11.
[2] Sic, instead of 5;30,34. Isolated error.
[3] Sic, instead of 6;48,23. Isolated error.
[4] Sic, instead of 17;33,28. Isolated error.
[5] Sic, instead of 4;35,29. Isolated error. Same error in MS O.
[6] Sic, instead of 11;32,35. Isolated error.

Fol. 120$^{r}$ at the upper margin: *Ad Annos Arabum super Toletum.* The tables on fol. 120$^{v}$ contain the following modifications. Section 1, line 30: 15;43,16 (in agreement with line 1 of section 3 on fol. 120$^{r}$); the original text had 15;43,14 as in Suter, Tab. 14. Section 2, line 24: 31,26 thus confirming the number of the original text (and Suter). Section 3 has a marginal column called *Secunde.* In line 1 we have to combine the entry of the new column with the superscript of the preceding *minutie* and read 2,36 (instead of 3 *minutie* of the original version). Line 2 gives 14 *secunde* (thus 5,14 *minutie*). Then there is only one more entry, namely in line 30, for 60 minutes or 1 hour: 34. Thus the hourly motion is assumed to be 0;1,18,34° corresponding to a daily motion of 0;31,25,36°. From the motion for 30 years (fol. 120$^{r}$ sect. 2) one deduces, however, 0;31,26,32,15, . .$^{o/d}$ which would lead to an hourly motion of 0;1,18,36,20°.

The epoch values given for the accumulated years (section 1 of fol. 119$^{r}$) agree exactly with the Toledan Tables.[1]

[1] Zinner [1] p. 751 M 20$^{v}$.

## Folio 121

### Tables for the Mean Anomaly of Venus

### A. The Original Text (CC)

fol. 121r — Veneris — **Plate 20**

Tabula ad cognoscendum motum medium heze Veneris

| | Anni Adiuncti | heza veneris in annis adiunctis | | | Anni Singulares | heza veneris in annis singularibus | | | menses Romani | heza veneris in mensibus | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | s. | g. p. m. | | | s. | g. p. m. | | | s. | g. p. m. | |
| | fundamentum | | | | | | | | | | | |
| 1. | | 3 | 10;48,42 | | 1 | 7 | 15; 2,25 | | October | 0 | 19; 6,47 | 1. |
| | 30 | 0 | 16;20,12 | | 2 | 3 | 0; 3,50[6] | | Novemb. | 1 | 7;36,34 | |
| | 60 | 9 | 21;51,42 | | 3 | 10 | 15; 7,15 | | Decemb. | 1 | 26;43,21 | |
| | 90 | 6 | 27;23,12 | | 4 | 6 | 0;46,40 | | Januar. | 2 | 15;50, 8 | |
| 5. | 120 | 4 | 2;54,43 | | 5 | 1 | 15;49, 5 | | Februar. | 3 | 3; 5,56 | 5. |
| | 150 | 1 | 8;25,12[2] | | 6 | 9 | 0;51,30 | | Martius | 3 | 22;12,43 | |
| | 180 | 10 | 13;56,43 | | 7 | 4 | 15;53,55 | | Aprilis | 4 | 10;42,30 | |
| | 210 | 7 | 19;28,14 | | 8 | 0 | 1;33,20 | | Maius | 4 | 29;49,17 | |
| | 240 | 4 | 22; 3,44[3] | | 9 | 7 | 16;35,45 | | Junius | 5 | 18;19, 4 | |
| 10. | 270 | 2 | 0;10,14[4] | | 10 | 3 | 1;38,10 | | Julius | 6 | 7;25,51 | 10. |
| | 300 | 11 | 6; 2,45 | | 11 | 10 | 16;40,35 | | Augus. | 6 | 26;32,38 | |
| | 330 | 8 | 11;34,15 | | 12 | 6 | 2;20, 0 | | Septemb. | 7 | 15; 2,25 | |
| | 360 | 5 | 17; 2,45[5] | | 13 | 1 | 17;22,25 | | | | | |
| | 390 | 2 | 22;20,16 | | 14 | 9 | 2;24,50 | | | | | |
| 15. | 320[1] | 11 | 18; 6,46 | | 15 | 4 | 17;27,15 | | Dies et hore | heza veneris in diebus et horis | | |
| | 450 | 9 | 3;38,16 | | 16 | 0 | 3; 6,40 | | | | | |
| | 480 | 6 | 10;10,47 | | 17 | 7 | 18; 7, 5[7] | | | s. | g. p. m. | |
| | 510 | 3 | 13;41,17 | | 18 | 3 | 3;10,30[8] | | | | | |
| | | | | | 19 | 10 | 18;11,55[9] | | Dies 1 | 0 | 0;36, 0 | 1. |
| | | | | 20. | 20 | 6 | 3;53,20 | | 5 | 0 | 3; 5, 0 | |
| | | | | | 21 | 1 | 18;55,45 | | 10 | 0 | 3;10, 0[12] | |
| | | | | | 22 | 9 | 3; 6,10[10] | | 15 | 0 | 9;15, 0 | |
| | | | | | 23 | 4 | 19; 0,35 | | 20 | 0 | 12;20, 0 | 5. |
| | | | | | 24 | 0 | 4;40, 0 | | 25 | 0 | 15;25, 0 | |
| | | | | 25. | 25 | 7 | 19;50,25[11] | | 30 | 0 | 17;20, 0[13] | |
| | | | | | 26 | 3 | 4;52,50 | | 60 | 1 | 7; 0, 0 | |
| | | | | | 27 | 10 | 19;55,15 | | hora 1 | 0 | 0; 1,32 | |
| | | | | | 28 | 6 | 5;20,40 | | 3 | 0 | 0; 4,38[14] | 10. |
| | | | | | 29 | 1 | 20;22, 5 | | 6 | 0 | 0; 9,15 | |
| | | | | 30. | 30 | 9 | 5;30,30 | | 12 | 0 | 0;18,30 | |

Notes to the transcription of fol. 121r:

[1] Sic, instead of 420.
[2] Sic, instead of 8;25,13.
[3] Sic, instead of 24;59,44. Isolated error.
[4] Sic, instead of 0;13,14. Isolated error.

[5] From here on all numbers contain some errors. One should have

| | |
|---|---|
| 17; 5,45 | 3;40,16 |
| 22;37,16 | 9;11,47 |
| 28; 8,46 | 14;43,17. |

[6] Sic, instead of 0;4,50. Isolated error.
[7] Sic, instead of 18;9,5. Isolated error.
[8] Sic, instead of 3;11,30. Isolated error.
[9] Sic, instead of 18;13,55. Isolated error.
[10] Sic, instead of 3;58,10. Isolated error.
[11] From here on all numbers contain an error in the middle digit. One should have

| | |
|---|---|
| 19;42,25 | 5;26,40 |
| 4;44,50 | 20;29, 5 |
| 19;47,15 | 5;31,30. |

[12] Sic, instead of 6;10,0.
[13] Sic, instead of 18;30,0.
[14] Should be 0;4,37.

Section 2 of fol. 121$^{r}$ should result, after correcting the errors of addition, in a motion of 9$^{s}$ 5;31,30° in 30 julian years. This is indeed the difference in section 1, with the slight modification that one more sexagesimal place shows its influence, such that we can assume as the basic parameter 9$^{s}$ 5;31,30,20°. The corresponding daily motion would then be 0;36,59,28,27, . .°. The table of daily motion given in section 1 of fol. 121$^{v}$, however, is based on 0;36,59,34$^{o/d}$.

The three sections of fol. 121$^{v}$ are identical with Suter Tab. 16, except for a 31st line in the daily motion (0$^{s}$ 19;6,46).[1] The section titles are: *heza veneris in diebus/ horis/punctis horarum.*

[1] A scribal error in line 12 is 7;33,55 instead of 7;23,55 as given in Suter's text.

**B. Interlinear Version (D)**

fol. 121[r] **Plate 21**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | | | 1 | 15;28,45[1] | | | 7 | 8;14,59 | | | 18;29,45 | Almuh. 1. |
| | | | 3 | 29;45,25 | | | 2 | 17; 6,59 | | 1 | 6;22,31 | |
| | | | 6 | 14; 2,14 | | | 9 | 25;21,58 | | 1 | 24;51,16 | |
| | | | | | | | 5 | 3;36,58 | | 2 | 12;45, 0 | |
| 5. | | | | | | | 0 | 12;28,58 | | 3 | 1;14,46 | 5. |
| | | | | | | | 7 | 20;43,57 | | 3 | 19; 7,31 | |
| | | | | | | | 2 | 29;35,57 | | 4 | 7;37,16 | |
| | | | | | | | 10 | 7;50,56 | | 4 | 25;30, 0 | |
| | | | | | | | 5 | 16; 5,56 | | 5 | 13;59,45 | |
| 10. | | | | | | | 0 | 24;57,55 | | 6 | 1;52,30 | 10. |
| | | | | | | | 8 | 3;12,55 | | 6 | 20;22,15 | |
| | | | | | | | 3 | 11;27,55 | | 7 | 8;15, 0 | Zilhege |
| | | | | | | | 10 | 20;29,54[2] | | | | |
| | | | | | | | 5 | 19;34,54[3] | | | | |
| 15. | | | | | | | 1 | 6;49,53 | | | | |
| | | | | | | | 8 | 15;41,53 | | | | |
| | | | 5 | 3;57,33 | | | 3 | 23;56,52 | | | | |
| | | | 7 | 18;14,21 | | | 11 | 2;48,52 | | | | |
| | 540 | | 10 | 2;31,10 | | | 6 | 11; 3,51 | | | | |
| 20. | 570 | | 0 | 16;47,58 | | | 1 | 19;18,51 | | | | |
| | 600 | | 3 | 1; 4,47 | | | 8 | 28;10,51 | | | | |
| | | | | | | | 4 | 6;25,50 | | | | |
| | | | | | | | 11 | 14;40,50 | | | | |
| | | | | | | | 6 | 23;32,50 | | | | |
| | | | | | 25. | | 2 | 1;47,49 | | | | |
| | | | | | | | 9 | 10;39,49 | | | | |
| | | | | | | | 4 | 18;54,48 | | | | |
| | | | | | | | 11 | 27; 9,48 | | | | |
| | | | | | | | 7 | 6;50,48[4] | | | | |
| | | | | | 30. | | 2 | 14;16,48 | | | | |

Notes to the transcription of fol. 121[r] (D):

[1] Sic, instead of 15;28,37 as given by the Toledan Tables (Zinner [1] p. 752 M 23[r]) and required by the 30-year motion given in section 2. The next line agrees with the Toledan Tables.

[2] Sic, instead of 20;19,54. Isolated error.

[3] Sic, instead of 28;34,54. Isolated error.

[4] Sic, instead of 6;1,48. Isolated error.

From the difference for 30 Arabic years in section 1 of fol. 121[r], namely 2[s] 14;16, 48,30°, one obtains a daily motion of 0;36,59,29,27, . .°. On the upper margin is written: *Ad annos arabum super Toletum.* The epoch values in section 1 agree with the Toledan Tables.

On the left margin of fol. 121[v] is written by a modern hand: 1277. In section 1, line 30, the interlinear number 18;29,46 is given, which would correspond to a daily motion of 0;36,59,32°. In section 2 line 1 the last place is changed to 33 (thus 0;1,33°

as hourly motion, 0;37,12° as daily motion!), line 24 in the second place 37 is repeated over the original text (thus 0;37,0° as daily motion). Section 3 has on the margin one more place, headed *Secunde*, with three entries at the first three lines, namely 4, 8, and 12, and two entries at the last two lines, namely 22 and 26. Thus the hourly motion would be 0;1,32,26°, the daily motion 0;36,58,24°.

## Folio 122

### 1. Tables for the Mean Anomaly of Mercury

#### A. The Original Text (CC)

fol. 122r **Plate 22**

Tabula ad cognoscendum motum medium heze mercurii.

| | Anni Adiuncti | heza mercurii in annis adiunctis s. | g. p. m. | | Anni Singulares | heza mercurii in annis singularibus s. | g. p. m. | | menses Romani | heza mercurii in mensibus romanis s. | g. p. m. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | fundamentum | | | | | | | | | | | |
| 1. | | 11 | 19;22,51 | | 1 | 1 | 23;56,48 | | Octob. | 3 | 6;18,28 | 1. |
| | 30 | 6 | 9;30,39 | | 2 | 3 | 17;53,36 | | Novemb. | 6 | 9;30,32 | |
| | 60 | 0 | 29;38,27 | | 3 | 5 | 11;50,24 | | Decemb. | 9 | 15;49, 0 | |
| | 90 | 7 | 19;46,15 | | 4 | 7 | 8;53,26[1] | | Ianuar | 0 | 22; 7,28 | |
| 5. | 120 | 1 | 29;54, 3 | | 5 | 9 | 2;50,24 | | Februa. | 3 | 19; 6,44 | 5. |
| | 150 | 8 | 20; 1,51 | | 6 | 10 | 26;47,12 | | Martius | 6 | 25;25,12 | |
| | 180 | 3 | 10; 9,39 | | 7 | 0 | 20;44, 0 | | April. | 9 | 28;37,16 | |
| | 210 | 10 | 0;17,27 | | 8 | 2 | 17;47,12 | | Maius | 1 | 4;55,44 | |
| | 240 | 4 | 20;25,15 | | 9 | 4 | 11;44, 0 | | Iunius | 4 | 8; 7,48 | |
| 10. | 270 | 11 | 10;36, 3 | | 10 | 6 | 5;40,48 | | Iulius | 7 | 14;26,16 | 10. |
| | 300 | 6 | 0;40,51 | | 11 | 7 | 29;37,36 | | Augustus | 10 | 20;44,44 | |
| | 330 | 0 | 20;48,39 | | 12 | 9 | 26;40,48 | | Septemb. | 1 | 23;56,48 | |
| | 360 | 7 | 10;56,27 | | 13 | 11 | 20;37,36 | | | | | |
| | 390 | 2 | 1; 4,15 | | 14 | 1 | 14;33,24[2] | | Dies et hore | heza mercurii in diebus et horis | | |
| 15. | 420 | 8 | 21;12, 3 | | 15 | 3 | 8;31,12 | | | | | |
| | 450 | 2 | 19;11, 3[8] | | 16 | 5 | 5;34,24 | | | s. | g. p. m. | |
| | 480 | 10 | 1;27,39 | | 17 | 6 | 29;31,12 | | | | | |
| | 510 | 4 | 21;35,27 | | 18 | 8 | 23;28, 0 | | | | | |
| | | | | | 19 | 10 | 17;24,48 | | Dies 1 | 0 | 3; 6,24 | 1. |
| | | | | 20. | 20 | 0 | 14;28, 0 | | 5 | 0 | 15;32, 0 | |
| | | | | | 21 | 2 | 8;23,48[3] | | 10 | 1 | 4, 0[5] | |
| | | | | | 22 | 4 | 2;21,36 | | 15 | 1 | 16;36, 0 | |
| | | | | | 23 | 5 | 26;18,24 | | 20 | 2 | 2; 8, 0 | 5. |
| | | | | | 24 | 7 | 23;21,36 | | 25 | 2 | 17;40, 0 | |
| | | | | 25. | 25 | 9 | 17;18,24 | | 30 | 3 | 3; 2, 0[6] | |
| | | | | | 26 | 11 | 11;15,12 | | 60 | 6 | 6; 4, 0 | |
| | | | | | 27 | 1 | 5;12, 0 | | hora 1 | 0 | 0; 7,46 | |
| | | | | | 28 | 3 | 2;14,12[4] | | 3 | 0 | 0;24,18[7] | 10. |
| | | | | | 29 | 4 | 26;11, 0 | | 6 | 0 | 0;48,36 | |
| | | | | 30. | 30 | 6 | 20; 7,48 | | 12 | 0 | 1;37,12 | |

Notes to the transcription of fol. 122r:

[1] Sic, instead of 8;53,36. Isolated error.

[2] Sic, instead of 14;34,24. Isolated error.

[3] Sic, instead of 8;24,48. Isolated error.

[4] Sic, instead of 2;15,12. Consequently the two last lines should be 26;12,0 and 20;8,48 respectively. In section 1, however, the incorrect value from the last line, 20;7,48, is used as difference.

[5] Sic, instead of 1s 1;4,0. Isolated error.

[6] Sic, instead of 3;12,0. Consequently the next line should be 6;24,0.

[7] Sic, instead of 0;23,18. Consequently the next two lines should be 0;46,36 and 1;33,12 respectively, as given correctly in Suter's Tab. 18.

[8] Sic, instead of 19;11,51. Isolated error.

The daily velocity which results from the value 6s 20;7,48° used in section 1 of fol. 122r for the 30-year interval is 3;6,23,36,56, . .° or 3;6,23,37, . .° if one corrects the error in section 2 (cf. note 4). Section 1 of fol. 122v, however, would give 3;6,24,8°/d.

The tables on fol. 122v are identical with Suter's Tab. 18 with the following exceptions:

Section 1 (daily motion in anomaly), line 4 has 12;25,37 as Suter's MS C, whereas O gives 12;25,36. Line 31: 3s 6;18,28° is not in Suter. Section 2 (hourly motion) line 2: 0;16,32 against Suter's correct value 0;15,32; line 21: 2;47,6 instead of 2;43,6.

**B. Interlinear Version (D)**

fol. 122r **Plate 23**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | | 2 | 13;46,18 | | | 0 | 19;16,32[1] | | | 3 | 3;12,11 | Almuh. 1. |
| | | 11 | 11; 7,19 | | | 1 | 12;39,28 | | | 6 | 3;17,45 | |
| | | 8 | 8;28,20 | | | 2 | 2;26, 0 | | | 9 | 6;29,50 | |
| | | 5 | 5;49,21 | | | 2 | 22;12,32 | | | 0 | 6;35,31 | |
| 5. | | | | | | 3 | 15; 5,28 | | | 3 | 9;47,36 | 5. |
| | | | | | | 4 | 4;52, 0 | | | 6 | 9;54,16[8] | |
| | | | | | | 4 | 27;44,56 | | | 9 | 13; 5,21 | |
| | | | | | | 5 | 17;31,28 | | | 0 | 13;11, 1 | |
| | | | | | | 6 | 7;17, 0[2] | | | 3 | 16;21, 6 | |
| 10. | | | | | | 7 | 0;10,56 | | | 6 | 16;24,47[9] | 10. |
| | | | | | | 7 | 19;57,28 | | | 8 | 18;40,52 | |
| | | | | | | 8 | 9;44, 1 | | | 0 | 18;16,32[10] | Zilhege |
| | | | | | | 9 | 2;36,58 | | | | | |
| | | | | | | 9 | 22;33,30[3] | | | | | |
| 15. | | | | | | 10 | 12;10, 2 | | | | | |
| | | | | | | 11 | 5; 2,28[4] | | | | | |
| | | 1 | 1;22,34 | | | 11 | 23;49,31[5] | | | | | |
| | | 9 | 28;43,35 | | | 0 | 17;42,28 | | | | | |
| | 540 | 6 | 26; 4,36 | | | 1 | 7;29, 0 | | | | | |
| 20. | 570 | 3 | 23;25,37 | | | 1 | 27;15,32 | | | | | |
| | 600 | 0 | 20;46,38 | | | 2 | 20; 8,28 | | | | | |
| | | | | | | 3 | 9;55, 1 | | | | | |
| | | | | | | 3 | 29;41,34 | | | | | |
| | | | | | | 4 | 22;34,30 | | | | | |
| | | | | 25. | | 5 | 12;31, 2[6] | | | | | |
| | | | | | | 6 | 5;13,58 | | | | | |
| | | | | | | 6 | 25; 0,30 | | | | | |
| | | | | | | 7 | 14;47, 3 | | | | | |
| | | | | | | 8 | 6;39,59[7] | | | | | |
| | | | | 30. | | 8 | 27;20, 1[11] | | | | | |

Notes to the transcription of fol. 122r (D):

[1] Sic, instead of 19;46,32. Isolated error. Cf. note 10.
[2] Sic, instead of 7;18,0. Isolated error.
[3] Sic, instead of 22;23,30. Isolated error.
[4] Sic, instead of 5;2,58. Isolated error.
[5] Sic, instead of 24;49,31. Isolated error.
[6] Sic, instead of 12;21,2. Isolated error.
[7] Sic, instead of 7;39,59. Isolated error.
[8] Sic, instead of 9;53,16. Isolated error.
[9] Sic, instead of 16;28,47. Isolated error.
[10] Sic, instead of 19;46,32. The latter number is used (incorrectly) as first number in section 2. Cf. above note 1.
[11] Cf. the commentary.

The tables for single years (fol. 122$^{r}$ sect. 2) are based on a difference of 19;46,32 for an ordinary year (354 days) and 19;46,32 + 3;6,24 = 22;52,56 for a leap-year (355 days). With the occasional addition of 0;0,1 this scheme is carried through up to year 29, thus leading to $8^{s}$ 7;39,59. Consequently the total for 30 years should be $8^{s}$ 27;26,31. The text, however, gives only $8^{s}$ 27;20,1, and this is also the difference used in section 1 for the accumulated years. Apparently this difference was found independently (from a daily motion of 3;6,24,7, . .°). The individual differences for single years are obviously too great; they should be 19;46,17,18° and 22;52,41,18° in order to produce the desired total for 30 years. The error probably comes from the use of 3;6,25$^{o/d}$ as daily motion in the table of months (section 3).

Fol. 122$^{v}$ contains only one interlinear entry, namely in section 1 (daily motion) in line 30: 3;12,5°. The corresponding daily motion would be 3;6,24,10°.

**Marginal Notes**

Fol. 122$^{r}$ has on the upper margin the following note:

1. *Ad Annos Arabum super Toletum per Algorismum et incipit dies Arabum secundum tabulas a meridie diei precedentis*
2. *et terminatur m(eridie) se(quentis) ita quod prima dies Arabum incipit die Jovis Idibus Julii Ista dies secundum has tabulas incipit die mercu*
3. *rii precedente et terminatur die Jovis ad meridiem.*

"For Arabic years and at Toledo according to al-Khwārizmī; and the Arabic day begins, according to (these) tables, from noon of the preceding day and ends at noon of the following. Thus, the first day of the Arabs begins on Thursday of the ides of July[1] (whereas) this day according to these tables begins on the preceding Wednesday and ends on Thursday noon".

Cf. for similar notes fol. 115$^{v}$, 118$^{r}$, and 123$^{r}$.[2]

[1] July 15; cf. fig. 1 p. 10.
[2] p. 151a (a), p. 161, and p. 181 respectively.

## Folio 123

### 1. Tables for the Ascending Lunar Node

### A. The Original Text (CC)

fol. 123r **Plate 24**

Tabula ad cognoscendum motum medium capitis Draconis

| | Anni Adiuncti | motus centri capitis draconis in annis adiunctis medius | | | Anni Singulares | motus centri capitis draconis in annis singularibus medius | | | Menses Romanorum | motu centri capitis draconis in mensibus medius | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | s. | g. p. m. | | | s. | g. p. m. | | | s. | g. p. m. | |
| | fundamentum | 2 | 21;18,10 | | 1 | 0 | 19;20,27 | | Octob. | 0 | 1;38,33 | |
| 1. | 30 | 10 | 1;53,57 | | 2 | 1 | 8;40,54 | | Novemb. | 0 | 3;13,56 | 1. |
| | 60 | 5 | 12;29,44[3] | | 3 | 1 | 23; 1,21[2] | | Decemb. | 0 | 4;52,29 | |
| | 90 | 0 | 23; 8,31[4] | | 4 | 2 | 17;24,59 | | Ianua. | 0 | 6;31, 2 | |
| 5. | 120 | 8 | 3;41,18 | | 5 | 3 | 6;45,26 | | Febr. | 0 | 8; 0, 6 | 5. |
| | 150 | 3 | 14;17, 5 | | 6 | 3 | 26; 5,53 | | Mart. | 0 | 9;38,39 | |
| | 180 | 10 | 24;52,52 | | 7 | 4 | 15;26,20 | | April. | 0 | 11;14, 2 | |
| | 210 | 6 | 5;28,39 | | 8 | 5 | 4;49,58 | | Maius | 0 | 12;52,35 | |
| | 260[1] | 1 | 16; 4,26 | | 9 | 5 | 24;10,25 | | Iunius | 0 | 14;27,58 | |
| 10. | 270 | 8 | 26;40,13 | | 10 | 6 | 13;30,52 | | Iulius | 0 | 16; 6,31 | 10. |
| | 300 | 4 | 7;16, 0 | | 11 | 7 | 2;51,19 | | August. | 0 | 17;45, 4 | |
| | 330 | 11 | 17;51,47 | | 12 | 7 | 22;14,57 | | Septemb. | 0 | 19;20,27 | |
| | 360 | 6 | 28;27,34 | | 13 | 8 | 11;35,24 | | | | | |
| | 390 | 2 | 9; 3,21 | | 14 | 9 | 0;55,51 | | | | | |
| 15. | 420 | 9 | 19;39, 8 | | 15 | 9 | 20;16,18 | | | | | |
| | 450 | 5 | 0;14,55 | | 16 | 10 | 9;39,56 | | | | | |
| | 480 | 0 | 10;50,42 | | 17 | 10 | 29; 0,23 | | | | | |
| | 510 | 7 | 21;26,29 | | 18 | 11 | 18;20,50 | | | | | |
| | | | | | 19 | 0 | 7;41,17 | | | | | |
| | | | | 20. | 20 | 0 | 27; 4,55 | | | | | |
| | | | | | 21 | 1 | 16;25,22 | | | | | |
| | | | | | 22 | 2 | 5;45,49 | | | | | |
| | | | | | 23 | 2 | 25; 6,16 | | | | | |
| | | | | | 24 | 3 | 14;29,54 | | | | | |
| | | | | 25. | 25 | 4 | 3;50,21 | | | | | |
| | | | | | 26 | 4 | 23;10,48 | | | | | |
| | | | | | 27 | 5 | 12;31,15 | | | | | |
| | | | | | 28 | 6 | 1;54,53 | | | | | |
| | | | | | 29 | 6 | 21;15,20 | | | | | |
| | | | | 30. | 30 | 7 | 10;35,47 | | | | | |

Notes to the transcription of fol. 123r:

[1] Sic, (*cclx*) instead of 240 (*ccxl*).

[2] Sic, instead of 28;1,21. Isolated error.

[3] Between the year number 60 and the 5 of the second column is written a small *32* in Roman numerals. There are dots under the 29 of 12;29,44 and beside the 44 is written in small Roman numerals *55*. Thus 12;29,44

was intended to be replaced by 12;32,55. This corresponds to one day's additional motion (0;3,11), as is indicated by the marginal note to this line: *adde diem* (cf. Pl. 26).

[4] Beside the dotted 31 is written with small Roman numerals *42*. This is consistent with the change in the preceding line. No change has been noted, however, for the preceding digit (from 8 to 11).

The daily (retrograde) motion of the lunar nodes is assumed to be 0;3,10,45,1 . .°, using the 30-year intervals. The table of daily motion on fol. 123$^{v}$ (section 1), however, is based on 0;3,10,48$^{o/d}$.

Section 2 of fol. 123$^{r}$ contains a sequence of letters *b* and of dots in front of the year numbers.[1] The dots represent the leap-years in the Islamic 30-year cycle and thus belong undoubtedly to the scribe of the interlinear version (D). The letters *b* indicate the julian leap-years (*bissextus*) and thus are only of interest for the computation of the original version (CC). Nevertheless these letters were certainly written by a later hand, perhaps by the person who made the changes in lines 3 and 4 of section 1.[2]

Fol. 123$^{v}$ is practically identical with Suter's Tab. 20. In section 1 line 3 occurs the isolated error 0;11,32 instead of 0;9,32 (correct in Suter). In section 3 (motion during pairs of seconds) Suter's text gives one more sexagesimal place than usual.[3] Our text,[4] in its original version, gives the customary three digits, the last being a not quite consistent rounding from Suter's numbers. The right margin, however, contains a column of one additional digit (called *Secunde*), written by a later hand using Hindu numerals, as in all the preceding tables of mean motions. These additions operate with a constant difference 0;0,0,16° except for the final result in line 30 (i.e. 60 minutes = 1 hour) which would be 0;0,8 but is given as 58, i.e. 0;0,7,58 in agreement with Suter.

[1] Cf. Pl. 26.
[2] Cf. the notes 3 and 4 to the transcription Pl. 24.
[3] Cf. Suter p. 131 note 7.
[4] Cf. Pl. 25.

fol. 123v, Sect. 3 **Plate 25**

| | Numerus Punctorum horarum | Motus centri capitis Draconis in singulis punctis horarum | | Secunde | Suter | | |
|---|---|---|---|---|---|---|---|
| | | s. | g. p. m. | | C & O | N[3] | |
| 1. | 2 | 0 | 0; 0, 0 | 16[1] | 0; 0, 0,16 | 0; 0, 0 | 1. |
| | 4 | 0 | 0; 0, 0 | 32 | 0,32 | 0 | |
| | 6 | 0 | 0; 0, 0 | 48 | 0,48 | 0 | |
| | 8 | 0 | 0; 0, 1 | 4 | 1, 4 | 0 | |
| 5. | 10 | 0 | 0; 0, 1 | | 1,19 | 1 | 5. |
| | 12 | 0 | 0; 0, 2 | | 1,35 | 1 | |
| | 14 | 0 | 0; 0, 2 | | 1,51 | 1 | |
| | 16 | 0 | 0; 0, 2 | 8 | 2, 7 | 1 | |
| | 18 | 0 | 0; 0, 2 | | 2,23 | 2 | |
| 10. | 20 | 0 | 0; 0, 3 | | 2,39 | 2 | 10. |
| | 22 | 0 | 0; 0, 3 | | 2,55 | 2 | |
| | 24 | 0 | 0; 0, 3 | 12 | 3,11 | 3 | |
| | 26 | 0 | 0; 0, 4 | | 3,27 | 3 | |
| | 28 | 0 | 0; 0, 4 | | 3,43 | 3 | |
| 15. | 30 | 0 | 0; 0, 4 | 0 | 3,58 | 4 | 15. |
| | 32 | 0 | 0; 0, 4 | 16 | 4,14 | 4 | |
| | 34 | 0 | 0; 0, 5 | | 4,30 | 4 | |
| | 36 | 0 | 0; 0, 5 | | 4,46 | 4 | |
| | 38 | 0 | 0; 0, 5 | 4 | 5, 2 | 5 | |
| 20. | 40 | 0 | 0; 0, 5 | | 5,18 | 5 | 20. |
| | 42 | 0 | 0; 0, 6 | | 5,34 | 5 | |
| | 44 | 0 | 0; 0, 6 | | 5,50 | 6 | |
| | 46 | 0 | 0; 0, 6 | 8 | 6, 6 | 6 | |
| | 48 | 0 | 0; 0, 6 | | 6,22 | 6 | |
| 25. | 50 | 0 | 0; 0, 7 | | 6,38 | 6 | 25. |
| | 52 | 0 | 0; 0, 7 | | 6,54 | 7 | |
| | 54 | 0 | 0; 0, 7 | 12 | 7,10 | 7 | |
| | 56 | 0 | 0; 0, 7 | | 7,26 | 7 | |
| | 58 | 0 | 0; 0, 7 7 | | 7,42 | 8 | |
| 30. | 60 | 0 | 0; 0, 8 | 58[2] | 7,58 | 8 | 30. |

Notes to the transcription of fol. 123v, Sect. 3:

[1] All numbers in this column are written with Hindu numerals, all other numbers in section 3 with Roman numerals, except for the 7 above line 30.

[2] To be combined with the preceding interlinear 7 to 0;0,7,58.

[3] Suter p. XIX note 9.

**B. Interlinear Version (D)**

fol. 123r **Plate 26**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | | | 7 | 24; 9,55 | | | | 18;45,48 | | 1;35,24 | Almu- | 1. |
| | 1146 | | 2 | 17;32,41 | | · | 1 | 7;34,47 | | 3; 7,38 | haram | |
| | adde diem | 1176 | 9 | 10;55,27 | | | 1 | 26;20,35 | | 4;42,59 | | |
| | 1206° | | 4 | 4;18,13 | b | | 2 | 15; 6,23 | | 6;15,12 | | |
| 5. | adde duas | 1236 | | | | · | 3 | 3;57, 2[2] | | 7; 1,36[7] | | 5. |
| | | | | | | | 3 | 22;39,55 | | 9;22,48 | | |
| | adde tres | | | | | · | 4 | 11;28,42 | | 10;58,12 | | |
| | | | | | b | | 5 | 0;13,17 | | 12;30,24 | | |
| | adde 4[or] | | | | | | 5 | 19;59,53[3] | | 14; 5,48 | | |
| 10. | | | | | | · | 6 | 7;48,39 | | 15;38,10 | | 10. |
| | | | | | | | 6 | 26;34,34 | | 17;13,34 | | |
| | | | | | b | | 7 | 15;19,50 | | 18;45,48 | Zilhege | |
| | | | | | | · | 8 | 4; 8,36 | | | | |
| | | | | | | | 8 | 22;54,12 | | | | |
| 15. | | | | | | | 9 | 11;39,40 | | | | |
| | | | | | b | · | 10 | 0;28,34 | | | | |
| | | | 8 | 8;14,12 | | | 10 | 19;14,10 | | | | |
| | | | 3 | 1;36,58 | | · | 11 | 8; 2,57 | | | | |
| | | 540 | 9 | 24;59,44 | | | 11 | 26;48,34 | | | | |
| 20. | | 570 | 4 | 18;21,30 | b | | 0 | 5;34, 8[4] | | | | |
| | | 600 | 11 | 11;45,17 | | · | 1 | 4;22,55 | | | | |
| | | 630 | 6 | 5; 8, 3[1] | | | 1 | 23; 8,32[5] | | | | |
| | | | | | | | 2 | 11;54, 7 | | | | |
| | | | | | b | · | 3 | 0;42, 3 | | | | |
| | | | | 25. | | | 3 | 19;28,28 | | | | |
| | | | | | | · | 4 | 8;17,14 | | | | |
| | | | | | | | 4 | 27;51,50 | | | | |
| | | | | | b | | 5 | 15;48,25 | | | | |
| | | | | | | · | 6 | 4;37,11[6] | | | | |
| | | | | 30. | | | 6 | 23;22,46 | | | | |

Notes to the transcription of fol. 123r (D):

[1] This line written by later hand.
[2] Before the 2 a smeared sign; after the 2 a marginal note: *dū est* (= *dubium est* ?).
[3] Sic, instead of 18;59,53. Isolated error.
[4] Sic, instead of 15;34,8. Isolated error.
[5] The 3 in 32 is crossed out or corrected; but 32 is correct.
[6] Below the 37 is written, in small signs, what looks like 3010 or 3.1.
[7] Sic, instead of 7;50,36. Isolated error.

From the 30-year motion given in section 2 (or 1) of fol. 123r one derives a daily motion of 0;3,10,46,42,31, . .° for the nodes. The epoch values given in section 1 agree with the Toledan Tables.[5]

[5] Zinner [1] p. 750 M 15v.

Fol. 123v shows only a few modifications of the original text. In section 1 the entry for day 1 is changed from 0;3,11 to 8 in the last digit and in line 30 the last digit 24 is modified to 9. In section 2 we have again 0;3,11 replaced by . .8. The changes and additions in section 3 are shown in Pl. 25 (p. 179).

## 2. Marginal Notes

### A. Transcription

(a) Fol. 123r upper margin:

1. *Primo anno arabum qui incepit feria 5. et tamen illa dies secundum tabulas incepit annum die mercurii hora meridia.*
2. *super meridiem toleti*

(b) Fol. 123v lower margin:

1. . .][6] *Secundum petrum qui dicit Christum natum die sabatum*[7] *uno anno antequam nos et incipit annum a nativitate domini anno domini 1224*[a][8] *prima die martii*
2. *Secundum nos vero qui incipimus a die annunciacionis. anno domini 1220tertio prima die martii incipiente ultima die februarii in meridie*
3. *et eodem anno iam transactus est dies bissextilis. et est illa dies prima martii fere in fine illius anni. Tunc erat is medius motus*
4. *octave spere* 1 24;57,13,27 *arcus quo augmentatur in computatione*

| | | *Caput draconis* | *medius motus solis* | *lune* | *ar (gumentum) lune* |
|---|---|---|---|---|---|
| 5. | § 0 8;47,28 *hic? . . ad* | 11 17;46,27 | 11 6;41,54 | 2 19;50,10 | 2 13;31,54 |
| 6. | *de* 8. | 11 26;33,55 | | 2 19;50,10 | |

(c) Fol. 123v section 2, below last number (which is *xi* with raised 8): *lond.*.(?) *diem octo.*

### B. Translation and Commentary

(a) "In the first Arabic year, which began on a weekday 5 (Thursday); and yet this day began the year according to the tables on a Wednesday, at noon hour, for noon at Toledo".

This is again a reference to the situation represented by fig. 1 p. 10; cf. the marginal notes fol. 115v, 118r, and 122r.

(b) "According to Petrus who says that Christ was born on a Saturday, one year earlier than we (say), and he begins the year from the nativity of the Lord in the year of the Lord 1224, on the first day of March. According to us, however, who begin with

[6] Partly cut away; probably paragraph-sign.
[7] Sic.
[8] Written with Roman numerals. Above the *4* is written an *a*, not an *o*.

the day of the annunciation, in the year of the Lord 1223 on the first day of March which begins at noon of the last day of February. And in the same year the intercalary day has already passed by and it is this first day of March about at the end of this year. – Then this was the mean motion of the eighth sphere 1 24;57,13,27 the arc by which (longitudes) should be increased in a computation".

Then follows a tabulation:

| | | *mean motion* | | *anomaly of* |
|---|---|---|---|---|
| | *asc. node* | *of the sun* | *of the moon* | *the moon* |
| $0^s$ 8;47,28 | $11^s$ 17;46,27 | $11^s$ 6;41,54 | $2^s$ 19;50,10 | $2^s$ 13;31,54 |
| | 11 26;33,44 | | 2 19;50,10 | |

Apparently this note deals with two independent items. The first is the counting of years: according to Petrus Alphonsi from Saturday, March 1, 1224, in contrast to "us" (the scribe of D?) who start with 1223 March 1 (counted from noon of February 28). Actually March 1 is a Friday[9] and 1224 is not "one year earlier" if "we" count from 1223. The day of annunciation is ordinarily reckoned March 25 and I do not know of any system which reckons it as March 1.

The second item concerns precession or rather the *motus accessionis et recessionis* as represented in the *Liber de motu octave sphere* of Thābit ibn Qurra.[10] In order to find according to this theory the difference between sidereal and tropical longitudes for the epoch date A.D. 1224 March 1 we have to find the argument Θ for the point on the small circle which controls the motion of the ecliptic (cf. the appendix at the end of this section, p. 183). Since A.D. 1224 March 1 equals A.H. 621 Safar 9 we have to operate with a time interval of 620 Arabic years, 1 month, 8 days elapsed since the beginning of the era Hijra. For this interval one finds by interpolation from the tables called *motus* $8^e$ *spere*[11] the value Θ = $1^s$ 24;57,13,39, thus only 0;0,0,12° more than in our text. If we round this result to Θ = 54;57° and enter with this argument the table called *equatio diuersitatis longitudinis capitis arietis ab equatore diei*[12] we find by interpolation the precessional correction of 8;47,28° exactly as given in the text.

The tabulation of epoch values concerns the epoch of A.D. 1224 March 1, as is proved by the most sensitive values, namely the lunar mean longitude and anomaly. In this way one finds

| | mean long. | anomaly |
|---|---|---|
| from al-Khwār.'s Tables: | $2^s$ 15;58,48 | $2^s$ 11;50,13 |
| from Toledan Tables: | $2^s$ 19;14,14 | $2^s$ 12;48,15 |
| text: | $2^s$ 19;50,10 | $2^s$ 13;31,54 |

[9] A.D. 1223 March 1 is a Wednesday.

[10] Cf. Millás-Vallicrosa, Azarquiel p. 487–509. and Delambre HAMA p. 75. The same tables are found in the Toledan Tables (Vindob. 2385 fol. 39v).

[11] Millás-Vallicrosa, Azar. p. 507; Delambre HAMA p. 75, left and middle table.

[12] Millás-Vallicrosa p. 508; Delambre p. 75, right table.

In spite of the negligible deviation between text and Toledan Tables it seems unlikely that the computation was based on the Toledan tables, because for the solar mean longitude and the ascending lunar node one obtains

| | sun | node |
|---|---|---|
| from al-Khwār.'s Tables: | 11$^s$ 4;52,38 | 12$^s$ 0;40, 4 |
| from Toledan Tables: | 11$^s$ 4;15,51 | 11$^s$ 29;20,15 |
| text: | 11$^s$ 6;41,54 | 11$^s$ 26;33,55 (tropical) |

### Accessio et Recessio

The model on which the "trepidation" of the vernal point is computed for the tables used here is probably explained in a treatise of Azarqiel "On the motion of the fixed stars", preserved in a Hebrew translation and published with a Spanish translation, by Millás-Vallicrosa in his Estudios sobre Azarquiel, ch. V. I assume that the model under consideration is the "first scheme" described by Azarqiel[1] and presented here as fig. 44[2].

HA is the celestial equator, HM the ecliptic with a fixed obliquity ε. The point

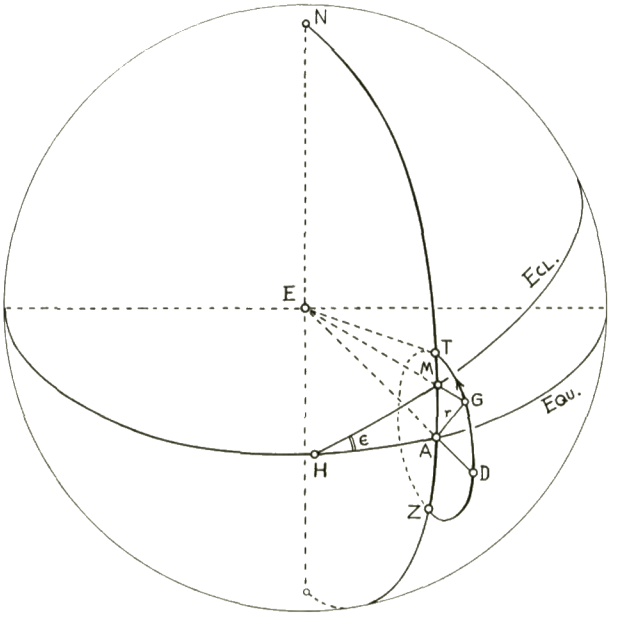


Fig. 44.

[1] Millás-Vallicrosa, Azarquiel, p. 284–287. Error on p. 285, second paragraph, line 2: read el arco ZT (not KT).

[2] A different possibility which produces practically the same result would consists in the use of a model described in Thābit ben Qurra's *De motu octaue spere* (Millás-Vallicrosa, Azar. p. 501 or now Carmody, Thâbit, p. 102–113).

A is the mean vernal equinox. In the plane of the circle of declination AN lies a small circle, with center A and radius $r = 4;18,43°$, on which a point G rotates with uniform velocity and a period of about 4181 $^1/_2$ Arabic years. The angle $\Theta$ = DAG is tabulated for the years of the era Hijra as *motus* $8^e$ *spere*.[1] The circle DG is so small that we may say that the declination AM of G is given by $r \sin \Theta$ which is tabulated as *equatio diametri dimidii circuli parui*.[2] As G moves from D to T, which we may consider as point of maximum declination of the small circle, the ecliptic point M moves from A to T, causing the vernal point H to recede from its mean position A.

The distance HM = $\Delta\lambda$ is the desired difference between the variable vernal equinox H and its mean position A. It can be found in the right spherical triangle AMH from $\sin \Delta\lambda = \sin \delta / \sin \varepsilon$ where the declination of M is known from AM = $\delta = r \sin \Theta$. The values of $\Delta\lambda$ as function of $\Theta$ are tabulated under the heading *equatio diuersitatis longitudinis capitis arietis ab equatione diei*.[2] The maximum deviation will occur for $\Theta = 90°$ and is given as 10;45,0°. Since we know that $\delta = r = 4;18,43$ for $\Theta = 90$ we can compute $\Delta\lambda_{max}$ and find approximately

| | $\varepsilon$ | $\Delta\lambda_{max}$ |
|---|---|---|
| Hindus: | 24; 0° | 10;39 |
| Ptolemy: | 23;51 | 10;43 |
| al-Ma'mūn: | 23;33 | 10;51 |

This shows that our tables are based on Ptolemy's value for $\varepsilon$.

For intermediary values of $\Theta$ one should have

$$\sin \Delta\lambda = \frac{\sin \delta}{\sin \varepsilon} = \frac{r \sin \Theta}{\sin \varepsilon} = \sin \Delta\lambda_{max} \sin \Theta$$

or approximately

$$\Delta\lambda = \Delta\lambda_{max} \sin \Theta.$$

The values in the table show small deviations from this relation.

## Folios 124 to 126

### Equations of Sun and Moon; Solar Declination; Lunar Latitude

#### A. The Original Text (CC)

The heading of each of the six pages from fol. $124^r$ to $126^v$ is *Tabula ad cognoscendum Directionem Solis et Lune* "Table for finding the equation of sun and moon". At the upper rim of the pages $124^v$ and $125^r$ is written *Sol et Lune*, similarly on $125^v$ and 126 *Sol et Lun.*

**Columns 1 and 2:** *Regule Numeri* "Columns for the number" subdivided in *signa* and *gradus* from $0^s\,1°$ to $6^s\,0°$ and backwards from $6^s\,0°$ to $11^s\,29°$.

[1] Millás-Vallicrosa p. 507.
[2] Millás-Vallicrosa p. 508.

**Column 3** fol. 124 to 125$^{r}$: *Directio Solis moram facientis eundo* "equation of the sun while in slow motion", fol. 125$^{v}$ and 126: *Directio Solis festinantis eundo* "equation of the sun while in fast motion".

**Column 4** similarly: *Directio Lune moram facientis eundo* and *festinantis eundo* respectively.

**Column 5:** *Inclinatio Solis ascendentis* and *descendentis* respectively for the solar declination.

**Column 6:** *Latitudo Lune ascendentis* and *descendentis* for the lunar latitude.

In columns 3 to 6 the entries are given as *gradus*, *puncta*, and *minutie*. The tables themselves are identical with Suter's Tables 21 to 26, except for the variants listed below.

In these and in the subsequent tables we find two types of corrections. A first revision was probably made after the completion of a page by comparing it with the original manuscript. This resulted in corrections consisting of writing the proper number above the incorrect digit. Twice (fol. 132$^{r}$ and 132$^{v}$, col. 8) a whole column of digits was misplaced, showing that the tables were copied in columns, digit by digit. One of these mistakes was then corrected in marginal notes (fol. 132$^{r}$, cf. below p. 197).

A second revision, however, is recognizable by the use of a curious algebraic notation. Three dots[1] in triangular arrangement indicate a positive correction, one dot a negative correction. For example[2]

has to be interpreted as

$$2+1 \quad 49 \quad 29-7$$

indicating in this fashion a correction of 2,49,29 to 3,49,22. Most of these corrections are found on the pages from fol. 124$^{r}$ to 130$^{r}$ (cf., e.g., the photograph of fol. 124$^{v}$, Pl. II). In the following a remark "corrected to" always means this type of correction. The majority of these changes must have been made on the basis of a manuscript belonging to the family C, whereas the original text usually belongs to the O family. In many cases the version C is worse than the original O.

For the addition of a new interlinear version of the table for the solar equation on fol. 124 to 126 cf. below p. 189 f.

**Deviations from Suter, Tab. 21 to 26**

**fol. 124$^{r}$:**

col. 1 line 30: $0^{s}\,0°$ instead of $1^{s}\,0°$.

col. 3 line 21: 0;46,48. Suter: 0;46,49.

26: corrected by erasure of 27 to Suter's value 22.

27: corrected by erasure of 28 to Suter's value 26.

28: 1;1,28. Suter: 1;1,29.

[1] Occasionally two dots, arranged like . ˙ , are used instead.

[2] fol. 124$^{r}$ col. 5 line 7.

col. 4 line 20: 1;38,39 corrected to 1;38,29 as in C.
col. 5 line 7: 2;49,24 corrected to 3;49,22 as (incorrectly) in C.
13: 5;13,7, corrected by original hand to 5;13,50 as in Suter.
16: corrected by erasure of 47 to Suter's 57.
30: 11;39,50 corrected to 11;29,50 as (incorrectly) in C.

**fol. 124$^{v}$** (photo: Pl. II):

col. 3 line 24: 1;47,27. Suter, correctly, 1;47,17.
col. 4 line 3: 2;37,3 corrected to 2;38,53 as (incorrectly) in O.
8: 2;59,50. Suter, correctly, 2;58,50.
10: 3;6,58 corrected to 3;6,58 + 3 instead of 58 − 3. Suter, correctly, 3;6,55.
23, 24: 3;53,49 and 3;57,0 corrected to 4; . . as (incorrectly) in C.
col. 5 line 8: 14;24,54. Over the 54 is written *xx*, thus read 24? Suter: 14;24,34.
lines 17 to 30 show the following corrections of the first digits: +1 in line 19 and 24, in all other lines −1. These correspond in most cases to the incorrect version C.
line 18: 17;29,14. Suter (incorrectly) 17;29,54.
21: 18;13,51. Suter (correctly) 18;18,51.
28: 20; 3,15. Suter 20; 3,55.
29: 20;16,44. Suter 20;16,45.
col. 6: the last digit in line *n* of the present text is identical with the last digit in line *n* + 2 of Suter, leaving the first and second digit unchanged. Exceptions: in line 13 the last digit is 54 as compared with 14 in Suter line 15. Lines 29 and 30 end in 25 and 46 respectively, not represented in Suter. A marginal note below this column says

*3*
*3*
*3*
*59*
*etc*
*51. ultima*

which are the last digits of Suter's text.

**fol. 125$^{r}$:**

col. 3 line 2: 1;57,0 as (incorrectly) C and O.
3: 1;58,38. Suter 1;58,39.
24: 2;13,14. Suter 2;13,15 (but cf. Tab. 24 col. 2 line 6).
col. 4 line 10: 4;37,19 corrected to 4;37,9 as in C.
14: 4;43,12. Sic, instead of 4;43,52. Suter: 4;44,12 but C and O: 4;43,12. Cf. Suter Tab. 24 col. 2 line 16.
17: 4;48,58: Suter (correctly) 4;47,58.
26: 4;55,52: changed by first hand to 4;55,22 (as in Suter).

col. 5 line 12: 22;38, 0: corrected to 22;37,0 as correctly in Suter.
col. 6 line 3: 3; 0,14: the last digit changed to 34. Suter correctly 4;0,34.
16: 4;22,57 corrected to 4;22,58. Suter correctly 4;21,58.
25: 4;28,57 corrected to 4;28,58 as in Suter.

**fol. 125$^{v}$:**

col. 3 line 4: 2;13,39 as in C. Suter: 2;13,41.
5: 2;13,28. Suter: 2;13,14 (but cf. Tab. 23 col. 2 line 25).
7: 2;12,57 as in C. Suter: 2;12,58 (but cf. Tab. 23 col. 2 line 23).
18: 2; 8, 3. Suter: 2;7,4.
21: 2; 4,36. Suter: 2;4,37 (but cf. Tab. 23 col. 2 line 9).
25: 1; 0,47. Suter (correctly) 2;0,47.
col. 4 lines 5 to 9 and 14: as in Suter but changed (by first hand?) to the values gives in C (except the error 52 in line 6).
line 21: 4;34,15 corrected to 4;35,15 as in Suter.
23: 4;31,12 corrected to 4;31,18 as in Suter.
col. 5 line 3: 23;48,25 changed to 23;48,55 as in Suter.
8: 23;37,15 corrected to 23;36,15 as in Suter.
19: 22;28,39 corrected to 22;28,19 as in Suter.
20: 22;19,58 corrected by −60 instead of −40 (*lx* instead of *xl*).
22: 22;50, 4 instead of 22;1,4.
29: 20;42,59 corrected to 20;42,39 as in Suter.
col. 6 lines 7 and 8: 4;26, . . corrected to 4;27, . . as in Suter.
line 29: 3;56,10 as in Suter but perhaps corrected to 3;57,10 as (incorrectly) in C.
30: 3;53,41 corrected to 3;53,51 as in Suter.

**fol. 126$^{r}$:**

col. 3 line 12: 1;38,13. Suter: 1;38,18.
17: 1;29,17. Suter, correctly, 1;29,56.
24: 1;17,15. Suter: 1;17,12.
lines 27 to 30: the last digits were by mistake replaced by the second digits of the corresponding numbers in col. 4.
col. 4 line 6: 4;57,0 instead of 3;57,0; cf. the following lines.
lines 7 to 21: all first digits 4 instead of 3, but corrected to 3 in each line.
line 12: 4;38, 2 corrected to 3;38,2 as (incorrectly) in C, instead of 3;37,2.
22: 2;59,50 instead of 2;58,50 as in Suter.
29: 2;28, 6 corrected to 2;29,6 as in Suter.
col. 5 line 2: 20; 8,14. Suter 20;4,14 (Tab. 22 col. 5 line 28: 20;3,55).
7: 19;50,24. Above line: 18, corresponding to Suter's value.
8: 18;34,49 corrected to 18;35,49 as in Suter.
15: 16;36,49 as in Suter but corrected to 16;46,49 as (incorrectly) in C.

27: 12;42,20 instead of 12;43,20 as in Suter.
28: 12;22,23: last digit changed to 10 as in Suter.
30: 11;39, 7: last digit changed to 50 as in Suter.
col. 6 lines 1 to 17: all first digits 4 instead of 3 but corrected to 3 in each line.
line 14: 4;14,13, but above the 13 is written 58. Suter: 3;13,58, but O gives 3;14,58.
18: 4; 0,40 instead of 3;0,40.

**fol. 126$^{v}$:**

col. 3 line 25: 0;11,22. Suter 0;11,21.
26: 0; 9, 8. Suter 0;9,5.
27: 0; 7,52. Suter correctly 0;6,49.
28: 0; 4,37. Suter: 0;4,33.
29: 0; 2, 2. Suter: correctly 0;2,17.
col. 4 line 4: 2; 6,43 as in O but corrected to 2;6,53 as in C.
6: 1;17,32. Suter correctly 1;57,32.
8: 1;48,20. Suter 1;48,10.
9: 1;43,22 as in C.
17: 1; 5,46. Suter correctly 1;4,46.
21: 0;45, 5. Suter 0;45,0.
28: 0;10, 2. Suter: 0;10,3.
col. 5 line 14: 6;23,58. Suter 6;23,57.
col. 6 line 1: 2;10,13. Last digit changed to 53 as in Suter.
11: 1;37,54. Suter correctly 1;27,54.
20: 0;46,13. Suter 0;46,18.
29: 0; 4,42. Last digit changed to 12 as in Suter.

**Marginal Notes**

On fol. 124$^{v}$, 125$^{v}$, 126$^{v}$ on the left margin beside the last lines one finds in very small writing the following remarks:

*ap]posui unam .s.*
*Et*(?) *hic appono unam s*
*] hic* (*u*)*nam appono .s.*

This refers to corrections made in the number of signs. The original text had I 0, III 0, V 0 respectively instead of II 0, IIII 0, VI 0. In each case one unit was therefore added to the number of the signs. The entries on fol. 125$^{r}$ and 126$^{r}$ were correct from the beginning.

**B. Interlinear Version (D): Tables of Sines**

Above the numbers of the third column of the original text fol. 124 to 126 are written Hindu numerals which constitute a table of sines with the Hindu norm $R = 150$. Exactly the same table is found in the Toledan Tables (T fol. $3^r$ to $4^r$) under the title *equatio sinus*. In our interlinear version only fol. $124^v$ mentions *Sinus* in the heading. But all columns refer to Azarqiel as author (cf. Pl. 27 and photo Pl. 30) and fol. $125^r$ indicates that this table was written only for one quadrant. The identity of errors in D and T shows, however, that D was copied from an archetype in which the table of sines extended over six signs.

The other columns of fol. $124^r$ to $126^v$ remained free from interlinear additions, except at the heading of column 4 on fol. $124^r$ (*directio lune*) where one finds added the words *equatio argumenti*. This is the term of the Toledan Tables for the epicyclic anomaly[3] which is the only anomaly considered in the lunar tables of al-Khwārizmī.

[3] Zinner [1] p. 750 M $14^r$, $15^r$.

fol. 124r | 124v | 125r | 125v | 126r | 126v **Plate 27**

| | Secundum Arzachele vero Algorum Secunde | Secundum Arzachel Sinus | Hic est finis secundum Arzach(el) | Secundum Arzachel Algorum | Secundum Arzachel | Secundum Arzachelem | |
|---|---|---|---|---|---|---|---|
| 1. | 2,37, 5 | 1,17,16,22 | 2,11,11,35 | 2,29,58,40 | 2, 8,34,30 | 1,12,43,17 | 1. |
| | 5,14, 5 | 1,19,29,17 | 2,12,26,32 | 2,29,54,32 | 2, 7,12,27 | 1,10,25,15 | |
| | 7,51, 2 | 1,21,41,45 | 2,13,39, 5 | 2,29,47,40 | 2, 5,48, 2 | 1, 8, 5,55[26] | |
| | 10,27,50 | 1,23,52,42[6] | 2,14,49,10 | 2,29,38, 5 | 2, 4,21,27[15] | 1, 5,45,20 | |
| 5. | 13, 4,25 | 1,26, 2,52 | 2,15,56,45 | 2,29,25,45 | 2, 2,52,22 | 1, 3,23,32 | 5. |
| | 15,40,45 | 1,28,10, 5 | 2,17, 1,15[12] | 2,29,10,42 | 2, 1,21,10 | 1, 1, 0,34[23] | |
| | 18,16,50 | 1,30,16,20 | 2,18, 4,32 | 2,28,52,55 | 1,59,47,47[24] | 0,58,36,37 | |
| | 20,52,35 | 1,32,20,17[20] | 2,19, 4,40 | 2,28,32,26 | 1,58,12, 5 | 0,56,11,27 | |
| | 23,27,55 | 1,34,23,15[28] | 2,20, 2,15 | 2,28, 9,12 | 1,56,34,20 | 53,45,19 | |
| 10. | 26, 2,50 | 1,36,25,50 | 2,20,57,15 | 2,27,43,57 | 1,54,54,25 | 51,18,10 | 10. |
| | 28,37,57 | 1,38,24,32 | 2,21,49,42 | 2,27,14,40 | 1,53,12,25 | 48,50, 6 | |
| | 31,11,52 | 1,40,22,10 | 2,22,39,32 | 2,26,43,20 | 1,51,28,20 | 46,21, 9 | |
| | 33,44,32 | 1,42,18, 0 | 2,23,26,44[13] | 2,26, 9,20 | 1,49,42,52[16] | 43,51,21 | |
| | 36,16,57 | 1,44,11,57 | 2,24,11,22 | 2,25,32,40 | 1,47,54, 2 | 41,20,44 | |
| 15. | 38,49,22 | 1,46, 3, 2[8] | 2,24,53,20 | 2,24,53,20 | 1,46, 3,17[25] | 38,49,22 | 15. |
| | 41,20,44 | 1,47,54,12 | 2,25,32,40 | 2,24,11,22 | 1,44,11,17[7] | 36,16,57 | |
| | 43,51,21 | 1,49,42,20 | 2,26, 9,20 | 2,23,26,45 | 1,42,58, 0[17] | 33,44,32 | |
| | 46,21, 9 | 1,51,28,25 | 2,26,43,20 | 2,22,39,32 | 1,40,22,10 | 31,11,52 | |
| | 48,50, 6 | 1,53,12,25 | 2,27,14,40 | 2,21,49,42 | 1,38,23,32[18] | 28,37,17 | |
| 20. | 51,18,10 | 1,54,54,20 | 2,27,43,57 | 2,20,57,15 | 1,36,25,50 | 26, 2, 7 | 20. |
| | 53,45,59[1] | 1,56,34, 5 | 2,28, 9,12 | 2,20, 2,15 | 1,34,23, 0[19] | 23,27,55 | |
| | 56,11,27 | 1,58,12,57[9] | 2,28,32,26 | 2,19, 4,40 | 1,32,20,57 | 20,52,35 | |
| | 58,36,37 | 1,59,47,10 | 2,28,52,55 | 2,28, 4,32 | 1,30,56,20[21] | 18,16,50[27] | |
| | 61, 0,37[2] | 2, 1,21,22[8] | 2,29,10,42 | 2,17, 1,55 | 1,28,10, 5 | 15,40,45 | |
| 25. | 63,23,32 | 2, 2,52,22 | 2,29,25,45 | 2,15,56,45 | 1,26, 2,56 | 13, 4,25 | 25. |
| | 65,45,30[3] | 2, 4,21,20 | 2,29,39, 5[14] | 2,14,49,10 | 1,23,52,45 | 10,27,50 | |
| | 68, 5,15[4] | 2, 5,43, 2[10] | 2,29,47,40 | 2,13,39, 5 | 1,21,11,45[22] | 7,51, 2 | |
| | 1,10,25,15 | 2, 7,12,27 | 2,29,54,32 | 2,12,26,32 | 1,19,29,57[5] | 5,14, 5 | |
| | 1,12,43,17 | 2, 8,34,30 | 2,29,58,40 | 2,11,11,35 | 1,17,16,22 | 2,37, 5 | |
| 30. | 1,15, 0, 0 | 2, 9,54,54[11] | 2,30, 0, 0 | 2, 9,54,14 | 1,15, 0, 0 | 0, 0, 0, 0 | 30. |

Notes to the transcription:

[1] Sic, instead of 54,45,19. Cf. 126v,9. T: 54;45,29.
[2] Note in lines 24 to 27 the writing 61, 62, etc. T writes all numbers ≥ 60 decimally.
[3] Sic, instead of 65,45,20. Cf. 126v,4.
[4] Sic, instead of 68,5,55. Cf. 126v,3.
[5] Sic, instead of 1,19,29,17. Cf. 124v,2.
[6] Sic, instead of 1,23,52,45. Cf. 126r,26.
[7] Sic, instead of 1,44,11,57. Cf. 124v,14.
[8] The last digits of all numbers in lines 15 to 24 should be moved one line down; cf. 126r, 15 to 5.
[9] The last digit should be 47 according to 126r,7.
[10] Sic, instead of 2,5,48,2. Cf. 126r,3. T: 125;48,2.
[11] Sic, instead of 2,9,54,14. Cf. 125v,30.
[12] Sic, instead of 2,17,1,55. Cf. 125v,24.

[13] Sic, instead of 2,23,26,45. Cf. 125v,17. T: 143;26,45.
[14] Sic, instead of 2,29,38,5. Cf. 125v,4.
[15] Sic, instead of 2,4,21,20. Cf. 124v,26.
[16] Sic, instead of 1,49,42,12. Cf. 124v 17 and 16.
[17] Sic, instead of 1,42,18,0. Cf. 124v,13.
[18] Sic, instead of 1,38,24,32. Cf. 124v,11.
[19] Sic, instead of 1,34,23,15. Cf. 124v,9. The correct value would be 1,34,23,53 (or . . , 55).
[20] Sic, instead of 1,32,20,57. Cf. 126r,22.
[21] Sic, instead of 1,30,16,20. Cf. 124v,7.
[22] Sic, instead of 1,21,41,45. Cf. 124v,3.
[23] Sic, instead of 1,1,0,37. Cf. 124r,24.
[24] T has here 119;47,40. The correct value would be 119;47,43. Cf. also note 9.
[25] Sic, instead of 1,46,3,57.
[26] T (incorrectly): 68;5,52.
[27] T (incorrectly): 18;16,7.
[28] Cf. note 19.

Except for the cases listed in notes 1, 10, 13, 26, and 27, all these errors are common to D and T, in particular the dislocation of a column of digits mentioned in note 8. In 12 cases of the above given list the error consists in an interchange of 10 and 50, thus suggesting an Arabic archetype.

## Folios 127 to 129

### Equation, Stations, and Latitude of Saturn

The heading of all six pages (which correspond to Suter Tab. 27 to 32) is *Directio Saturni* "Equation of Saturn". The column headings are:

**Columns 1 and 2:** *Regule Numeri* "Columns for numbers", subdivided in *signa* and *gradus* for the first and the second six signs respectively.

**Column 3:** fol. 127: *Sublimatio examinata*, fol. 128 and 129: *Directio auc*, giving *signa*, *gradus*, *puncta*. Tabulated is the quantity *s* as defined above p. 98.

**Column 4:** *Directio heze* "equation of anomaly" giving *gradus* and *puncta*. Tabulated is the quantity $|\sigma|$ as defined on p. 99.

**Column 5:** *Directio centri*, giving *gradus* and *puncta* of the "equation of center" i.e. $|\mu|$ (p. 100).

**Column 6:** fol. 127r, 128, 129v: *Prima statio Saturni*; fol. 129r: *Statio Saturni*; fol. 127v incorrectly *Directio centri*. Given: *signa*, *gradus*, *puncta*.

**Columns 7 and 8:** fol. 127 and 128r: *Latitudo Saturni ascendentis*, fol. 128v and 129: *Latitudo Saturni descendentis*, subdivided in two sections: *prima* (*gradus* and *puncta*) and *secunda* (*gradus*, *puncta*, *minutie*).

### Deviations from Suter, Tab. 27 to 32

The corrections made, above single digits, employ the same device as explained on p. 185: three (or occasionally two) dots for a positive correction, one dot for a negative correction.

**fol. 127$^{r}$:**

col. 1 line 30: $0^{s}\,0°$ instead of $1^{s}\,0°$.
col. 4 line 6: 0;36. Above line: 23, as in C.
16: 1;26 corrected to 1;27 as in C.
col. 5 line 2: 0;18. Suter: 0;17.
11: 1;27 corrected to 1;37 as in Suter.
16: 2;21 corrected to 2;22 as in C.
24: 3;28 corrected to 3;38 as in C.
25: 3;36 corrected to 3;37 as in C.
col. 8 line 1: 0; 5,13. Suter: 0;5,14.
6: 0;29,21. Sic, instead of Suter's 0;31,21.
7: 0;26,33 corrected to 0;36,33 as in Suter.
17: 1;17,43 corrected to 1;27,43 as in Suter.
22: 1;55,23. Sic, instead of Suter's 1;52,23.
24: 2; 2,50. Sic, instead of Suter's 2;2,1.
29: 2;25,26. Suter: 2;25,27.

**fol. 127$^{v}$:**

col. 1 lines 4 to 28: degrees omitted.
line 30: $1^{s}\,0°$ instead of $2^{s}\,0°$.
col. 2 lines 2 to 24: degrees omitted.
col. 3 line 3: 3;20 instead of Suter's 3;30.
4: 3;22 instead of Suter's 3;27.
28: 2;37 instead of Suter's 2;38.
col. 4 line 11: 3;28 instead of Suter's 3;27.
col. 5 line 1: 4;25 corrected to 4;35 as in C and $M_1$.
2: 4;22 corrected to 4;32 as in Suter.
3: 4;39 corrected by +10 to 4;49 which is probably an error for −10 since C has 4;29.
9: 5;22 corrected to 5;32 as in C.
12: 5;42. Suter: 5;43.
20: 6;20 instead of Suter's 6;32.
col. 6 lines 6 to 25: signs omitted.
col. 8 line 6: 2;56,20 corrected to 2;57,20 as in Suter.
7: 3; 0,24 corrected to 3; 0,33 as in Suter.
15: 3;22, 8 corrected to 3;32, 8 as in Suter.
17: 3;39,24 corrected to 3;39,25 as in Suter.
28: 4;14,25 corrected to 4;15,22 as in C.
29: 4;17, 9 corrected to 4;18, 9 as in C.

**fol. 128$^{r}$:**

col. 1 lines 4 to 28: degrees omitted.
line 30: $2^{s}\,0°$ instead of $3^{s}\,0°$.

col. 2 lines 3 to 24: degrees omitted.
col. 4 line 4: 4;53 corrected to 4;54 as in Suter.
lines 21 to 24: the last digits are corrected by + 2, thus yielding Suter's values.
lines 25 and 26: Suter's values are higher by one unit of the last digit, but C, N, and O agree with CC.
col. 5 lines 6 and 7: 7;47 and 7;51 respectively, incorrectly corrected to 8;47 and 8;51.
lines 8 and 9: 8;54 and 8;58 respectively, instead of 7;54 and 7;58.
col. 6 lines 5 to 30: signs omitted.
line 20: 23;59 corrected to 23;58 as in Suter.
lines 23 to 30: all degrees are given as 8 (as in column 5), instead of 24.
line 27: 8;7 corrected to 8;8. Suter: 24;8.
30: 8;9 corrected to 8;10. Suter: 24;11.
col. 7 line 25: 1;19 corrected to 1;20. Suter: 1;18.
col. 8 line 3: 4;20,18 instead of 4;27,18.
5: 4;31,43 corrected to 4;31,53 as in Suter.
11: 4;43,39 corrected to 4;43,29 as in C.
30: 4;0,0 as in C and O, instead of 5;0,0.

**fol. 128$^{v}$:**

col. 1 lines 6 to 28: degrees omitted.
line 30: $3^{s}\,0°$ instead of $4^{s}\,0°$.
col. 2 lines 2 to 25: degrees omitted.
line 26: $8^{s}\,5°$ instead of $8^{s}\,4°$.
line 27: $8^{s}\,4°$ instead of $8^{s}\,3°$.
col. 3 lines 23 to 27: all last digits corrected by − 1, thus yielding Suter's numbers.
col. 6 lines 4 to 27: signs omitted.
col. 7 line 1: 1;18 corrected to 1;28 as in C.
lines 8 to 10 and 17 to 20: all last digits corrected by − 1, thus yielding Suter's numbers.
col. 8 line 9: 4;56,18 corrected to 4;57,18 as in C.
17: 4;48,23 corrected to 4;48,43 as in C.
18: 4;46,53 changed to 4;46,13 as in C.
20: 4;44,39 instead of Suter's 4;43,39 which is correct.
27: 4;27,18 corrected to 4;26,18 as in C.

**fol. 129$^{r}$:**

col. 1 lines 6 to 28: degrees omitted.
line 30: $4^{s}\,0°$ instead of $5^{s}\,0°$.
col. 2 lines 2 to 24: degrees omitted.
col. 4 line 14: 4;27 which is the correct value. Suter: 4;29.
col. 5 line 7: 6;49 corrected to 6;48 as in Suter.
9: 6;36 corrected to 6;37 as in Suter.

col. 6 lines 5 to 25: signs omitted.
col. 7 lines 1 to 4: 1;14 corrected to 1;24 (perhaps 1;25 in line 1), thus following C; cf. Suter's apparatus p. 142 note f–f.
lines 11 to 14: 1;13 as in N and O.
col. 8 line 2: 4;14,25 corrected to 4;14,24 as in Suter.
9: 3;53,28 changed to 3;53,9 as in Suter.
16: 3;28,24 corrected to 3;29,24 as in Suter.
21: 3; 7,48 corrected to 3; 8,48 as in Suter.
23: 2; 0,33 corrected to 3; 0,43 as in Suter.
26: 2;47,45 corrected to 2;46,45 as in C.

**fol. 129$^{v}$:**

col. 1 line 30: $5^{s}\,0°$ instead of $6^{s}\,0°$.
col. 3 line 5: 3;35 corrected to 3;25 as in C.
9: 3;46. Sic, instead of Suter's 3;47.
col. 4 line 3: 2;42 corrected to 2;52 as in Suter.
5: 2;41. Sic, instead of Suter's 2;40.
6: 2;35. Sic, instead of Suter's 2;34.
line 18 to 20: beside the 1°–s are written (incorrectly) zeros.
col. 5 line 3: 3;56. Sic, instead of Suter's 3;53.
10: 2;52. Sic, instead of Suter's 2;54.
15: 2;14 corrected to 2;13 as in Suter.
16: 2;4 corrected to 2;3 as in Suter.
29: 0;8 as in O.
col. 7 line 23: 1;11. Suter: 1;10.
col. 8 line 1: 2;25,17 corrected to 2;25,27 as in Suter.
5: 2; 6,47 corrected to 2; 6,46 as in Suter.
6: 2; 3, 1 corrected to 2; 2, 1 as in Suter.
7: 2;54,13 corrected to 1;57,13 as in Suter.
8: 1;52,23 as in N and O.
9: 1;46,31 corrected to 1;47,31 as in Suter.
15: 1;16,39 corrected to 1;17,39 as in Suter.
19: 0;54,15 corrected to 0;57,15 as in Suter.
21: 0;43,56 corrected to 0;46,56 as in Suter.
23: 0;33,33 corrected to 0;36,33 as in Suter.
27: 0;12,42 corrected to 0;15,42 as in Suter.

## Folios 130 to 132

### Equation, Stations, and Latitude of Jupiter

These tables correspond to Suter Tab. 33 to 38. The heading is *Directio Iovis* "Equation of Jupiter". The columns are headed as follows:

**Columns 1 and 2:** *Regule Numeri*
**Column 3:** *Directio Auc*
**Column 4:** *Directio heze*
**Column 5:** *Directio centri*
**Column 6:** *prima Statio Iovis*
**Columns 7 and 8:** fol. 130$^{v}$, 131$^{r}$: *Latitudo Iovis ascendentis*; fol. 131$^{v}$, 132: *Latitudo Iovis descendentis*. The subtitles *prima* and *secunda* for column 7 and 8 respectively are omitted on fol. 130. Fol. 130$^{r}$ has by mistake the heading *Prima Statio Iovis*.

The number of tabulated digits is the same as in the tables for Saturn (cf. p. 191). From fol. 131$^{v}$ on, the words *signa*, *gradus*, and *puncta* in the headings are frequently replaced by *s*, *g*, and *p* respectively. Only *minutie* is never abbreviated.

**Deviations from Suter, Tab. 33 to 38**

**fol. 130$^{r}$:**

col. 1 lines 21 to 29: degrees omitted.
line 30: $0^{s}\,0°$ instead of $1^{s}\,0°$.
col. 2 lines 13 to 17: degrees omitted.
col. 3 line 3: 22;59 changed to 22;18 as in Suter.
6: 22;4. Suter: 22;3.
col. 4 line 11: 1;48. Sic, instead of Suter's 1;45.
col. 5 line 17: 1;28. The 28 is written *XX*IVIII
18: 1;33. Suter: 1;34.
28: 2;23 corrected to 2;33 as in C.
col. 6 lines 8 to 12: 4;7. Suter 4;6.
line 16: 4;7. Suter: 4;8.
19: 4;8. Suter: 4;9.
col. 7 line 24: 1;30 changed to 1;29 but again corrected to 1;30 as in Suter.
col. 8 line 3: 0; 7,51 changed to 0;7,11 as in Suter.
7: 0;18,17 as in Suter. Correction above last digit: + 30.
10: 0;25, 3. Sic, instead of Suter's 0;26,3.
14: 0;36,47. Above last digit is written 26(?). Correct in Suter: 0;36,17.
18: 0;46,21 corrected to 0;46,28 as in Suter.
22: 0;56,40 changed to 0;56,11 as in Suter.
lines 24 to 30: beside the 1° – s are written (incorrectly) zeros.
line 24: 1; 1,50 changed to 1;1,41 as in Suter.
25: 1; 3,25 corrected to 1;2,24 as in Suter.
28: 1;10,25 corrected to 1;10,29 as in Suter.

**fol. 130$^{v}$:**

col. 1 lines 6 to 27: degrees omitted.
line 30: $1^{s}\,0°$ instead of $2^{s}\,0°$.

col. 2 lines 3 to 24: degrees omitted.
col. 3 line 5: 19;41. Sic, instead of Suter's 19;51.
7: 19;48. Sic, instead of Suter's 19;43.
26: 28;32 as in N and O. Suter (C) has 28;31.
col. 4 line 8: 5;47. Suter: 5;48.
col. 5 line 5: 2;53. Suter 2;54.
22: 3; 2. Sic, instead of Suter's 4;2.
col. 6 lines 9 to 27: signs omitted.
line 20: 4;38. Suter: 4;39.
col. 7 line 18: 1;25. Suter: 1;26.
col. 8 line 4: 1;23,33. Sic, instead of Suter's 1;23,53.
11: 1;38,25. Corrections above the last digit: +3. Sic, instead of −3. Suter: 1;38,22.
lines 24 to 30: first digit 1 instead of 2. Same error in C and O.
line 28: 1;7,12. Suter (incorrectly) 19 for the last digit.

**fol. 131$^{r}$:**

col. 1 lines 6 to 28: degrees omitted.
line 30: $2^{s}\,0^{\circ}$ instead of $3^{s}\,0^{\circ}$.
col. 2 lines 2 to 24: degrees omitted.
col. 3 line 4: 18;7. Suter (incorrectly) 18;8.
col. 4 line 12: 9;35. Suter (incorrectly) 9;36.
29: 10;35. Sic, instead of Suter's 10;38.
col. 5 line 20: 5;59. Sic, instead of 4;59.
29: 5; 5. Suter: 5;6.
col. 6 lines 3 to 28: signs omitted.
col. 7 lines 18, 22, and 26: last digit one unit less than in Suter.
col. 8 line 5: 2;15,58. Suter: 2;15,57.
13: 2;22,26. Sic, instead of 2;23,26.
14: 2;23,11. Sic, instead of 2;24,11.

**fol. 131$^{v}$:**

col. 1 lines 3 to 28: degrees omitted.
line 30: $3^{s}\,0^{\circ}$ instead of $4^{s}\,0^{\circ}$.
col. 2 lines 3 to 26: degrees omitted.
col. 6 lines 3 to 30: signs omitted.
line 21: 6;20. Sic, instead of 6;10.
col. 7 lines 9, 21, 29, and 30: last digit one unit higher than in Suter.
col. 8 lines 10, 12, 14, and 15: second digit one unit higher than in Suter's (correct) version.
line 17: 2;24,9. Suter: 2;24,11.
lines 17 to 20: second digit one unit higher than in Suter's (correct) version.
line 24: 2;17,2, following C. Suter: 2;17,1.

**fol. 132$^{r}$:**

col. 1 lines 4 to 28: degrees omitted.
line 30: $4^{s}\,0^{\circ}$ instead of $5^{s}\,0^{\circ}$.
col. 2 lines 3 to 27: degrees omitted.
col. 3 line 10: 17;54 corrected to 17;53 as in Suter.
21: 18;34 corrected to 18;33 as in C.
23: 18;53. Sic, instead of 18;43.
col. 6 lines 3 to 29: signs omitted.
col. 8 line 4: 2;4,21 changed to 2;4,12 as in C.
lines 5 to 25: all second digits should be moved one line up (line 5 should read 2;2,52 and line 25: 1;26,2).
The marginal notes near line 5 refer to this error:

3 *deleatur*
*et incipiat ibi*
2.1.59. *e[tc* (cf. Suter lines 5 to 7, second digit)

and near line 26:

26.*i[bi*(?)
28 *et[c*
23 (which should, however, have been 28/26/23 for the second digits in the lines 24 to 26)

line 6: last digit 21, changed to 12 as in C.
12: last digit 18 instead of 28 as in Suter.
16: second digit 46. Thus the number in line 15 would be 1;46,4 where Suter has incorrectly 1;45,4.
26: 1;23,54 corrected to 1;23,53 as in Suter.

**fol. 132$^{v}$:**

col. 1 lines 6 to 28: degrees omitted.
line 30: $5^{s}\,0^{\circ}$ instead of $6^{s}\,0^{\circ}$.
col. 2 lines 3 to 24: degrees omitted.
col. 3 lines 25: 21;51 instead of Suter's 21;55.
col. 4 line 12: 4;13. Suter: 4;14.
16: 3;21, as in O. Suter: 3;22 (C).
19: 2;49 instead of Suter's 2;40.
26: 1; 0. Suter (incorrectly) 0;0.
col. 6 lines 5 to 30: signs omitted.
col. 8 line 11: 0;48,50. Suter: 0;48,7.
lines 13 to 29: all last digits should be moved two lines up (line 13 should read 0;43,49 and line 27: 0;7,37, line 28: 0;5,5, line 29: 0;2,31).
line 25: second digit 13 as in C and O. Suter: 12.
26: last digit (belonging to line 24): 23. Suter: 28 in O, 48 in C.

From fol. 130$^{v}$ on the corrections with + and – no longer appear, except for a few cases on fol. 132$^{r}$ and 138$^{r}$.

## Folios 133 to 135

### Equation, Stations, and Latitude of Mars

The main heading is *Directio Martis* "Equation of Mars". These tables correspond to Suter Tab. 39 to 44. The subtitles of columns 1 to 5 are the same as for Jupiter (cf. p. 195). Column 6 *Prima Statio Martis* is on fol. 133$^{r}$ still called *prima Statio Iovis.* Col. 7 and 8 are *Latitudo Martis ascendentis* and *descendentis* respectively (fol. 133, 134$^{r}$ and 134$^{v}$, 135) and *prima* (col. 7) and *secunda* (col. 8). Except for fol. 135$^{v}$ the abbreviations *s g p* are used in most cases for the units.

### Deviations from Suter, Tab. 39 to 44

**fol. 133$^{r}$:**

col. 1 lines 6 to 28: degrees omitted.
line 30: 0$^{s}$ 0° instead of 1$^{s}$ 0°.
col. 2 lines 3 to 27: degrees omitted.
col. 4 line 8: 4;8. Sic, instead of 3;8.
col. 6 lines 4 to 27: signs omitted.
lines 15 and 16: 7;41 instead of 7;40 and 7;42 respectively in Suter.
line 19: 7;46 instead of Suter's 7;47.
lines 25, 70, 30: first digit 7° instead of 8°.
col. 7 line 26: 2;9. Suter: 2;10.
col. 8 line 12: 0;46,46. Suter: 0;46,47.

**fol. 133$^{v}$:**

col. 1 lines 6 to 28: degrees omitted.
line 30: 1$^{s}$ 0° instead of 2$^{s}$ 0°.
col. 2 lines 3 to 27: degrees omitted.
col. 3 line 14: 29;59. Sic, instead of Suter's 29;52.
col. 4 lines 6 and 9: the first digit should be 1° higher.
line 10: 15;39. Sic, instead of Suter's 15;32.
lines 12, 14, 17, 20, 22, 25: the first digit should be 1° higher.
col. 6 lines 3 to 28: signs omitted.
col. 8 line 1: 1;55,54. Suter: 1;55,53.
2: 2;59,14. Sic, instead of 1;59,14.
4: 2; 5,39. Suter: 2;5,49. N and O: 2;5,19. C: 2;5,30.
14: 2;35,18. Sic, instead of 2;36,18.
23: 3;59,41. Sic, instead of 2;59,41.
25: 3; 4,18. Suter: 3;4,19.

**fol. 134$^{r}$:**

col. 1 lines 4 to 28: degrees omitted.
line 30: $2^{s}\,0°$ instead of $3^{s}\,0°$.
col. 2 lines 3 to 26: degrees omitted.
col. 3 line 2: 26;33. Suter: 26;32.
17: 24;56. Sic, instead of 23;56.
28: 22;29. Sic, instead of 22;11. Cf. line 26.
col. 4 line 4: 30;27. Sic, instead of Suter's 24;28.
21: 30;15. Suter: 30;14.
30: 33; 4. Suter: 33; 3.
col. 4 line 1: 9;56 corrected by *.x.* above line presumably to 9;46 as in Suter.
col. 6 lines 4 to 27: signs omitted.
line 11: 11;29. Sic, instead of Suter's 11;25.
14: 11;44. Suter: 11;43.
25: 12;52. Suter: 12;53.
col. 8 line 26: 3;44,27. Suter: 3;44,26 but C: 3;44,27.

**fol. 134$^{v}$:**

col. 1 lines 4 to 28: degrees omitted.
line 30: $3^{s}\,0°$ instead of $4^{s}\,0°$.
col. 2 lines 2 to 27: degrees omitted.
col. 3 line 2: 21;31. Sic, instead of Suter's 21;35.
12: 20;22. Sic, instead of Suter's 20;13.
19: 19;23. Suter: 19;24.
21: 19;14. Suter: 19;12. C: 19;16. O: 19;17.
24: 19;56. Sic, instead of 18;56.
28: 18;37. Suter (incorrectly) 18;39.
col. 6 lines 3 to 29: signs omitted.
col. 7 line 10: 1;28 instead of Suter's 1;27.
23: 3;27,30 instead of Suter's 3;27,7.

**fol. 135$^{r}$:**

col. 1 lines 6 to 28: degrees omitted.
line 30: $4^{s}\,0°$ instead of $5^{s}\,0°$.
col. 2 lines 3 to 27: degrees omitted.
col. 4 lines 4 and 5: 40;16 instead of 40;21 and 40;26 respectively.
col. 6 lines 5 to 30: signs omitted.
col. 8 line 12: 2;48,12. Sic, instead of Suter's 2;47,12.
13: 2;44,33. Suter (incorrectly) 2;44,13.
14: 2;41,41. Sic, instead of Suter's 2;41,51.
28: 1;59,14. Suter 1;59,15.
29: 1;55,33. Sic, instead of Suter's 1;55,53.

**fol. 135$^{v}$:**

col. 1 line 28: $5^{s}\ 29°$ instead of $5^{s}\ 28°$.
line 30: $5^{s}\ 0°$ instead of $6^{s}\ 0°$.
col. 3 line 26: 4;40. Suter (incorrectly) 3;40.
col. 4 line 10: 39;21 instead of 29;21.
col. 7 line 9: 0;38. Sic, instead of Suter's 0;37.
col. 8 line 10: 1;13;35 as in C and O. Suter: 1;13,15.
13: 1;5,46. Suter: 1;5,47.

## Folios 136 to 138

### Equations, Stations, and Latitude of Venus

Main heading: *Directio Veneris* "Equation of Venus", corresponding to Suter Tab. 45 to 50. The subtitles for the columns 1 to 5 are the same as for Jupiter (p. 195). Fol. 136$^{r}$ has in col. 6 the erroneous title *Directio centri* instead of the heading *prima Statio*[1] *Veneris* of the subsequent pages. Cols. 7 and 8 are called *Latitudo Veneris ascendentis*[2] or *descendentis* (fol. 137$^{v}$, 138) subdivided in *prima* and *secunda*. Abbreviations *s*, *g*, *p* for the units occur only on fol. 136$^{r}$. Fol. 136$^{v}$ col. 8 has *minuta* instead of the usual *minutie*.

### Deviations from Suter, Tab. 45 to 50

**fol. 136$^{r}$:**

col. 1 lines 3 to 29: degrees omitted.
line 30: $0^{s}\ 0°$ instead of $1^{s}\ 0°$.
col. 2 lines 2 to 25: degrees omitted.
col. 3 line 6: 19;19. Sic, instead of Suter's 19;59.
col. 4 line 19: 8;50 as in C. Suter (correctly) 8;1.
28: 11;48. Suter: 11;47.
col. 5 line 21: 0;48. Sic, instead of Suter's 0;47.
col. 6 lines 4 to 30: signs omitted.
line 15: 15;55. Suter 15;56.
col. 7 line 22: 2;18. Suter: 2;19.
col. 8 line 1: 0;5,14. Suter: 0;5,16.
lines 24 to 30: first digit 1 instead of 2.
line 27: 1;16,14. Suter: 2;16,12.

**fol. 136$^{v}$:**

col. 1 lines 2 to 28: degrees omitted.
line 30: $1^{s}\ 0°$ instead of $2^{s}\ 0°$.

[1] Beginning with fol. 138$^{r}$ spelled *stacio*.
[2] The last word omitted on fol. 136$^{v}$.

col. 2 lines 2 to 25: degrees omitted.
col. 3 line 28: 9;40. Suter: 9;11 (misreading *xl* of *xi*).
col. 4 line 8: 15;56. Suter: 15;57.
9: 16;20. Suter: 16;22.
10: 16;46. Suter: 16;47.
18: 20;4. Suter: 20;3.
col. 5 line 15: 1;34. Suter: 1;35.
col. 6 lines 2 to 28: signs omitted.
lines 26 and 27: 16;26. Suter 16;25 and 16;27 respectively.
col. 7 lines 13, 15, 17, 19, 21, 23, 25: all digits are one unit higher than in Suter.
col. 8 line 4: 2;48,45. Sic, instead of Suter's 2;47,45.
7: 3;0,33. Suter: 3;0,32. C: 3;0,33.
23: 4;59,39. Sic, instead of Suter's 3;59,35.

**fol. 137$^{r}$:**

col. 1 lines 2 to 28: degrees omitted.
line 30: $2^{s}\ 0^{\circ}$ instead of $3^{s}\ 0^{\circ}$.
col. 2 lines 2 to 27: degrees omitted.
col. 4 line 19: 31;19. Sic, instead of 32;19.
21: 32;4. Sic, instead of 33;4.
col. 5 lines 1 to 4: first digit 2 instead of 1.
col. 6 lines 2 to 27: signs omitted.
line 13: 16;45. Suter (incorrectly) 16;44.
col. 8 line 5: 4;39,53. Sic, instead of 4;31,53.
8: 4;37,9. Sic, instead of 4;38,9.
30: 4;0,0. Sic, instead of 5;0,0.

**fol. 137$^{v}$:**

col. 1 to 30: $3^{s}\ 0^{\circ}$ instead of $4^{s}\ 0^{\circ}$.
col. 2 line 10: $8^{s}\ 10^{\circ}$ instead of $8^{s}\ 20^{\circ}$.
14: $8^{s}\ 10^{\circ}$ instead of $8^{s}\ 16^{\circ}$.
lines 15 to 24: degrees omitted.
col. 3 line 8: 1;46 as in N and O, but 1;45 in C.
col. 6 lines 10 to 14: signs omitted.
line 12: 17;24. Suter: 17;23.
col. 8 line 12: 4;53,26. Suter: 3;53,27.
27: 4;28,18 as in C. N and O: 4;27,18.

**fol. 138$^{r}$:**

col. 1 line 30: $4^{s}\ 0^{\circ}$ instead of $5^{s}\ 0^{\circ}$.
col. 3 line 2: 28;19. Sic, instead of Suter's 28;29.
24: 28;57. Suter (incorrectly) 27;55.

col. 4 line 2: 45;34 as in O. C: 45;33.
lines 25 and 26: 47;26 and 47;12 instead of 46;26 and 46;12.
line 27: 45;54. Sic, instead of Suter's 45;58.
col. 5 line 1: 1;54 corrected to 1;53 as in Suter.
2: 1;53 corrected to 1;52 as in Suter.
col. 7 line 26: 0;47. Sic, instead of Suter's 0;46.
col. 8 line 14: 3;34,48 corrected to 3;35,48 as in Suter.
15: 3;28,8 as in Suter. Above the 8 are written two dots : which refer to the marginal notes at the same line:

: *36*
*44 et cetera per*
*ordinem*

The modified values are the values from C and O, quoted by Suter in the apparatus to Tab. 49.
19: 3;16,49. Suter: 3;16,48.
23: 3; 0,33. Suter: 3;0,32 but C and O: 3;0,4 (sic).
25: 2;52, 4. Suter: 2;52,3 but C and O: 3;52,23 (sic).
28: 2;38,59. Suter: 2;39.0.
29: 2;38,31. Sic, instead of Suter's 2;34,31.
30: 2;30, 0. Suter: 2;30.1

**fol. 138$^{v}$:**

col. 1 line 30: $5^{s}\ 0^{\circ}$ instead of $6^{s}\ 0^{\circ}$.
col. 4 lines 2 and 6: first digit omitted, since identical with preceding line.
line 28: 5;40. Sic, instead of Suter's 5;37 or 5;17 in C and O.
col. 7 line 1: 0;40. Sic, instead of Suter's 0;41.
col. 8 lines 1 to 6: first digits 1 instead of 2.
line 1: 1;26,27. Sic, instead of Suter's 2;25,27.
7: 1;58,13. Sic, instead of Suter's 1;57,13.
9: 1;48,31. Sic, instead of Suter's 1;47,31.
11: 1;36,40. Sic, instead of Suter's 1;37,40.
18: 1; 2,22. Suter (incorrectly) 0;2,22.
19: 0;57,16. Suter 0;57,15.
24: 0;31,29. Sic, instead of Suter's 0;31,21.
29: 0; 5,54. Sic, instead of Suter's 0;5,14.

## Folios 139 to 141

### Equation, Stations, and Latitude of Mercury

These tables for the *Directio Mercurii* "Equation of Mercury" correspond to Suter Tab. 51 to 56. The subtitles are the same as in the preceding tables for the columns 1 to 5. Then follows col. 6 *prima stacio Mercurii* and cols. 7 and 8 *Latitudo Mercurii ascendentis* and *descendentis*, divided in *prima* and *secunda*. No abbreviations occur for the units.

### Deviations from Suter, Tab. 51 to 56

**fol. 139$^{r}$:**

col. 1 line 30: $0^{s}\ 0°$ instead of $1^{s}\ 0°$.
col. 4 line 2: 0;42. Sic, instead of Suter's 0;32.
3: 0;38. Sic, instead of Suter's 0;48.
6: 1;30. Sic, instead of Suter's 0;36.
7: 1;52. Suter (incorrectly) 1;58.
11: 2;56. Sic, instead of Suter's 2;57.
17: 5;32. Sic, instead of 4;32.
25: 6;37 as in C. Suter, following O: 6;38.
col. 5 line 9: 0;38. Sic, instead of Suter's 0;37.
12: 0;59. Sic, instead of 0;49.
29: 1;55 as in C. Suter, following O: 0;54.
col. 6 line 10: 27;10. Suter (incorrectly): 27;11.
col. 8 line 3: 0;19,39. Suter: 0;19,38.
4: 0;26,14 Sic, instead of Suter's 0;26,10.
12: 1;17,48. Sic, instead of Suter's 1;17,58.
20: 2; 8,12. Suter: 2;8,15.
21: 2;14,13. Sic, instead of Suter's 2;14,23.
27: 2;50,15. Suter (incorrectly) 2;56,15.

**fol. 139$^{v}$:**

col. 1 line 30: $1^{s}\ 0°$ instead of $2^{s}\ 0°$.
col. 3 line 11: 9;35. Suter: 9;34. C: 9;32.
28: 7;36. Suter: 7;37.
30: 7;23. Suter: 7;24.
col. 4 line 11: 10;29. Sic, instead of Suter's 10;39.
17: 12;3. Sic, instead of Suter's 12;6.
30: 15;50 as in O, instead of Suter's 15;1 (confusion of *l* and ı).
col. 5 line 14: 2;42 (as in line 13), instead of Suter's 2;45.
20: 3;9. Sic, instead of Suter's 3;1.
col. 6 line 13: 26;53. Sic, instead of Suter's 26;4 (confusion of *l* and ı).
14: 26;50. Sic, instead of Suter's 26;1 (confusion of *l* and ı).

col. 7 lines 23, 26, 29, and 30: last digit one unit higher than in Suter.
col. 8 line 6: 3;42,24. Sic, instead of Suter's 3;40,25.
19: 4;43,50. Sic, instead of Suter's 4;43,1 (confusion of *l* and ı).
20: 4;46,16. Sic, instead of Suter's 4;47,16.
24: 4;3,23. Sic, instead of 5;3,23.
26: 5;10,23 as in C and O. Suter: 5;10,53.

**fol. 140$^{r}$:**

col. 1 line 30: $2^{s}\ 0°$ instead of $3^{s}\ 0°$.
col. 3 line 6: 6;57. Sic, instead of Suter's 6;47.
14: 5;1. Sic, instead of Suter's 6;1.
col. 4 line 18: 18;17. Sic, instead of Suter's 18;27.
lines 22 to 24: first digit one unit lower than in Suter.
col. 5 line 11: 3;44. Sic, instead of Suter's 3;45.
13: 3;46. Sic, instead of Suter's 3;47.
23: 3;57. Suter: 3;56.
col. 7 line 10: 1;29 as in C. O: 1;30.
col. 8 line 4: 5;37,4. Suter: 5;37,3.
10: 5;52,22. Suter: 5;52,23.
16: 6;3,52. Suter: 6;3,51.
24: 6;12,47. Suter: 6;12,57.

**fol. 140$^{v}$:**

col. 1 line 30: $3^{s}\ 0°$ instead of $4^{s}\ 0°$.
col. 3 line 7: 4;27. Sic, instead of Suter's 4;28.
col. 4 line 13: 21;18. Suter: 21;17.
15: 21;22. Suter: 21;23.
18: 21;28. Suter: 21;27.
21: 21;30. Suter: 21;29.
28: 21;21. Suter: 21;19.
29: 21;18. Suter: 21;17.
col. 6 line 25: 24;30 as in O. C: 24;29.
col. 7 line 23: 1;9. Sic, instead of Suter's 1;10.
col. 8 line 6: 6;12,17. Sic, instead of Suter's 6;12,57.
8: 6;9,21. Sic, instead of Suter's 6;11,21.
16: 5;0,28. Sic, instead of 6;0,28.
20: 5;52,20. Suter: 5;52,23.
25: 5;38,12. Sic, instead of Suter's 5;39.52.
29: 5;27,49 as in C and O. Suter: 5;27,59.

**fol. 141$^{r}$:**

col. 1 line 30: $4^{s}\ 0°$ instead of $5^{s}\ 0°$.

col. 3 line 2: 4;22. Suter: 4;21.
col. 4 line 6: 20;44. Suter: 20;43.
8: 20;26. Suter: 20;27.
13: 19;42. Suter: 19;41.
24: 17;8. Suter: 17;7.
col. 5 line 16: 2;41. Suter: 2;45.
col. 8 line 22: 3;50,52 as in C. O: 3;50.51.
26: 3;28,42. Sic, instead of Suter's 3;29,42.

**fol. 141$^{v}$:**

col. 1 line 30: $5^{s}\ 0°$ instead of $6^{s}\ 0°$.
col. 3 line 7: 8;46. Suter: 8;47.
15: 10;45. Suter: 10;44.
col. 4 line 4: 13;35. Suter: 13;34.
21: 5;6. Suter: 5;7.
col. 5 line 2: 1;51. Suter: 1;52.
20: 0;45. Sic, instead of Suter's 0;41.
col. 6 line 5: 24;38. Suter: 24;37.
lines 7 and 8: 24;39. Suter: 24;38.
col. 7 line 24: 49 written *xlxı*.
col. 8 line 9: 2;12,23. Sic, instead of 2;14,23.
lines 12 to 20: first digit, as in O, 2. In C: 1.
line 20: 2;5,6. Suter: 1;5,7.

## Folio 142r

### 1. Table, Corresponding to Suter Table 2

#### A. Transcription

fol. 142r

**Plate 28**

| | Anni Arabum collecti | Note | Anni Plani | Note | (blank) | (blank) | Menses | Note | (blank) | | Version D of col. 4 [5] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 30 | 5 | 1 | 4 | cekebi | A | Elmuharram | 1 | | 1. | 11 |
| | 60 | 3 | 2 | ·1 | keb | RΘ | zafar | 3 | | | 22 |
| | 90 | 1 | 3 | 6 | keb | A | Rabe primus | 4 | | | 3 |
| | 120 | 6 | 4 | 3 | keb | RΘ | Rabe secundus | 6 | | | 14 |
| 5. | 150 | 4 | 5 | ·7 | keb | A | Iemedi primus | 7 | | 5. | 25 |
| | 180 | 2 | 6 | 5 | keb | RΘ | Iumedi secundus | 2 | (blank) | | 6 |
| | 210 | 7 | 7 | ·2 | keb | A | Regeb | 3 | | | 17 |
| | 240 | 5 | 8 | 6[3] | keb | RΘ | Scabin | 5 | | | |
| | 270 | 3 | 9 | 4 | keb | A | Ramham | 6 | | | 9 |
| 10. | 300 | 1 | 10 | ·1 | keb | R[4] | Scauuel | 1 | | 10. | 20 |
| | 330 | 6 | 11 | 6 | keb | A | Dulkada | 2 | | | 1 |
| | 360 | 4 | 12 | 3 | | RΘ | Dulheia | 4 | | | 12 |
| | 390 | 2 | 13 | ·7 | | | | | | | 23 |
| | 320[1] | 7 | 14 | 5 | | | | | | | 4 |
| 15. | 450 | 5 | 15 | 2 | | | | | | 15. | 15 |
| | 480 | 3 | 16 | ·6 | | | | | | | 26 |
| | 510 | 1 | 17 | 4 | | | | | | | 7 |
| | 540 | 6 | 18 | ·1 | | | | | | | 18 |
| | 570 | 4 | 19 | 6 | | | | | | | |
| 20. | 600[2] | 2 | 20 | 3 | | | | | | 20. | 10 |
| | 630 | 7[2] | 21 | ·7 | | | | | | | 21 |
| | | | 22 | 5 | | | | | | | 2 |
| | | | 23 | 2 | | | | | | | 13 |
| | | | 24 | ·6 | | | | | | | 24 |
| | | 25. | 25 | 4 | | | | | | 25. | 5 |
| | | | 26 | ·1 | | | | | | | 16 |
| | | | 27 | 6 | | | | | | | |
| | | | 28 | 3 | | | | | | | 8 |
| | | | 29 | ·7 | | | | | | | 19 |
| | | 30. | 30 | 5 | | | | | | 30. | |

Notes to the transcription:

[1] Sic, instead of 420.
[2] The last two entries are later additions.
[3] Sic, instead of 7; correct in fol. 114r and in Suter.
[4] Sic, instead of R Θ.
[5] The numbers of Version D of column 4 are written with Hindu numerals above the corresponding numbers of the original text. The hand is distinctly different from the hand in which the Hindu numerals on fol. 115v to 126v are written.

**B. Translation and Commentary**

No heading for the whole table, only column titles:

Column 1: "Accumulated Arabic years". Column 2: "Weekdays". Column 3: "Single years". Column 4: "Weekdays". Column 7: "Months". Column 8: "Weekdays".

This table is a duplicate of Suter's Table 2 and of the table on fol. 114$^{r}$ of the present manuscript (cf. above p. 135).

The copyist did not understand the use of the table. In column 5 he wrote eleven times the word "*keb*"[1] (i.e. *kebice* = leap year) but he associated it with the table of the months instead of with the proper years of the preceding column.

The next column should give the alternating numbers 30 and 29 of days of the Arabic months. The scribe must have copyied a manuscript which used Greek capitals Λ and ΚΘ which the copyist misinterpreted as A and RΘ respectively.

A later owner noted the leap years by dots[2] in column 4 and wrote above the *notae* (with Hindu numerals) all multiples of 11 modulo 30 (cf. the last column in our transcription), omitting the residues $\geq 27$. This scheme indicates the position of leap years by those places where a number is followed by a smaller one, e.g.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 11 | 22 | 3 | 14 | 25 | 6 | 17 | (–) | 9 |
| year | 1 | 2* | 3 | 4 | 5* | 6 | 7* | 8 | 9 |

because then one full day has accumulated from the fraction 11/30 over 354 days in an ordinary year[3].

## 2. Text, written by First Hand

**A. Transcription**

1. § *Calle autem quia in paginam notarum et in paginam conjunctionis.*
2. *et oppositionis fit introitus secundum tempus in ceteras vero omnes secundum praeteritum.*
3. [5] § *Anno ab incarnatione domini .1126. die ianuarii* $^{aa}$*.26. prima fuit dies.*
4. *almuharram et feria tercia ali(ter)*[6]*. Annus arabicus .510. planus .10. anno igitur ab.*
5. *incarnatione domini* $^{oo}$ $^{o}$*.1133. eclipsis solis* $^{o}$*2 die augusti mensis feria* $^{a}$*.4.*
6. *ciclo* $^{oo}$*.91.13. luna vigesima* $^{a}$*7 .2. kalendis novembris primus dies. Elmuharram. feria* $^{a}$*.3.*

[1] In the first line written *cekebi*, obviously originating from a writing $^{ce}$*kebi* in the original.
[2] The same device is used in the marginal column of fol. 116$^{r}$ (cf. Pl. 9).
[3] Cf. chapter 1 (p. 11). I do not know why the intercalation was placed earlier when the accumulated fraction became $\leq 27/30$ of one day. Has it to do with the use of evening epoch for the civil calendar?
[4] All numbers are written with Roman numerals.
[5] The lines 3 to 7 were transcribed by Haskins SHMS p. 23.
[6] Haskins: *ali*, (without further explanation).

7. *annus arabicus adjunctus 510 ʃ 18 planus ʃ in anno sequenti .12. kalendis*
   *novembris .1.* (a) *feria.*

Above line 7: *transactus plene . ⁘ iste presens*

On margin: ⁘ § . . . . *am* 510 . . . (referring to the same sign above line 7)

.. *ille etiam 19*$^{us}$ } (referring to the same signs
.. § *primus dies elmuh(arram)* } near the end of line 7)

8. § *Notandum vero quod solis eclipsis non fit nisi sol et luna in capite sive in cauda sive in*
9. *proximo gradu simul convenerint. Eclipsis autem lune fieri potest etiam si oppositi*
10. *fuerint sol et luna .10. gradibus a capite vel a cauda draconis distantes. Sed tunc.*
11. *luna non tota sed in aliqua parte sui obscurabitur. Simili modo sol si a capite vel a cauda.*
12. *draconis aliquantulum luna remota fuerit: non ex toto sed ex parte obscura.*
13. *bitur. Sepe etiam evenit ut in uno climate obscurari videatur et in alio minime.*
14. *Sed luna nunquam nisi .15.* (aa) *patitur eclipsin. Tricesima vero vel prima facit solis eclipsin;*
15. § *Caput draconis non nisi .19. gradus in anno peragit. Sol et luna et caput draconis sive.*
16. *cauda si convenerint in eodem gradus eclipsis fiet solis. Si vero sol et caput sive cauda draco.*
17. *nis conveniunt in eodem gradu et luna fuerit in opposito gradu et ipsa patietur eclipsin;*

## B. Translation and Commentary

### Section 1: Lines 1 and 2

"Understand that the entry in the table of weekdays and in the table of conjunction and opposition is made with the (present) time, in all other (tables) with the past (i.e. completed) time".

A similar statement is found in a gloss of MS C to Suter, Table 2 (above p. 84).

### Section 2: Lines 3 to 7

"In the year of the Lord 1126 January 26 was the first day of Muḥarram, and a Tuesday, otherwise Arabic year 510 (plus) single (years) 10".

It is indeed correct to say that A.D. 1126 Jan. 26 = A.H. 520 Muḥarram 1 (reckoning, as always in our text, A.H. 1 Muḥ. 1 = A.D. 622 July 15). The same statement is found in MS O at the end of ch. 4 (cf. above p. 15).

"In the year of the Lord 1133 a solar eclipse occurred on August 2, Wednesday, in the 13th (year) of the 19(-year) cycle, lunar day 27".

Many contemporary reports of this eclipse are preserved since its path of totality

crossed Europe between Scotland and the Aegaean Sea[8]. According to the mediaeval lunar cycle 1133 is indeed the 13th year and Wednesday August 2 is the 27th day of a cyclically computed lunation[9] – a rather bad result for an eclipse which should only happen on a 30th or 1st lunar day, as stated in the very next section. Such a discrepancy was occasionally noted even by mediaeval annalists[10].

"2nd Kalends of November (= Oct. 31) was the first day of Muḥarram, Tuesday, of the accumulated Arabic year 510 and 18 single (years). In the next year the 12th Kalends of November (= Oct. 21), Sunday, (was) the first day of Muḥarram". Indeed:

A.H. 528 Muḥ. 1 = A.D. 1133 Oct. 31, Tuesday
A.H. 529 Muḥ. 1 = A.D. 1134 Oct. 21, Sunday.

**Section 3: Lines 8 to 14**

"It should be known that a solar eclipse can not occur unless sun and moon come simultaneously either to the ascending or the descending node, or to a nearby degree. A lunar eclipse can occur even if sun and moon at opposition are 10° distant from the ascending or from the descending node. But then the moon will not be totally obscured but only in part. Similarly the sun will be obscured only in part if the moon is a little bit removed from the ascending or descending node. It often happens that it (the sun) will appear obscured in one region and very little in another. But the moon never suffers an eclipse except in the 15th (lunar day), while the sun makes eclipses on the 30th or on the first".

**Section 4: Lines 15 to 17**

"The ascending node travels only 19° in one year. A solar eclipse happens when sun and moon and ascending and descending node come together in the same degree. If, however, the sun and the ascending or descending node come together in the same degree while the moon is located in the opposite degree, then the moon itself suffers an eclipse".

A (retrograde) nodal motion of 19° per year is approximately correct. Walcher in his "*De Dracone*" says[11] that Petrus Alphonsi assumed a daily nodal motion of 0;3,10,50° thus 19;21,42° in one julian year; cf. also Tab. 19 and 20 (above p. 94).

## 3. Marginal Notes by Later Hands

### A. Transcription

(a) Fol. 142[r], upper margin:[12]

First line and left column:

1. *Si diuidas dies unius cicli id est* 30 *annorum per dies septimane unius inuenientur residui dies* 5.

[8] Cf. Schröter, Kanon, Karte 64b and Ginzel [2] Taf. III. Ginzel [2] p. 697 to 705 lists 78 references to this eclipse. For Byzantium also Mentz [1] p. 475.

[9] Cf., e.g., the table in Ginzel, Hdb. III p. 136, according to which 1133 July 7 = *luna* 1.

[10] Ginzel [2] p. 702 No. 49, p. 703 No. 55. The corresponding Arabic date is A.H. 527 Ramadan 29, thus considerably better than the *luna*.

[11] Cf. Millás-Vallicrosa [1] p. 26 and in particular p. 29f.

[12] All numbers written with Hindu numerals.

2. . . *qualibet istarum notarum designat principium.*
3. *cuiuslibet septimane con . . . te in suo ciclo.*
4. *notat ergo feriam prime diei. de* 5. *diebus.*
5. *residuis quatuor sequentes qui terminant primum.*
6. *ciclum ponuntur (?) pro nota prima annorum pla.*
7. *norum.*

Middle column:

1. *Residui sunt* 5. *sed prima dies.*
2. *notatur per notam annorum collectorum.*
3. *quatuor residui hoc pro nota. et quia(?) in.*
4. *crastino incipit Almuharam ideo habet.*
5. *ille mensis pro nota* .1.

Right column:

1. *Ad habendum ergo feriam principii cuiusque mensis accipienda est nota annorum collectorum penitus transactorum*
2. *sed in annos planos et menses secundum presens tempus intrandum est. Unum non habent locum haec regula.*
3. *id est prius* 30 *annis transactis regula non habet locum to . . . igitur in principio tabule ante(?) quam(?) trans . . . .*
4. *essent primi* 30 *anni ad habendum ferias cuiuslibet mensis principii operandum est sicut annis.*
5. *collectis accipe ergo de anno plano primo* 4. *haec enim est nota annorum precedentium preteritorum.*
6. *id est ultima dies. deinde accipe notam primi mensis scilicet* .1. *et habebis* 5. *cicli feria incipit primus ciclus et.*
7. *sic pro sequente per totum primum ciclum.*
8. *siue annis collectis.*

(b) Lower margin:[13]

1. *Unus ciclus arabum id est* 30. *anni arabum tot continet dies* .10631. § *Differentia annorum Christi et arabum a principio nativitatis Christi usque ad principium.*
2. *regni mahum et annorum arabum sunt* 621. *anni. et dies sequentes.* 202. *quibus peractis die sequenti incepit primus annus arabum idibus Julii.*
3. *Continet ergo praedicta differentia dies tot* .227022. *Dies a nativitate Christi secundum Petrum Anfulsum usque ad principium huius operis* 407534.
4. *Subtrahe differentiam remanentes dies* 180512. *Subtrahe (numer)(?)um unius cicli et invenies quae de annis arabum transeunt* 16 *cicli et remanentes.*
5. *dies* 10416 *qui faciunt annos* 29 *et remanentes dies* 139 *qui faciunt menses* 4 *Almuharam zafar Rabe primus Rabe secundus.*

[13] All numbers in this note are written with Hindu numerals unless stated otherwise.

6. *remanent dies* 21. ⊣·[14] *Subtrahe differentiam toleti ab Aren et est Radix super Aren secundum zodiacum stellarum. Solis.*
7. *Signa sex gradus* 7 *minute* 42 *secunde* 45 *in principio huius operis. Adde motum* 8 *spere et erunt* 6 15 19 20.
8. ⊣·[15] § *Nota Petrus hic addit unam diem igitur incipit primam diem octobris a*
9. *meridie ipsius diei. non a meridie diei precedentis sicut arabes. secundum hoc computa et invenies secundum zodiacum stellarum signa* 6. *gradus* 8 *puncta* 41 *minute* 56. *Sub-*
   *trahe longitudinem toleti ab* $80^{o}$ *gradibus et subtrahes* 4 *horas.*
10. *et quintam* $10^{m}$[16] *unius et* 21 *minute. Subtrahe ergo super aren de predicta mora quae est super toletum* $10^{16}$ *puncta et* 51 *minute. et erit signa* 6. *gradus* 8 *puncta* 31 *minute* 5.
11. *Addat(ur) motus octave spere qui est signa* 0. *gradus* 7. *minute* 36 *secunde* 35. *et est et erunt* 6. 16 7 40.

**B. Translation and Comments**

(a) Fol. 142[r], upper margin:

These marginal notes are in part very difficult to read and thus my translation is only fragmentary and often doubtful. Nevertheless this much is clear: we are dealing here with the rules for finding the weekday of the beginning of Arabic years and months[17].

First line, left and middle column:

"If you divide the (number of) days of one cycle, i.e. 30 years, by the days of one week and 5 days are found as remainder. . . . each one of these *notae* designates the beginning of whatever week . . . in its (30-year) cycle, (and) thus denotes the weekday of the first day (of the 30-year cycle). Of the 5 remaining days the four following ones which conclude the first cycle are taken as first *nota* for the single years". Obviously the fact is here expressed that each 30-year cycle contributes the *nota* 5, each single year the *nota* 4, modulo 7. This is once more the theme of the middle column: "The remaining (days) are 5. But the first day is denoted by the *nota* of the collected (sic) years which have the four of the remainder for (their) *nota*. And because the (month) Muḥarram begins on the next day, this month has 1 as its *nota*".

Right column:

"In order thus to obtain the weekday of the beginning of an arbitrary month one has to take the *nota* of the fully completed years, but for the single years and for the

[14] Sign of omission, corresponding to the remarks of lines 8 to 11.

[15] Lines 8 to 11 correspond to the omission sign in line 6 after *dies* 21. These four lines are written in a slightly different hand. Note also the change in terminology: in line 7 *gradus-minute-secunde*, in line 9 and 10 *gradus-puncta-minute*, in line 11 again *gradus-minute-secunde*.

[16] Written with Roman numerals.

[17] Cf. above Tab. 2 (p. 84).

months one has to enter (the table) according to the current time[18]. But these rules are not always valid, namely before 30 years are completed this rule does not hold'' – because the first entry in Tab. 2 is the year 30 instead of the year 0 to which should be given the *nota* 0 (or 7). This is probably expressed in lines 3, 4: ''. . . in the beginning of the table before (?) the first 30 years are completed (?)''.

''In order to obtain the weekday of the beginning of an arbitrary month one has to proceed as with the collected years. Thus take for the first single year (the *nota*) 4. This, namely, is the *nota* of the preceding years that have passed, i. e. the last day. Then take the *nota* of the first month, namely 1, and you will get 5 as the weekday of the cycle. (So) it begins the first cycle and so for the following, through the whole first cycle that is for the accumulated years''.

(b) Lower margin.

Lines 1 to 6:

''One Arabic cycle i. e. 30 Arabic years, which contains 10631 days.[19]

The difference between Arabic and Christian years, from the beginning of (the years from) the nativity of Christ to the beginning of the reign of Muhammed and of the Arabic years, amounts to 621 (julian) years and an additional 202 days; these having counted off (beginning with the nativity), the first year of the Arabs began on the next day, on the ides of July (= July 15). The above-mentioned difference thus contains 227022 days. The days from the nativity of Christ (year 0 Dec. 25) to the beginning of this work (A.D. 1116 Oct. 1) are 407534[20] according to Petrus Anfulsus.

Subtract the difference, 180512 remaining days. Subtract (from it, as often as possible, the number of days of) one cycle and you will find that 17 cycles of Arabic years have gone by and 10416 remaining days, which make 29 (Arabic) years and 139 remaining days which make 4 months (namely) Muḥarram, Zafar, first Rabe, (and) second Rabe, and there remain 21 Roman days''.

In lines 7 to 11 (cf. below) a later remark to this last number of 21 days is added.

The computations described so far are the equivalent of:

$$407543 - 227022 = 180512$$
$$180512 = 16 \cdot 10631 + 10416$$
$$10416 = (10631 - 354) + 139 = 29 \text{ Arabic years} + 139 \text{ days}$$
$$139 = 118 + 21.$$

Thus we have the date

$$\text{A.H. } 1\ \text{I}\ 1 + (16 \cdot 30 + 29) \text{ years } 4 \text{ months } 21 \text{ days}$$
$$= \text{A.H. } 510\ \text{V}\ 22 = \text{A.D. } 1116 \text{ Oct. } 1.[21]$$

[18] Cf. the example above p. 85.
[19] Cf. above p. 85.
[20] Cf. above p. 139.
[21] The tables of Wüstenfeld-Mahler would give Oct. 2 since they count July 16 as epoch (cf. above p. 11).

Lines 6 and 7:

"Subtract the difference of Toledo from Arin. And the *radix* for Arin according to the zodiac of the (fixed) stars is for the sun: six signs 7 degrees 42 minutes 45 seconds at (the time of) the beginning of this work. Add the motion of the 8th sphere and it gives $6^s$ 15;19,20°".

These two lines are the last lines of the text written by a first hand on the lower margin. How much this scribe assumed as time difference between Arin and Toledo we can not tell. His text ends abruptly with the statement that for the *radix* of this work (i.e. for A.D. 1116 Oct. 1, noon) at Arin the sidereal longitude of the (mean) sun was $6^s$ 7;42,45°, the tropical longitude $6^s$ 15;19,20°. Thus a motion of precession of 7;36,35° was assumed, a number which we will find again in the subsequent note.

As in the previous case (fol. 123$^v$ lower margin, cf. above p. 182) this value for the difference between sidereal and tropical longitudes is based on Thābit ibn Qurra's theory of trepidation. The epoch date A.D. 1116 Oct. 1 falls 509 Arabic years, 4 months and 21 days later than the epoch of the era Hijra. For this interval one finds for the argument $\Theta$, the *motus* $8^e$ *spere*, the amount of 45;25,20°. Rounding it to $\Theta = 45°$ we find for the *equatio diuersitatis* in the second table $\Delta\lambda = 7;36,35$, exactly as given in the text.

We now turn to the note added to the "21 days" mentioned in line 6 (cf. p. 211 note 15). We read in lines 8 to 11: "Note that Petrus adds to this one day; consequently he begins the first of October at noon of this very day, not at noon of the preceding day as the Arabs. Compute accordingly and you will find with respect to the zodiac of the (fixed) stars 6 signs 8 degrees 41 minutes 56 seconds. Subtract the longitude of Toledo from 80 degrees and you subtract 4 hours and the fifteenth (part) of one (hour) and 21 minutes. Thus subtract for(?) Arin from the above-mentioned delay for Toledo 10 minutes and 51 seconds and it will make 6 signs 8 degrees 31 minutes 5 seconds. If the motion of the 8th sphere is added, which is 0 signs 7 degrees 36 minutes 35 seconds, it is and will be $6^s$ 16;7,40°."

First we are told that Petrus reckoned "day $n$" of a month from noon of the calendar date $n$ to noon of $n+1$, whereas the Arabic astronomers begin day $n$ at noon of the calendar date $n-1$ and end it at noon of day $n$. For this reason, the sidereal solar longitude at epoch, namely $6^s$ 7;42,45°, has to be augmented by one day's solar motion in order to obtain the epoch value used by Petrus Alphonsi, namely $6^s$ 8;41,56. Thus we see that a daily motion of 0;59,11° had been assumed. This is probably a simple error caused by the fact that the mean motion of the sun is $0;59,8,11°/^d$ (cf. above p. 148).

Finally the longitudinal difference between Arin and Toledo is taken into account. Assuming for Arin a longitude of 90° from the West, for Toledo 28;30°, we would have a longitudinal difference of 61;30° or of $4;6^h = 4\ 1/10^h$. These well-known parameters[22] were probably garbled in our note to 80° for Arin and a time difference of "$4\ 1/15^h$ and 21 minutes" i.e. $4;25^h$. From al-Khwārizmī's Tab. 5 one finds for $4;25^h$

[22] Cf. Millás-Vallicrosa, Azarquiel p. 49 or Zinner [1] p. 757 MS B.

a solar motion of 0;10,52,30° whereas our text uses 0;10,51°. This value is now subtracted from the longitude valid for Arin, instead of being added. Finally, as before, the precessional motion of 7;36,35° is added in order to obtain tropical longitudes. Thus the whole procedure can be summarized as follows:

| | | |
|---|---|---|
| Arin, Arabic norm: | $6^s$ 7;42,45 | sidereal |
| $1^d$ solar motion: | +0;59,11 | |
| Arin, norm of Petrus: | $6^s$ 8;41,56 | sidereal |
| solar motion for $4;25^h$: | −0;10,51 | |
| Toledo, norm of Petrus: | $6^s$ 8;31, 5 | sidereal |
| precession: | +7;36,35 | |
| | $6^s$16; 7,40 | tropical. |

It is not surprising that this value is not found in our MS[23].

## Folios 142$^v$ and 143$^r$

### Al-Khwārizmī's Chapters 7 to 17

In the following I list only the deviations from Suter's text. Spelling variants like *-cio/ -tio* or *dekaice/ dakaicae* are ignored.

**Chapter 7.** Title omitted. Otherwise CC follows the version O (Suter's right column). Ch. 7 line 1: [*est*]: om. CC
2: [*medialitas*]: *methalitas*
5: [*tabulam*]: *paginam*
p. 8 line 3: [*signis signa*]: *signa signis.*
5/6: [*dakaicas, dakaikas*]: *dekacas dekaicis.*
p. 9 and 10: [LXX]: *septuaginta*
13: [*signa, ea negligemus*]: *singna* (sic) *ea neglilemus* (sic)
14: [*ipsum*]: om.
21: [*nostrum locum*]: *locum nostrum.*
22: [*ergo*]: *igitur*
24: [*gradibus*]: *gradus*
26: [*attingit*]: *continget*
27: [*fiuntque*]: *fientque.*
p. 9: om. CC.

**Chapter 8.**[24] Title om. Ch. 8 line 8: [*igitur*]: om.
10: [*gradu arietis*]: *arietis gradu.*
p. 9 note 30: [*sullimatio*]: *sublimatio*; [*duo scilicet*]: *haec scilicet duo.*
Note 33: [*Hoc*]: *Haec.* Note 40: [VI]: *sex*; [*addenda*]: *addendum*; [*examinatio... sin*]: *examen medialitati. Si*

[23] Cf., in particular, p. 154.
[24] Mr. Toomer discovered that this chapter, in the form of version O, is also extant in Par. lat. 16208 fol. 95$_r$ II, 6 to 34.

p. 10 line 1: [*gradum . . . addendo*]: *gradibus prioribus addendo unum gradum.*

**Chapter 9.** Title om. Ch. 9 line 3: [*quodque*]: *quodque vero.*

5: [*sex . . . maius*]: *maius sex signis.*

**Chapter 10.** Title om.

p. 10 note 2: [*et secundum*]: *secundumque*; [*priori examinatione*]: *priore examine*; [*dakaicis praedictis*]: *predictis dekaicis*; [*examinationi*]: *examini.* Note 7: [*argumento*]: om. Note 14: [*est*]: om. Note 17: [*etiam*]: *et.* Note 22: [om.]: *Partesque sicut supradictum est in sua tota reducendae.* Note 29: [*simul*]: *similiter.* Note 35: as CM. Notes 38 and 39: *et in examen ipsius sublimationis examinate.* Note 43: [*examinationen*]: *examen.*

p. 11 line 1: [*examinatione*]: *examine.*

6: [*si centrum ipsum*]: *ipsum centrum si.*

14: [*elhacil*]: *elbahebil.*

p. 11 note 1: [*intrandum est*]: om. Note 27: [*examen erit*]: *erit examen.* Note 33: [*vero obtendum*]: *optendum vel centrum ultimum.* Note 34: [*obtento*]: *ultimo centro*; [*sullimatio*]: *sublimatio*; [*tibi . . . planetae*]: *planete tibi locum.* Note 40: [*Nota . . . duodecim*]: om.

**Chapters 11 and 12:** Titles om.

**Chapter 13.** Title om. Lines 1 and 2: [*quibus*]: *quibusque.*

**Chapter 14.** Title om. Line 1: [*retrogradi*]: *retrograde.*

4: [*praesens*]: om.

p. 12 note 12: [*intelligemus*]: *intelligimus.* Note 13: [*etiam*]: *est etiam.* Note 15: [*inventum pagina*]: *inventi in paginam.* Note 22: [*ipsi . . . primae*]: *prime ipsi stationi.* Note 24: [*stationi*]: *stacione stacione* (sic). Note 26: [*Hoc . . . fiet*]: om.

p. 13 line 1: [*autem*]: *enim.*

6: [*secundam*]: *secundam et*

11: [*autem sunt*]: *sunt autem.*

**Chapter 15.** Title om. Line 3: [*ibidem reperitur*]: *reperietur*

right column line 3: *shemeli*]: *shemedi*

8: [*duodecimum*]: XII.

p. 13 note 3: [*argumentum*]: *argumentum inventum.* Note 8: [*stationem*]: om. Note 14: [*ad*]: *ab.*

**Chapter 16.** Title om.

**Chapter 17.** Title om. Line 1: [*latitudinis . . . planetarum*]: *superiorum planetarum latitudinis.*

2: [*elheca*]: *argumentum*

p. 14 note 1: [*praedicta*]: om. Note 3: [*enim*]: om. Note 12: [*cuiuslibet*]: om. Note 25: [*Quodque*]: *Quod.*

**Fol. 143$^{v}$ lines 7 to 11** are written as if it were the first chapter of a new section. A blank space is left between the end of chapter 17 and line 7. The first letter

is written as large as the initials in the preceding chapters. Actually, however, it has nothing to do with the following and is merely a trivial remark on computing with longitudes:

> *Quotienscumque aliquis numerus signorum vel gra*
> *duum auferendus est de alio. Si auferendus.*
> *maior fuerit: addantur* $\overset{cim}{.12.}$ *signa minori numero.*
> *et sic qui maior fuerit prius: de predictis signis et numero auferetur.*

"Whenever a certain number of signs and degrees has to be subtracted from another one, and if the subtrahend is the greater (of the two) then 12 signs should be added to the smaller number and in this way what previously had been larger can be subtracted from the given signs and number (of degrees)".

## Folios 143$^{v}$ to 145$^{r}$

### *Dixit Petrus Anfulsus*

### 1. The Main Text

#### A. Introduction

*Transcription*

Fol. 143$^{v}$, left column:

12.[1] *Dixit petrus anfulsus seruus*
13. *ihesu christi translatorque*[2] *huius libri. Gratias.*
14. *deo omnipotenti et domino nostro qui creauit mun-*
15. *dum sua sapiencia et disposuit suo intellectu*
16. *omnia. Qui celum decorauit angelis*[3] *et sideribus et terram*
17. *impleuit diuersis animalium generibus quam uestiuit herbis fru-*
18. *ticibus atque arboribus. Quique celestes creaturas*
19. *in duas partes diuisit unam scilicet spiritualem alteram quoque*
20. *corporalem. Et posuit in eis potestatem super terrestres*
21. *creaturas ad perficiendum mandatum ipsius et ad complen-*
22. *dam uoluntatem eius. Sicut dauid utrumque distin-*[4]
23. *guens ait. Benedicite domino omnes angeli eius*
24. *p. v. f. i. ad. a. v.*[5] *sermonem eius. Benedicite domino uir-*
25. *tutes. m. e. qui. v. e.*[6] *Celestibus ergo creaturis*

[1] Transcribed from line 12 to 15 (*sapiencia*) by Coxe, Catal. II p. 122, from line 12 to 16 (*omnia*) by Haskins SHMS p. 117, and from line 12 to 16 (*sideribus*) by Millás-Vallicrosa [1] p. 75. This text also appears as part of a version found in Brit. Mus. MS. Arundel 270, fol. 40$^{v}$ to 45$^{r}$, published by Millás-Vallicrosa [1] p. 97 ff., our text beginning p. 101.

[2] Millás-Vallicrosa: *que* omitted.

[3] Millás-Vallicrosa: *angelibus.*

[4] Corrected from original *extin.*

[5] Abbreviation from Psalm 103,20: *potentes virtute (qui) facitis (verbum) illius ad audiendum universos.*

[6] Abbreviation of: *ministri eius qui (facitis) voluntatem eius.*

*spiritualibus quidem potentiam imperandi terrestribus attribuit*
*corporalibus uero eisdemque celestibus naturam exequen-*
*di in prefatis uim inseruit. Veluti cum sol per .4.*$^{or}$
*celi uagatur partes quatuor etiam temporum mutat qualita-*
*tes et sicut idem sol temporum mutat qualitates: sic etiam naturas*
*et uarietates terrestrium mutat animalium nec non et terre*
*inherentium*[7]*. Porro quemadmodum luna mutatur per natura-*
*lis diei .4.*$^{or}$ *partes similiter occeani mutat etiam fluc-*
*tus in accessus et recessus et ut eadem per .4.*$^{or}$ *celi uagatur par-*
*tes ita etiam maris estus mutat in malina et ledones ma-*
*iores scilicet et minores cursus*[8]*. Quamdiu saturnus in igneis*
*moratur signis, tamdiu siccitas perseuerat. Et plura.*

fol. 143$^{v}$, right column:

*huiusmodi terrestrium euentuum subdi celestium potenciis*
*dinoscunt. Animalium uero terrestrium genus in dua species*
*diuisit deus: rationalem*[9] *scilicet et irrationabilem. Et quo maius*
*de terrestribus animal melius est homo et subtilius et ideo ingens ratione*
*eo magis creator de sua sapientia in eo infudit ad cog-*
*noscendum opus operatoris: et ut presertim per cogni-*
*cionem operacionis ab eodem operator cognosceretur.*
*Uerum ut dixi celestes creature in duo diuiduntur. in*
*spirituales scilicet et corporales. At non poterat homo pertingere ut*
*spirituales cognosceret: nisi per spiritum prophetice. Corporales*
*autem: huic sensuum prouenticione. cum rerum significatione*
*tunc experimentorum assuefactione. Haec*[10] *autem trina cognitio: uo-*
*catur stellarum scientia. Que in tres partes diuiditur. in cogi-*
*tacione mirabiles et in rerum significatione notabiles*
*et in experimento approbabiles. Quarum prima est scientia qualita-*
*tis et quantitatis circulorum firmamenti cum his que in eo sunt.*
*Ad quam uiuacitas humani ingenii peruenit geometrali*
*figura, numero et mensura. Secunda est scientia motuum firma-*
*menti circulorum et stellarum que per numerum sciri potest. Tertia*
*uero est scientia*
*nature circulorum et stellarum et significationes eorum in rebus terrenis. Que*
*contingunt eorum ex nature uirtute*[11] *et suorum motuum diuersi-*
*tate. Que experimento cognoscuntur. fuit etiam ex animi mei*[12]

[7] From line 29 (*quatuor*) to here: a much longer version in Millás-Vallicrosa [1] p. 102f.
[8] The version of MS Arundel 270 differs from here on from CC; cf. Millás-Vallicrosa [1] p. 103f. The rest of this line until fol. 143$^{v}$ left, line 33, is transcribed by Millás-Vallicrosa [1] p. 104f.
[9] Millás-Vallicrosa: *rationabilem.*
[10] From here to line 33 transcribed by Haskins SHMS p. 117f.
[11] Millás-Vallicrosa: *ueritate.*
[12] *mei* omitted by Millás-Vallicrosa.

23. *sententia: ut inde*[13] *librum ederem: et ut per ipsius noticiam*
24. *eiusdem utilitas cognosceretur, scilicet numerus et motus circulorum et*
25. *stellarum pertinentibusque*[14] *cum ipsis annis videlicet*[15] *mensibus*
26. *diebus horis ipsarumque punctis. Itaque primum nescessa-*
27. *rium est quota feria annus vel mensis incipiat nosse. Hoc autem*
28. *opus magno labore desudatum et summo studio ab ara-*
29. *bicis persicis egipciacis translatum: latinis benigne*
30. *impartiri uolui. Et quia uolo ut hic liber predictis omnibus*
31. *clareat: ideo sub eorundem*[16] *numero intitulaui et prout in ordine in*
32. *eorum lingua repperi: sic seriatim in latinam linguam*
33. *digessi.*

*Translation*

Said Petrus Alfonsus, servant of Jesus Christ and translator of this book: Thanks to god, the almighty and our lord who created the world by his wisdom and arranged all by his insight; who adorned the heaven with angels and stars and populated the earth with diverse kinds of animals and clothed it with bushes, plants, and trees. And he divided the celestial creatures into two classes, one spiritual, the other corporeal. And he gave them power over the terrestrial creatures in order to perform his orders and to do his will. Thus David said in distinguishing them: "Praise the lord, you his angels, powerful through virtue, for hearing his word. Praise the lord, you his servants, who do his will". Thus he gave to the celestial spiritual creatures indeed power to rule the terrestrial ones; and he endowed the corporeal yet celestial (creatures, i.e. the celestial bodies) with the ability to execute the preordained.

Just as, when the sun moves through the four parts of the sky, it also changes the qualities of the seasons, and as the same sun changes the qualities of the seasons, so does it also change the nature and the types of terrestrial animals as well as those which live within the earth. Furthermore, just as the moon is changed during the four parts of the natural day, so does the moon similarly change the flow of the ocean as it comes in and recedes, and as she moves through the four parts of the heavens she also changes the swell of the sea in spring and neap tide, i.e., in stronger and weaker currents. And as long Saturn stays in a fiery sign,[17] so long will drought prevail and in this way more of the terrestrial events are seen to be subject to the celestial powers.

The genus of the terrestrial animals god divided into two classes: the rational and the irrational. And in the same measure as man among the terrestrial (animals) is the better animal and finer and also of vast intelligence, the creator endowed him from his own wisdom that he might know the work of the maker and especially that by understanding of the work the maker would be recognized by him.

[13] Millás-Vallicrosa: *in*.
[14] Millás-Vallicrosa: *pertinentibus que*.
[15] Haskins: *uidelicet et*.
[16] From here to line 33 transcribed by Coxe, Catal. II p. 122, but reading, at the beginning, *de numero*.
[17] ♈, ♌, ♐; cf., e.g., Bīrūnī, Astrol. § 445, ed. Wright.

As I have said, also the celestial creatures are divided in two (classes), namely, the spiritual and the corporeal. And man could not attain knowledge about the spiritual ones, except by the spirit of prophecy; the corporeal, however, he might know by the help of the senses, by the signification of the objects and then by the practice of experiment. This three-fold knowledge is called the science of the stars, which is divided into three parts, miraculous in insight, notable in signification of objects, and provable through experiment.

Of these the first one is the science of the quality and the size of the circles of the heavens and those things which are in the heavens.[18] This science is obtained through human ingenuity by means of geometrical figures, number, and measure. The second is the science of the motion of the celestial circles and stars which can be known by means of numbers. The third one is the science of the circles and stars and their significations for terrestrial things, which result by virtue of their nature and the diversity of their motions, and which become known by experiment.

It was by the decision of my soul that I should publish this book and by its study its usefulness might become known, that is to say number and motions of the circles and of the stars and whatever else is relevant, namely their years, months, days, hours and their minutes. Thus it is first required to know on what weekday a year or a month begins. This work was performed with much labor and with extreme industry, translated from the Arabs, Persians, and Egyptians; I intended it for the benefit of the Latins. And since I wish that this book may be clear in all the above mentioned (matters) I have given them titles according to their number[19] and in the order in which I found them in their language. Thus I have arranged them in the same order in Latin.

**B. Chapter 1**

*Transcription*

fol. 143$^{v}$, right column:

34. *Primus itaque titulus est arabicus de annis lune*
35. *et mensium. Lunaris autem est .354.* *torum* *dierum*
36. *et undecim partium unius diei in .30. diuisi id est sexta*
37. *et quinta pars diei. Et cum plus dimidio die hee par-*
38. *tes compleuerint tunc annus computatur .355.* *torum* *dierum.*

fol. 144$^{r}$, left column:

1. *Si uero minus .354.* *tos* *dies perficiet. Hii autem dies*
2. *per .12. diuiduntur menses ita scilicet ut unus mensis .30.*
3. *alius .29. habeat dies. Primus quorum uocatur almo-*

[18] This, probably, means the epicycles or eccenters which are associated with each of the main planetary spheres.

[19] Referring to the numbering of the subsequent sections.

*harram 30. dierum. Secundus zaphar .29. die-*
*rum. Tercius rabe 30. Quartus iterum rabe 29. Quintus*
*iumet .30. Sextus iterum iumet 29. Septimus Rabieb.*
*30. Octauus sahben 29. Nonus ramazan 30.*
*Decimus sauhel 29. Vndecimus thilched 30.*
*Duodecimus thilhegge 29. Hic ultimus anno bis-*
*sextili est 30. dierum. Cum autem scire uolueris quota feria*
*quilibet annus inchoabit scito quot anni de numero arabi-*
*co transierint excepto eo anno cuius initium scire cupis. Et pro*
*uno quoque anno computa .4.*[or] *dies et quintam et sextam partem*
*unius diei. Et ex minutiis his scilicet senariis et quinariis*
*idemtidem collige dies. Et quos ex hac collectione re-*
*manserit quod diem non perfecerit si ultra dimidium*
*diem excreuerit pro die computa. Si autem diem dimidium uel*
*minus dimidio die habuerit pro nichilo habeto.*
*Et super id quod de minutiarum coadunatione proue-*
*nerit quinque adice dies. Et hanc totam summam inuni*(?)
*redige omnes septenarios inde abiciendo quod mi-*
*nus uero fuerit retine et inuenies quota feria annus presens in-*
*trabit a dominica die sequenti computando. Verbi gratia*
*si sex fuerit computa eundem in an*[er][um](?). *Scilicet pro die dominica*
*.1. § pro die lune .2. pro die Martis .3. § pro die Mercurii*
*4. § pro die iouis .5. § pro die ueneris .6. Sexta enim*
*feria annus de quo queris intrabit. Postquam autem cog-*
*noueris quota feria presens intrabit annus si scire uolue-*
*ris quota feria quilibet mensis eiusdem anni intrabit scito quot*
*menses eodem anno preterierint excepto eo de quo queris. Dein-*
*de mensibus supradictis de his scilicet qui .30. dies habent su-*
*me duos dies. De his uero qui .29. unum et pariter col-*
*lige et ipsis numerum ferie inchoationis anni presentis*
*adice. Et quot inde excreuerit per .7. partire. Si ultra*
*septem habundauerit .7. abice et quod superfuerit*
*retine et tota feria inicium eiusdem mensis erit. Si scire*
*uolueris annos arabicos secundum computacionem*
*tabule quota feria incipiant numera preteritos an-*

fol. 144[r], right column:

*nos cum presenti in quo es et colligens in summam eandem*
*quere in linea compositorum annorum in supradicta tabula.*
*Numerum uero annorum collectum. Si in predicta linea non inue-*
*neris eidem primo proximum conpositum accipe. Quod uero*

5. *superfuerit in linea singularium annorum in tabula quere.*
6. *Deinde quod in utraque linea inueneris compone deinde*
7. *diuide per septenarium. Quod super .7. inueneris retine et*
8. *quotus fit numerus tota erit feria presentis anni. Iterum si sci-*
9. *re uolueris quota feria quilibet mensis presentis anni intret*
10. *quere eiusdem anni quamlibet mensem uolueris in predicta tabula*
11. *et considera numerum sibi oppositum. Quem assumens a princi-*
12. *pio anni ipsius computare incipe et in quoto numero desierit*
13. *in tota feria mensem inchoare scito.*

**Summary of Contents**

The four chronological chapters present no serious difficulties and repeat essentially the material known from al-Khwārizmī's chapters 1 to 5, Suter's Tab. 2 and 2a, and the tables CC fol. 114r to 115r and fol. 142r. Thus a short summary of the contents will suffice for these four chapters.

The length of the lunar year is assumed to be 354 11/30 = 354 1/5 1/6 days; thus the calendar year is either 354 or 355 days long, depending on accumulated fractions $< 1/2$ or $> 1/2$. Then the corresponding list of months in the Arabic calendar is given.

The main problem concerns the determination of weekdays. If $r$ is the residue mod. 7 of the sum of days which give 4 days to each ordinary year, 5 days to each intercalary year, then $w \equiv 5 + r$ mod. 7 gives the weekday for the first day of a year, $w = 1$ being Sunday.[20] From this result the weekdays of the months can be found by making use of the relations $30 \equiv 2$, $29 \equiv 1$ mod. 7.

Finally rules are given how to find the weekdays by means of a table arranged like Suter's Tab. 2 or CC fol. 114r.[21]

**C. Chapter 2**

*Transcription*

fol. 144r, right column:

14. *Secundus uero titulus est ad cognoscendum persicos*
15. *annos et menses. Sciendum quia persicus annus*
16. *constat .365.tis diebus qui diuiduntur per 12. menses. Sin-*
17. *guli quorum constant .30. diebus excepto octauo ceteris*
18. *5. diebus exuperando id est .35. Primus quorum uocatur af-*
19. *foridmah .30. dierum. Secundus ardenhastamah.*
20. *30. dierum. Tercius ardemmah .30. dierum. Quartus*
21. *hirmah .30. Quintus mardamah .30. Sextus*
22. *zaherermah .30. Septimus maharmah .30.*

[20] For the reason for this rule cf. above p. 85.
[21] Cf. p. 84 and p. 135.

23. *Octauus habenmah .35. Nonus adarmah.*
24. *30. Decimus zimah .30. Vndecimus Barhar-*
25. *mah .30. Duodecimus hazphidarmah .30.*
26. *dierum. Si ergo scire uolueris quota feria incipiat quilibet*
27. *annus persarum scito quot anni transierint a tempore gezde-*
28. *girdi regis eorum excepto eo anno de quo queris. Et cum*
29. *scieris quot fuerint pone .3. desuper. Quos simul iunc-*
30. *tos diuide per .7. et quod super excreuerit retine. Et per eundem*
31. *numerum incipe numerare a dominica die sumens inicium.*
32. *Et in quo numero finierit tota feria presens annus intrabit*
33. *Si autem scire uolueris cuiuslibet alterius mensis inicium*
34. *scias quot presentis anni menses transierint excepto*
35. *eo de quo queris. Et de singulis mensibus preteritis duos accipe*
36. *dies excepto octauo qui si inter preteritos fuerit nichil*
37. *pro eo sumes. Et scito quot fuerint predicti dies. De-*
38. *inde ipsis adiunge numerum ferie inchoationis anni.*

fol. 144$^{v}$, left column:

1. *Deinde per .7. partire. Quod superfuerit pro feria habeto*
2. *a dominica die ut supra numerando. Si uero scire uolue-*
3. *ris cuiuslibet mensis cuiuscumque anni quotam feriam secundum compo-*
4. *sitionem tabule scias annos preteritos cum anno presenti*
5. *a tempore gezdegirdi regis et hos per .7. omnes diuide.*
6. *Quod super habundauerit uel si .7. fuerint in superiori*
7. *parte tabule inquire. Deinde ab eodem inferius*
8. *respice et considera e regione mensis et quod in angula-*
9. *ri linea eorundem inueneris pro feria eiusdem mensis habeto.*

**Summary of Contents**

The "Persian year" contains 11 months of 30 days and one month, the eighth, of 35 days. The weekday of the first day of the year $n+1$ of the era Yazdigerd is found as $w \equiv n+3$ mod. 7 (because $365 \equiv 1$ mod. 7 and $w = 3$ for $n = 1$).[22] For the single months one has $30 \equiv 2$ and $35 \equiv 0$ mod. 7. The corresponding tables are Suter Tab. 2a and CC fol. 114$^{r}$.[23]

**D. Chapter 3**

*Transcription*

fol. 144$^{v}$, left column:

10. *Tercius uero titulus est ad cognoscendum annos ro-*
11. *manorum et menses. Annus romanus est solaris annus.*

[22] Cf. above p. 86.
[23] Cf. above p. 86 and p. 136.

*Dies cuius sunt .365. et quadrans unius diei. Qui si duplicaturus fuerit pro die non reputabitur. Si uero triplicatus pro uno die habetur et tunc erit bissextilis annus .366. Qui per 12. menses diuiduntur. Et eorum primus uocatur. thesserin id est october .31. dierum. Secundus iterum thesserin id est .30. nouember. Tercius chenun id est december .31. et quadrante unius diei more chotensium christianorum et si fuerit bissextilis annus erit tunc 32. Quartus iterum chenun id est ianuarius .31. Quintus sabath id est Februarius 28. et quaddrante unius diei more romanorum christianorum. Et si fuerit bissextilis annus erit tunc .29. Sextus adar id est marcius .31. Septimus nizan id est aprilis .30. Octauus iar id est maius .31. Nonus haziren id est iunius. 30. Decimus thamuz id est iulius .31. Vndecimus ab id est augustus .31. Duodecimus eiul id est september .31.*[24] *Si itaque cupis scire quota feria incipiat quilibet annus romanorum scito quot anni preterierint a tempore alexandri magni absque eo anno de quo queris et scito quot fierint. Deinde hinc collectioni quartam partem totius summe adice. Qui si ad unum uel ad duo excreuerint pro nichilo ducito. Si uero ad tria pro uno habeto et tote summe superiori duos adice. Deinde eandem per .7. diuide. Quod superfuerit pro feria habeto a dominica die numerando et in quoto numero finierit tota feria erit inicium presentis anni id est primus dies octobris. Si scire uolueris de quolibet alio mense presentis anni scito quot menses eodem anno preterierint*

fol. 144v, right column:

*ab octobri numerando. Et pro singulis mensibus .30. dierum duos dies accipe pro illis uero qui .31. habent dies 3. dies sume Februarium autem intactum relinque. Deinde scito summam quae inde excreuerit. Deinde ipsis appone numerum ferie inchoationis anni. Deinde per .7. partire. Quod super .7. fuerit pro feria habeto. Quod si bissextilis annus fuerit pro octobri .4. sume dies. Iuxta illos qui in octobri bissextum ponunt si idem preterierit .1. accipe diem et ut supradictum est computa. Si uero scire uolueris quando bissextilis annus erit sume annos preteritos a tempore alexandri excepto presenti et diuide per 4. Quod superfuerit si ad duo excreuerit sequens annus bissextilis erit. Si uero minus uel amplius non erit.*

[24] Sic, instead of 30.

**Summary of Contents**

The "Roman year" of 365 1/4 days is considered to be a "solar year". The names of the months are the Syrian ones, beginning with Tishri = October. The intercalary day is placed at the end of the first Kanūn = December, which has 32 days. This is the rule of the "Chotensian Christians"[25] whereas the "Roman Christians" intercalate a 29th day in February. In al-Khwārizmī's chapter 2 (above p. 11) and Tab. 3a (cf. p. 89) the "Romans" are said to intercalate a 32nd day in December.

For the determination of the weekdays one has to reckon the years "from the time of Alexander" i.e. in the Seleucid era. To find the weekday for the first day of the year $n+1$ one has to form the number

$$\begin{aligned} m &= n+\frac{1}{4}(n-1) && \text{if } n \equiv 1 \bmod 4 \\ &\quad n+\frac{1}{4}(n-2) && n \equiv 2 \\ &\quad n+\frac{1}{4}(n+1) && n \equiv 3 \\ &\quad n+\frac{1}{4}\,n && n \equiv 0. \end{aligned}$$

Then

$$w \equiv m+2 \qquad \bmod 7.$$

For the reason cf. above p. 89. From $w$ for the year the weekdays for the months can be found using the remainder mod. 7 of 30, 31, etc.

The leap-years are the years $n$ of the Seleucid era for which $n \equiv 3 \bmod 4$. Since S.E. 1 = −311/10, the leap-years are the years $-309/8+4k$, which is correct.

**E. Chapter 4**

*Transcription*

fol. 144v, right column:

14. *Quartus uero titulus est ad cognoscendum egiptios*
15. *annos et menses. Annus itaque egiptius constat*
16. *355.*[26] *diebus et quaddrante unius diei sicuti romanus id est*
17. *solaris. Qui quaddrans si duplicatur non reputabitur pro die.*
18. *Si autem triplicitur pro uno die habebitur et tunc erit bissextilis*

[25] I do not know who these "Chotensian Christians" are. A derivation from the district Χορζάνη or Κορτζηνή in the area of the sources of the Euphrates (about 39° north, 40° west of Greenwich, according to Honigmann, Ostgrenze, p. 198ff.) seems rather unlikely. An equally uncertain possibility might be represented by a passage in al-Bīrūnī, Chronol. p. 69f., where he says that "the Christians in Syria, Iraq, and Khurāsān" use the names of months that are listed in our text, but he states that Shebāṭ (February) is the intercalary month, i.e. he describes the procedure called *more romanorum christianorum* in our text (line 21). Suter p. 34 considers an intercalation of a Dec. 32 a procedure used by Spanish Christians. This seems very doubtful, however, in view of a statement of Ibn Ezra (Millás-Vallicrosa p. 74,26) according to which intercalation in December is the practice of the "*christiani in terra sarrecenorum*".

[26] Sic, instead of 365.

*annus .356.*[27] *dierum. Qui diuiduntur per .12. menses singuli .30.*
*dierum excepto ultimo qui constat .35. diebus et quad-*
*drante unius diei. Primus omnium uocatur tut .30*
*dierum. Secundus Baabe .30. Tercius aton .30. Quar-*
*tus kihac .30. Quintus toba .30. Sextus amsir.*
*30. Septimus barmahat .30. Octauus Bar-*
*muda .30. Nonus Besnez .30. Decimus thona.*
*30. Vndecimus abib .30. Duodecimus mashare.*
*35. et quaddrante unius diei. Et si bissextilis annus*
*fuerit tunc erit 36 dierum. Si ergo scire uolue-*
*ris de quolibet anno egiptio quota feria incipiat.*
*Scias quot anni transierint a tempore declatiani re-*
*gis excepto eo de quo queris. Sume predictorum annorum*
*summam unum adice. Deinde quartam partem eiusdem sume*
*superiori adice collectioni. Quod si ad unum uel ad duo*
*excreuerint pro nichilo habeto. Si uero ad tria pro uno du-*
*cito. Deinde omnia coniunge. Deinde .5. appone.*
*Deinde omnia per .7. partire. Quod superfuerit pro feria tibi sit*
*a dominica die numerando et quota feria numerando tibi occur-*
*rerit tota erit in initio anni de quo queris. Si uero*

fol. 145r, left column:

*scire uolueris inicium alicuius mensis eiusdem anni scias quot*
*menses eodem anno preterierint excepto eo de quo queris.*
*et pro singulis mensibus duos accipe. Deinde scito quot*
*preterierint et ipsis adice feriam initii anni. Deinde*
*omnia per .7. partire. Quod super excreuerit inicium*
*mensis eiusdem habeto. Si autem scire uolueris quando*
*bissextus erit scias quot anni transierint a tempore di-*
*clatiani regis excepto eo anno de quo queris.* *or*
*Deinde omnibus unum adice. Deinde totam summam per .4.*
*diuide. Si duo residui fuerint in sequenti an-*
*no bissextus erit. Si uero amplius uel minus non erit.*
*Notandum quod secundum compositionem tabularum anni ro-*
*manorum et egiptiorum uniformi regula repperiuntur.*
*Si enim scire uolueris quota feria incipiat quilibet*
*mensis cuiuscumque anni romanorum uel egiptiorum si secundum*
*romanos queris scias quot anni transierint a tempore*
*alexandri magni. Si uero secundum egiptios a tempore*
*diclatiani queres. Deinde secundum utrosque per .28.*
*partire. Deinde quod superfuerit quere in linea anno-*

[27] Sic, instead of 366.

20. *rum singularium utrorumque. Deinde illinc angulariter*
21. *sub mense de quo queris curre. Et secundum numerun*
22. *ibi repertvm scito feriam inicii eius mensis*
23. *DE QVO QVERIS* – · –

**Summary of Contents**

The years under discussion are the "Egyptian years", i.e. the years of the Alexandrian calendar, counted from the Diocletian era which begins A.D. 284 Aug. 29. The weekday of the first day of the year $n+1$ is found from

$$w \equiv m + 5 \qquad \text{mod. } 7$$

where

$$m = n + 1 + \frac{1}{4}\, n \qquad \text{if } n \equiv 0 \text{ mod. } 4$$

$$n + 1 + \frac{1}{4}\,(n-1) \qquad n \equiv 1$$

$$n + 1 + \frac{1}{4}\,(n-2) \qquad n \equiv 2$$

$$n + 1 + \frac{1}{4}\,(n+1) \qquad n \equiv 3$$

For $n = 0$ we obtain correctly $w = 6$, i.e. Friday, for Diocletian 1 Thoth 1 = A.D. 284 Aug. 29.

The intercalary years, with 36 days given to the last month Mesore ("Mashare"), are the years $n+2$ of the era Diocletian for which $n+1 \equiv 2$ mod. 4, or simply, the years $n \equiv 3$ mod. 4. Indeed a Mesore 36 = August 29 occurs in the years Diocletian $3+4k$ or in the years A.D. $287+4k$.

If the day of the week of the first of a month of the Egyptian (i.e. Alexandrian) or of the Roman (i.e. Syriac) calendar should be found by means of a table, one has to reduce mod. 28 the year number of the era Diocletian or of the Seleucid era respectively. Then the table on fol. 115$^{r}$ or 114$^{v}$ respectively will give the desired number.[28]

## 2. Later Additions on fol. 145$^{r}$

### A. Chronologica

On the lower part of the left column of fol. 145$^{r}$ we find two sections, divided by a blank space, dealing with chronological matters. Both are written by the same new hand and the reference to Petrus (line 10) shows that it is not part of the preceding treatise.

[28] Cf. above p. 142, p. 86 and p. 140, p. 89 f. respectively.

*Transcription*

1. *Nota quod primus dies anni egiptiorum semper est quarto kalendis sep-*
2. *tembris in die decollationis beati Johannis baptiste secundum quod*
3. *testatur beda in libro de temporibus et de hoc multas in-*
4. *troducit auctoritates Item de eodem inuenitur in be-*
5. *stiario quod onager duodecies clamat in die et du-*
6. *odenos in nocte 22. die mensis famenoth quod est*
7. *octauo die mensis martii. Hic mensis uocatur in*
8. *superiori tabula egiptiorum barhamec.*[29]

9. *Nota quod ad principium huius operis petri erat ciclus anno-*
10. *rum egiptiorum.*[30] *.8. Set Nota quod petrus incipit an-*
11. *num eorum ọcṭaụọ*[31] *kalendis julii scilicet in ḍiḍ*[31] *Natiuitatis sancti*
12. *Johannis baptiste: prima igitur dies mensis qui uocatur thot*
13. *erat prima feria ut demonstrat ciclus octauus.*

**Translation and Commentary**

"Note that the first day of the Egyptian year always falls on the 4th calends of September (= August 29), the day of the beheading of St. John the Baptist (= August 29), about which Beda testifies in the book "*De Temporibus*" and on which he adduces many authorities. About the same (the Egyptian New Year) one also finds in the Bestiary that the donkey brays 12 times during the day and 12 times during the night on the 22nd day of the month Famenoth (VII) which is on the eighth day of March. This month is called Barhamec in the above-given table of the Egyptian (months).

It is correct that Alexandrian Thoth 1 = August 29. The corresponding passage in Beda, *De Temporibus*, is V, 8 f., ed. Charles W. Jones p. 296, but no reference to John the Baptist is made, nor are "*auctoritates*" quoted.

The story about the donkey comes from the "Physiologus" and thus goes back to antiquity. According to the Greek Physiologus the donkey announces the vernal equinox on the 25th of Phamenoth, which indeed corresponds to the 21st of March. The same date is found in the Latin Physiologus, version Y[32]. Our text gives instead Phamenoth 22 which, however, would be March 18 and not March 8[33].

"Note that at the beginning of this work of Petrus the cycle of the Egyptian years

[29] Actually written *barhamech* fol. 115[r] and *barmahat* fol. 144[v] right column, line 24.

[30] The 8 is preceded by a correction mark ∧.

[31] Dotted in MS.

[32] For the Greek text cf. F. Sbordone, Physiologus (Milano 1936) p. 139; also Max Wellmann, Der Physiologus, Philologus Suppl. 12,1 (1930) p. 65 ff. For the Latin version F. J. Carmody, Physiologus Latinus Versio Y, Univ. of California Publ. in Classical Philol. vol. 12 No. 7 (1941) p. 121.

[33] The Latin bestiarium, published in Migne, Patrol. Lat. 177, gives col. 62 cap. XI the 11th "*Farmochi id est Martii*". Cf. also B. E. Perry in Pauly-Wissowa RE 20,1 col. 1122,13 ff.

was (in the?) 8th (year?). But note that Petrus begins their year on the 8th calends of July (= June 24), i.e. on the birthday of St. John the Baptist (= June 24), the first day of the month called Thoth; it was *feria* 1 (= Sunday), as the eighth cycle shows".

A coincidence of Thoth 1 with June 24 would only be possible for the movable Egyptian years and would then occur in A.D. 240 to 243 ± 1460 years. Furthermore from A.D. 240 to 243, June 24 would move from Wednesday to Saturday but never fall on Sunday. Obviously the scribe confused *nativitas* (June 24) with *decollatio* (August 29), the date of Thoth 1 of the Alexandrian calendar, and he added that it fell on a Sunday, probably because the epoch date of Petrus Alphonsi, A.D. 1116 Oct. 1, is a Sunday.

How an 8-year cycle should give the weekday, I do not know. Perhaps 8 refers to the indictio 8 of the year A. D. 1115. I do not think that there exists any relation to the *ciclus lunaris grecorum* of 8 years which is mentioned in the marginal note (b) on fol. 114$^{v}$ (above p. 140f).

**B. Alchemica**

Below the two chronological sections is written, in a more cursive hand, a list of astrological and alchemical terms. A second column, written by a different hand, makes some corrections. The line numbering in the subsequent transcription corresponds to the arrangement of the two columns.

Left column:
1. (blank)
2. *Saturnus alkimicus plumbum est astronomicus ferrum.*
3. *Jupiter alkimicus* (blank)
4. *Mars alkimicus Cuprum est. Astronomicus aes.*
5. *Sol aurum*
6. *Venvs alkimica* (blank) *Astronomica stagnum*
7. *Mercurius alkimicus Viuum argentum. Astronomicus plumbum.*
8. *Luna argentum.*

Right column, in a different hand:
1. *melius secundum alkimiam*
2. *Plumbum*
3. *Stagnum*
4. *Ferrum*
5. *Aurum*
6. *Aes*
7. *Argentum uiuum*
8. *Argentum.*

Below the left column is written by the same hand:
*Forma humana est calamentum*(?)[34] *sanguinis humani alia stant ad letteram.*

[34] Or *colam̄tiū* (?).

The original list associates with each planet a metal, once according to "Alchemy", once according to "Astronomy". The alchemical list then was corrected by a later hand. Thus:

| | | | | | |
|---|---|---|---|---|---|
| ♄ | Alch.: | lead | Astr.: | iron | Alch. corr.: lead |
| ♃ | | – | | – | tin |
| ♂ | | copper | | brass | iron |
| ☉ | | gold | | – | gold |
| ♀ | | – | | tin | brass |
| ☿ | | quicksilver | | lead | quicksilver |
| ☾ | | silver | | silver | silver |

Then follows the sentence "The human figure is .... of human blod; other (things) stand by the letter (?)". I can not offer any explanation.

### C. Geographical and Personal Data

(a) In a much bolder hand than the preceding additions is written at the lower margin:

*Ric(ardus?) de sancto egidio. Nepos Nicolai* (?) *de sancto albano.*

This Nicolas of St. Albans is perhaps Nicholas of Wallingford, Monk of St. Albans; cf. Luard, Annales Monastici II p. 250, for A.D. 1195: *Obiit Warinus abbas Sancti Albani; successit Nicholaus prior de Walingeford.*[35]

I can not identify the Richard of St. Gilles. The Annales de Wintonia mention, however, for A.D. 1196, the *Comes Sancti Egidii*, i. e., Reymond VI, count of Toulouse.[36]

(b) In the same hand as the line (a) is also written in the upper right column:

1. *Distancia tholeti a Wintonia* .9. *gradus* .36. *minute*[37]
2. *Longitudo tholeti* 28. *gradus.* 39. *minute ab occidente*
3. *Longitudo Wintonie* 19. *gradus.* 30$\frac{3}{}$. *minute ab occidente et* 3 *minute.*

Then follows in a slightly different hand:

4. *magister Ansalmus contradixit. dixit quod toletum proprius est occidentii.*

The first note says:

"The distance of Toledo from Winchester is 9 degrees 36 minutes. The longitude of Toledo is 29 degrees 39 minutes from the West, the longitude of Winchester 19 degrees 30$\frac{3}{}$ minutes from the West and 3 minutes".

The corrected longitude of Winchester, 19;3° instead of 19;30°, agrees with the

[35] Cf. also: Chronica Jocelini de Brakelonda de rebus gestis Samsonis abbatis monasterii Sancti Edmundi, ed. J. G. Rockewode. London 1840, Camdon Soc., No. 13, p. 16: "... *remoti sunt alieni tres scilicet* (X), *et Nicholas monachus Sancti Albani, postea abbas Malmberiensis, et* (Y)". Also H. E. Butler, The Chronicle of Jocelin of Brakelond, London 1949, p. 22 (text and translation).

[36] Luard, Ann. Mon. II p. 64 n. 3. Cf. also D. Knowles, Monastic Order, p. 502 n. 1.

[37] This line was transcribed by Coxe, Catal. II p. 122, but he misread the last word as *mill.*

difference of 9;36° between Toledo and Winchester (cf. line 1), assuming for Toledo the longitude of 28;39°.[38]

Finally: "Magister Anselmus denies it; he says that Toledo is near to the West". Toledo is in fact 2;43° west of Winchester.[39] From medieval sources the following longitudes for Toledo may be quoted:

| | | |
|---|---|---|
| Ibn Ṣāʿid and Zarqali:[40] | 28° from W. | 62 from Arin |
| Zarqali:[41] | 28;30 from W. | 61;31 from Arin |
| from CC fol. 142r:[42] | | 65 or 66 from Arin. |

For Winchester one finds in MS Arundel 377 fol. 56v I,19–21:
*Andegauensium*[43] *uero longitudine ab occidente est* 24 *graduum fere. Eadem est Wintonie.*
The author of this treatise which is called *Magistri Habrahe de tabulis planetarum*, probably written about 1160, was Abraham ibn Ezra, as Millás-Vallicrosa has shown.[44] Our text apparently follows a different tradition.

Who Magister Anselmus is, I do not know. Roger of Hoveden mentions an *epistola fratris Anselmi Wigornensis monasterii* (i.e. of Worcester) *de supradicta* (A.D. 1186) *planetarum conjunctione*,[45] but there is no proof for an identity with our magister.[46]

All numbers in the lines 1 to 3 are written with Hindu numerals. This might indicate a connection with the interlinear tables (version D) and thus a relation to the Toledan tables.

## Summary

**1.** The major part of that section of the MS Corpus Christi College 283 considered here (folios 114r to 145r) is undoubtedly based on the tables of al-Khwārizmī in the version edited by Suter. In particular fol. 142v to 143r contain chapters 7 to 14 of al-Khwārizmī's introductory text, fol. 121r to 141v the tables for the planetary equations (Suter's Tab. 21 to 56), and, although in a modified form, the tables of mean motions fol. 115v to 123v (corresponding to Suter Tab. 4 to 20).

At the beginning and near the end, we have chronological tables (fol. 114r to 115r and fol. 142r) which are also found in Suter (Tab. 1 to 3a). To the same type of tables refer four chronological chapters (fol. 143v to 144v) by Petrus Alphonsi which, however, are not in Suter.

[38] Cf. the note, reported by Zinner [1] p. 757 MS B (14th cent.): *longitudo toleti* 28°30′, *long. wigton est* 19°30′ *ab occidente mundi*.
[39] Toledo: 4;2° W. of Greenwich (φ = 39;52), Winchester: 1;19° W. of Greenwich (φ = 51;4).
[40] Trsl. Blachère p. 122.
[41] Millás-Vallicrosa, Est. Azar. p. 49.
[42] Above p. 212.
[43] Angers, actually about 0;30° West of Greenwich (φ = 47;30).
[44] Millás-Vallicrosa, Tab. astron. p. 60 and p. 61.
[45] Magistri Rogeri de Hoveden Chronica, Rolls Series, vol. 51,II p. 293ff.
[46] Cf. perhaps Luard, Annales Monastici II p. 69: *Magister Anselmus, monachus Radingae, factus est abbas de Persora* (A.D. 1198).

Finally several groups of later notes are found on fol. 145$^{r}$ which relate our manuscript to the abbeys of Winchester and St. Albans.

**2.** The impression of a more than accidental connection between the chronological tables at the beginning and at the end with the chronological chapters of Petrus Alphonsi is further supported by the repeated mention of Petrus in marginal notes on fol. 114$^{r1}$ and 142$^{r}$ as well as in a table itself (fol. 114$^{r2}$). These remarks concur in giving A.D. 1116 Oct. 1 as date of the epoch used by Petrus Alphonsi[3].

**3.** The tables of mean motions (fol. 115$^{v}$ to 123$^{v}$) are not identical with al-Khwārizmī's tables since the latter were based on the Arabic calendar which was replaced, in our manuscript, by the julian calendar, leaving, however, the original arrangement in 30-year groups unchanged although this makes no sense except for Arabic lunar years. Thus it was only the framework of al-Khwārizmī's tables that had been taken over. The tables themselves were recomputed to fit the new calendar. The underlying parameters, however, are not identical with al-Khwārizmī's but seem to be close to al-Battānī's[4].

**4.** Thanks to these rather meaningless modifications of al-Khwārizmī's tables, a later owner inserted, with Hindu numerals, an interlinear version, here called D, which is taken from the Toledan Tables[5]. Again Petrus is mentioned in marginal notes[6]. But also "*algorismus*" is connected with Toledo in notes on fol. 118$^{r}$ and fol. 122$^{r}$[7]. The scribe of the marginal notes on fol. 123$^{v}$ makes remarks about differences between his norm and the norm of Petrus Alphonsi[8]. Who this scribe is – the scribe of D? – I do not venture to guess.

**5.** Fol. 124$^{r}$ to 141$^{v}$ are tables identical with Tab. 21 to 56 of Suter, except for minor variants which seem to suggest an original text close to Suter's version O; the same affiliation exists for the introductory chapters 7 to 14 (fol. 142$^{v}$, 143$^{r}$), as Haskins has observed. A considerable part of the tables underwent a collation with another set that belonged to class C. This resulted in many changes, made with Roman numerals and using a peculiar notation for + and − signs[9], often introducing incorrect values from C into the correct version O.

**6.** In the table of the equation of center for the sun (fol. 124 to 126, col. 3), an interlinear table for Sin $\Theta$ = 2,30 sin $\Theta$ - with Hindu numerals – was inserted between the original numbers[10]. These interpolated tables, agree exactly with corresponding Toledan Tables. In our manuscript they are ascribed to Azarqiel[11]; with how much justification, I do not know.

[1] Cf. p. 138, 140, 226.
[2] Cf. p. 138 (c). Similarly fol. 116$^{r}$, marginal notes (p. 151) with 1116 as *radix*; cf. p. 153.
[3] Cf. also p. 139, 143 ff., 212.
[4] Cf. p. 148.
[5] Cf. p. 151 f., 161, 187 etc.
[6] Cf. p. 151.
[7] Cf. p. 161 and p. 176.
[8] Cf. p. 181.
[9] Cf. p. 185.
[10] Cf. p. 190.
[11] Cf. Pl. 27 p. 190.

**7.** With fol. 142$^{r}$ we are back to the chronological material of the beginning (fol. 114$^{r}$ to 115$^{r}$), duplicating a table of fol. 114$^{r}$ but adding explanatory text. At the end of ch. 4 of the version O of al-Khwārizmī's text is found the statement that A.H. 520 Muḥarram 1 equals A.D. 1126 January 26[12]. Suter took this as an indication of the date of translation by Adelard of Bath[13] and in this he was followed by Millás-Vallicrosa[14]. The same date occurs, however, also in CC fol. 142$^{r}$ but combined with similar examples for A.D. 1133 and 1134 and a reference to a solar eclipse of 1133 Aug. 2[15]. This is followed by trivial remarks on eclipses in general[16]. It seems to me rather unlikely that just these examples are related to Adelard's translation.

The marginal notes to fol. 142$^{r}$ are in part calendaric, in part concerned with the epoch date used by Petrus Alphonsi, thus confirming the date A.D. 1116 Oct. 1. This seems, indeed, to be the only secure date for the tables composed by Petrus, in spite of marginal notes on fol. 123$^{v}$ which ascribe to him an epoch of 1224 March 1[17]. Equally obscure is a reference[18] to a cycle ending in A.D. 1200.

**8.** Pages 142$^{v}$ and 143$^{r}$ contain the version O of al-Khwārizmī's chapters 7 to 14, except for a short insignificant addition about computing with longitudes[19]. The direct continuation is a treatise beginning with "*Dixit Petrus Anfulsus*", with a long and pious introduction leading tortuously over to four calendaric chapters (fol. 143$^{v}$ to 145$^{r}$)[20] supposedly translated from the original languages by Petrus Alphonsi. These chapters explain the determination of the weekday for the four calendars: Arabic, Persian, Syrian, and Alexandrian.

This introductory treatise, however, is only an excerpt from a much longer introduction[21] which has the form of a letter to Petrus' "fellow-students" in France, "nourished by the milk of Philosophy". According to a hypothesis of Millás-Vallicrosa's we have here the beginning of a translation or free version of al-Khwārizmī's work by Petrus Alphonsi, later revised by Adelard of Bath[22].

**9.** The last page, fol. 145$^{r}$, contains notes by at least four different hands[23]. In part these notes are again calendaric, while others deal with the association of metals to the planets. Finally persons are named in connection with St. Giles, St. Albans, and Winchester which is compared to Toledo with respect to geographical longitude. Thus our manuscript seems to have been located, around 1200, in Winchester.

Somewhere one should be able to recognize the ordering hand of Robert of

[12] Above p. 15.
[13] Suter p. 37.
[14] Millás-Vallicrosa [1] p. 82f.
[15] Cf. p. 208.
[16] Cf. p. 208.
[17] Cf. p. 181.
[18] Cf. p. 141.
[19] Cf. p. 215. Note that a scholion at the end of ch. 7 (Suter p. 43, above p. 19) is omitted in CC, found on the margin of O and C, and in the text of M.
[20] Cf. p. 218f.
[21] This has been shown by Millás-Vallicrosa [1] p. 97ff. Cf. also Thorndike, HM II p. 70.
[22] Millás-Vallicrosa [1] p. 82f. Cf. also Nuevos Estudios p. 107.
[23] Cf. p. 227ff.

Chester, mentioned in the title of the version N[24]. Suter sees his influence mainly in replacing Arabic headings in tables by Latin ones[25] and in some marginal notes, in particular one concerning the epoch values of the planetary mean motions corresponding to a transformation of the zero meridian to the meridian of London (above p. 94). No evidence seems to exist, however, of any serious changes in text or tables due to Robert of Chester.

**10.** Suter, of course, had fully realized the composite character of the work he edited. Later additions, such as chronological tables with Spanish or other western elements, as well as astrological tables computed for Cordoba, could not have been mistaken. It was also clear that al-Khwārizmī's work itself contained elements of widely different origin: in part, Hindu (in particular the planetary theory); in part, Hellenistic – Arabic.

Nevertheless I think it is useful to underline more heavily than Suter did the inhomogeneous character of the work as we have it. In principle neither the addition of new tables, adjusted for a new region, nor the assimilation of different procedures would reduce the usefulness of an astronomical work. But in many mediaeval works – Islamic, Byzantine, or western – this process of accretion and modification has led to a chaos which no longer allows a meaningful use of works of this type, regardless of the fact that tolerably accurate results may still be obtained in performing the ritual of computation. In the following I shall quote a few examples of structural inconsistencies which show that no competent astronomer could have assembled such a text. Of course in most cases we have no means of saying whether this process began already with al-Khwārizmī himself in an attempt to combine Hindu and Hellenistic elements, or whether it is due to the Spanish or later editors.

A typical case of the combination of incongruous elements can be found in the treatment of the planetary stations. It is a long established fact that al-Khwārizmī's planetary theory is based on Hindu procedures which in turn go back to Greek models of a decidedly pre-Ptolemaic type[26]. Nevertheless we find here (cf. p. 101) Theon's Handy Tables used for the determination of the planetary stations, i.e. tables which are based on a different model and on different parameters than the tables for the planetary longitudes.

The planetary latitudes are computed according to rules which are probably of Hindu origin (p. 34ff.) but apparently incorrectly transmitted. Probably the (partial) change from the basic Hindu radius $R = 150$ to the hellenistic norm $R = 60$ has to do with the resulting discrepancies.

The rules given in ch. 25 for the determination of the rising times of ecliptic arcs do not agree with Tab. 59 to 59b, which follow the rules in the Handy Tables (cf. p. 48 and p. 108). Also the tables for the equation of time follow a Hellenistic prototype rather than the Hindu procedure in the related sections (cf. p. 108).

[24] Suter p. 1, above p. 9 note 2.
[25] Suter p. XIV.
[26] The references to the Almagest (above p. 10 and p. 45) are empty phrases without any relation to specific problems or methods.

A particularly drastic case concerns the inclination of the lunar orbit in the eclipse tables 73 to 76. One finds that for the apogee the Ptolemaic value $i = 5°$ has been used, for the perigee, however, the Hindu value $i = 4;30°$ (p. 120).

To this may be added the inconsistencies in correcting for new geographical locations elements which were originally computed for India (the meridian of "Arin") and then used with or without proper correction perhaps in Baghdad or in Cordoba or Toledo (cf., e.g., p. 111), not to mention countless scribal errors and variants which nobody could any longer have been in a position to adjust since there was no solid basis of empirical and theoretical data still extant. And it is clear, that no "oral tradition" could have replaced the lack of a clear theoretical insight into the structure of these tables – otherwise each codex should have carried along over centuries its own peculiar oracle. I think no other conclusion is possible but that the majority of mediaeval astronomical manuscripts, from India to England, owe their existence much more to the prestige they gave to their owners than to a real desire to master the underlying astronomical theory.

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Longitudes are filed under the number of the degrees within a sign, the zodiacal sign itself being ignored.

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621 = 10,21
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3725 = 1,2,5
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10416 = 2,53,36
10631 = 2,57,11
98069 = 27,16,29
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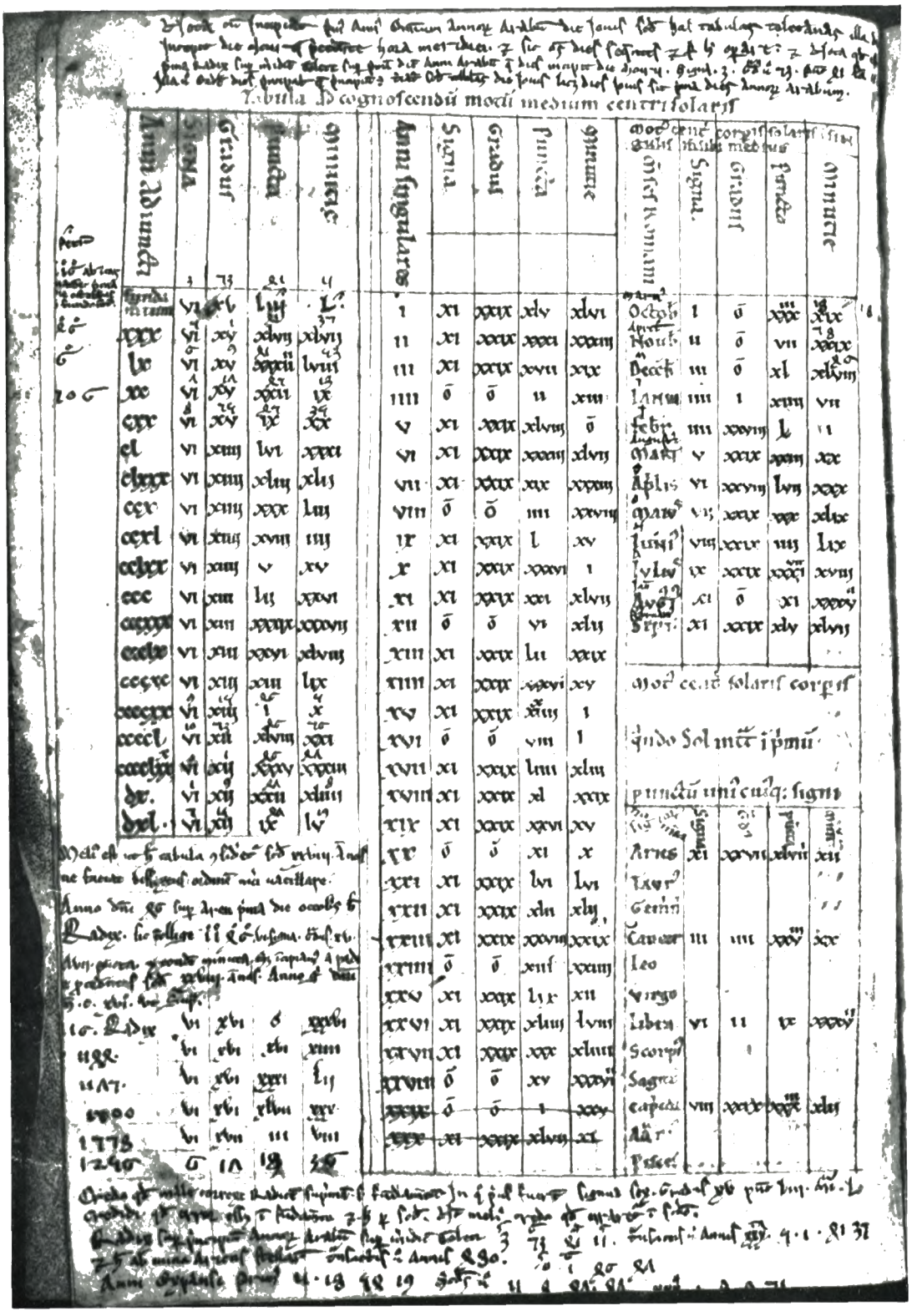
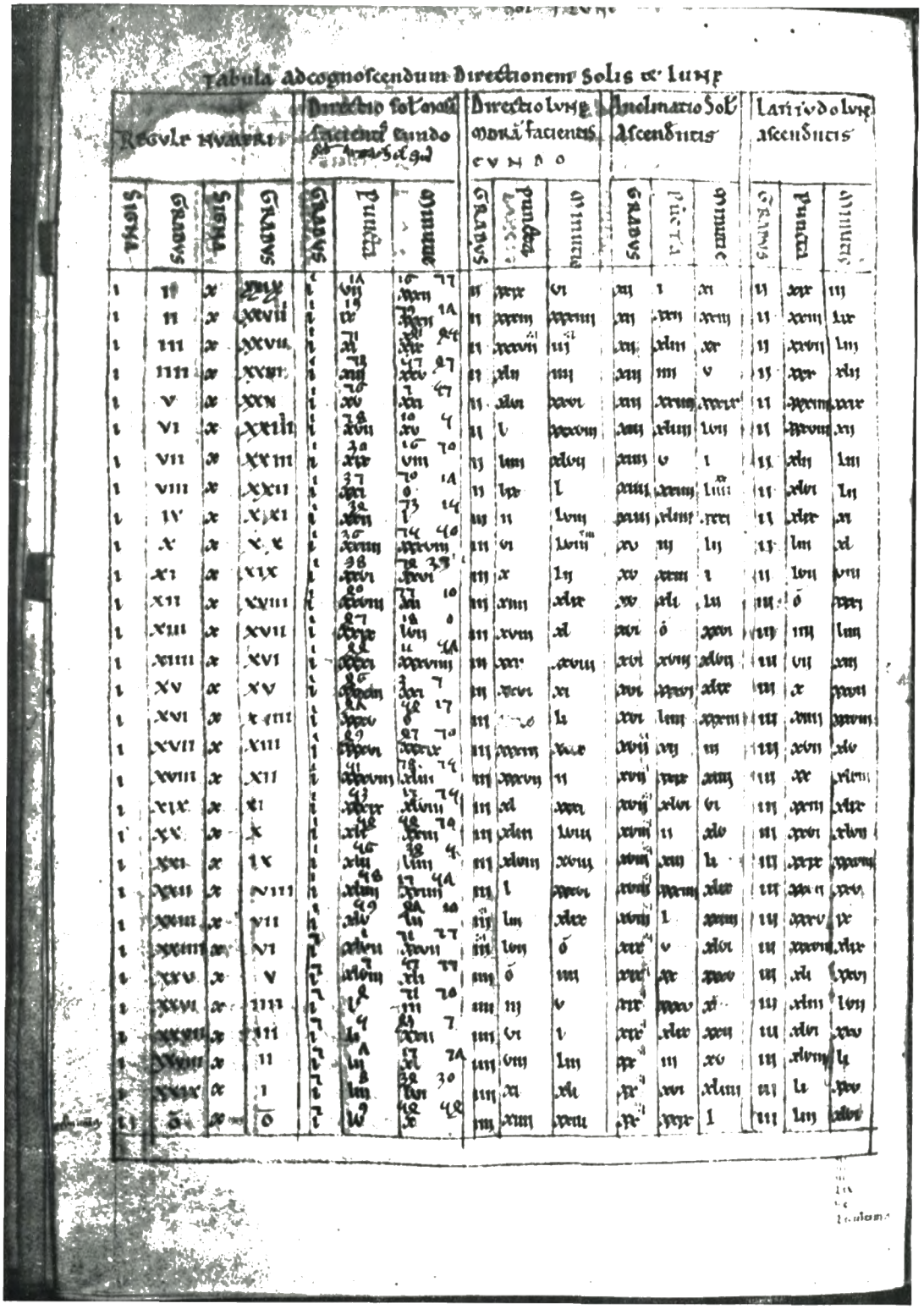
Indleveret til Selskabet den 9. december 1960.
Færdig fra trykkeriet den 24. februar 1962.



Tabula ad cognoscendū motū medium centri solaris



Tabula adcognoscendum directionem solis & lunę

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| signa | gradus | signa | gradus | gradus | puncta | minute | gradus | puncta | minute | gradus | puncta | minute | gradus | puncta | minute |
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| i | xx | x | x | | | | | | | | | | | | |
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